Modeling Short Crack Growth Using a Local Yield Zone Model

James A. Harter, Senior Consultant

LexTech

Motivation

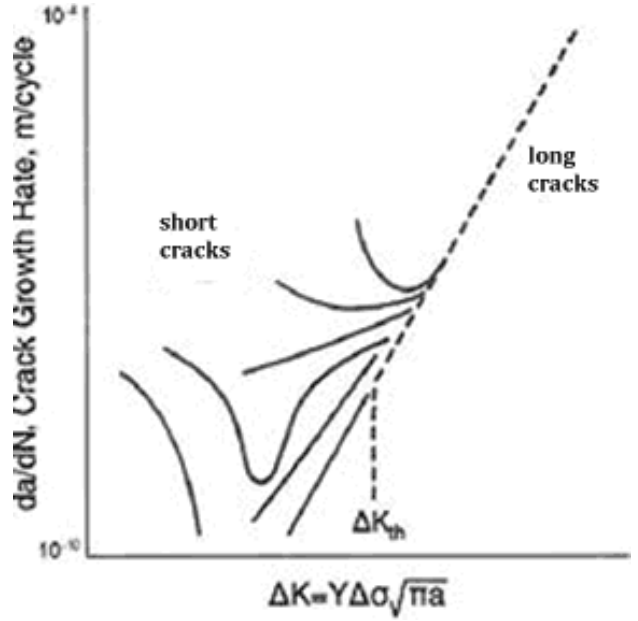
Over the years, many researchers/analysts have proposed extending long crack growth rate curves from Region II backwards to resolve the issue where “short” cracks demonstrate crack growth below the long crack threshold.

Potential Shortcomings of this approach:

Similitude? (Can we use LEFM for short cracks?)

Lack of a physical basis?

When K >= short/long crack intersection point, there is no short crack effect?



There may be another option to model short crack growth that is based on known material properties and physical phenomena

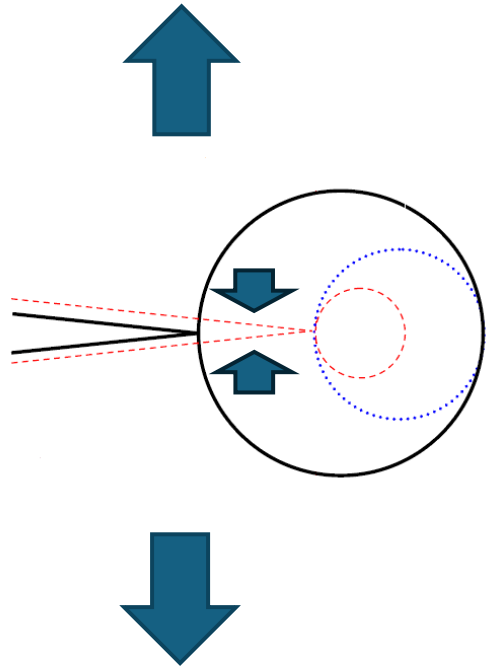
Consider a Yield Zone Based Model?

Long cracks have grown into the compressive plastic wake ahead of the crack tip

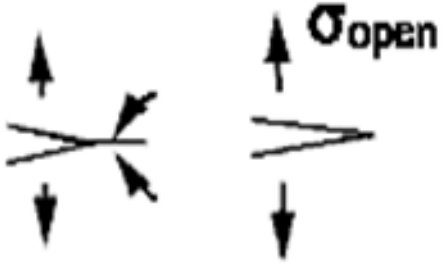
Constant amplitude long crack growth rate data includes the effect of residual compressive stresses (crack closure) that exist in this plastic wake

Short cracks are not long enough to be affected by the residual crack closing force to the same extent

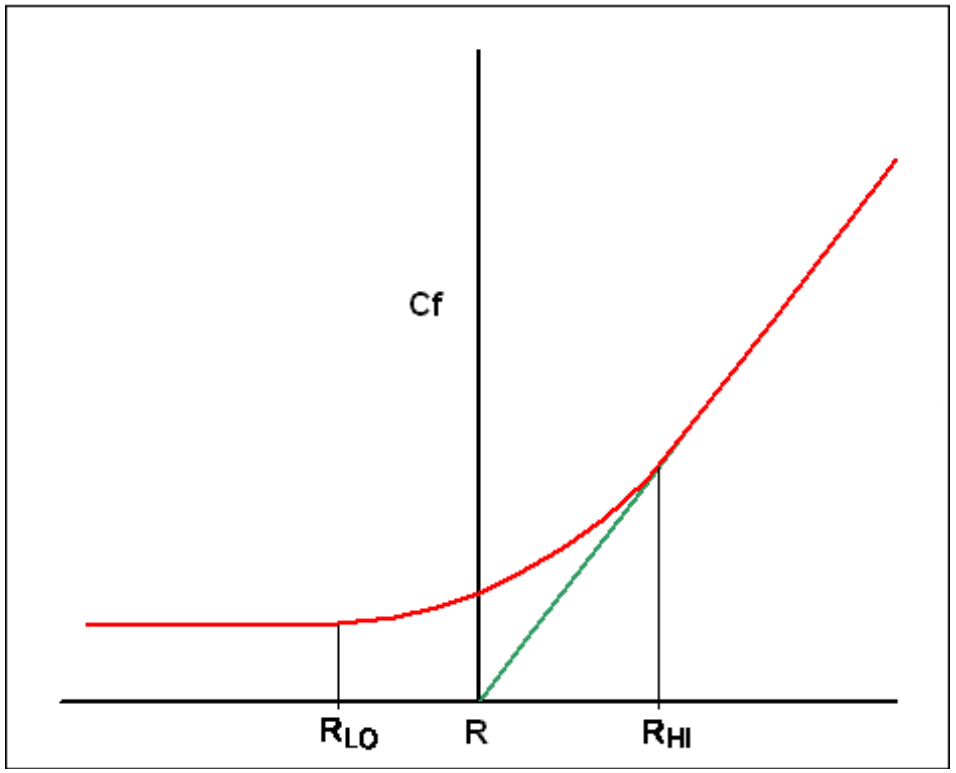
Differences in crack growth rate behavior between short and long cracks may be quantifiable using a yield zone model



Crack Closure Concept



$$C_f = \frac{\sigma_{open}}{\sigma_{max}} \qquad 0 < Cf <= 1$$



When R >= Rhi, the crack is open during the entire loading cycle

We will assume short cracks are open during the entire loading cycle since they have not yet grown into the plastic wake

$$\Delta K_{Applied} = K_{MAX} - K_{MIN} \qquad \Delta K_{Effective} = K_{MAX} - K_{OPEN}$$

Yield zone size is a function of several parameters

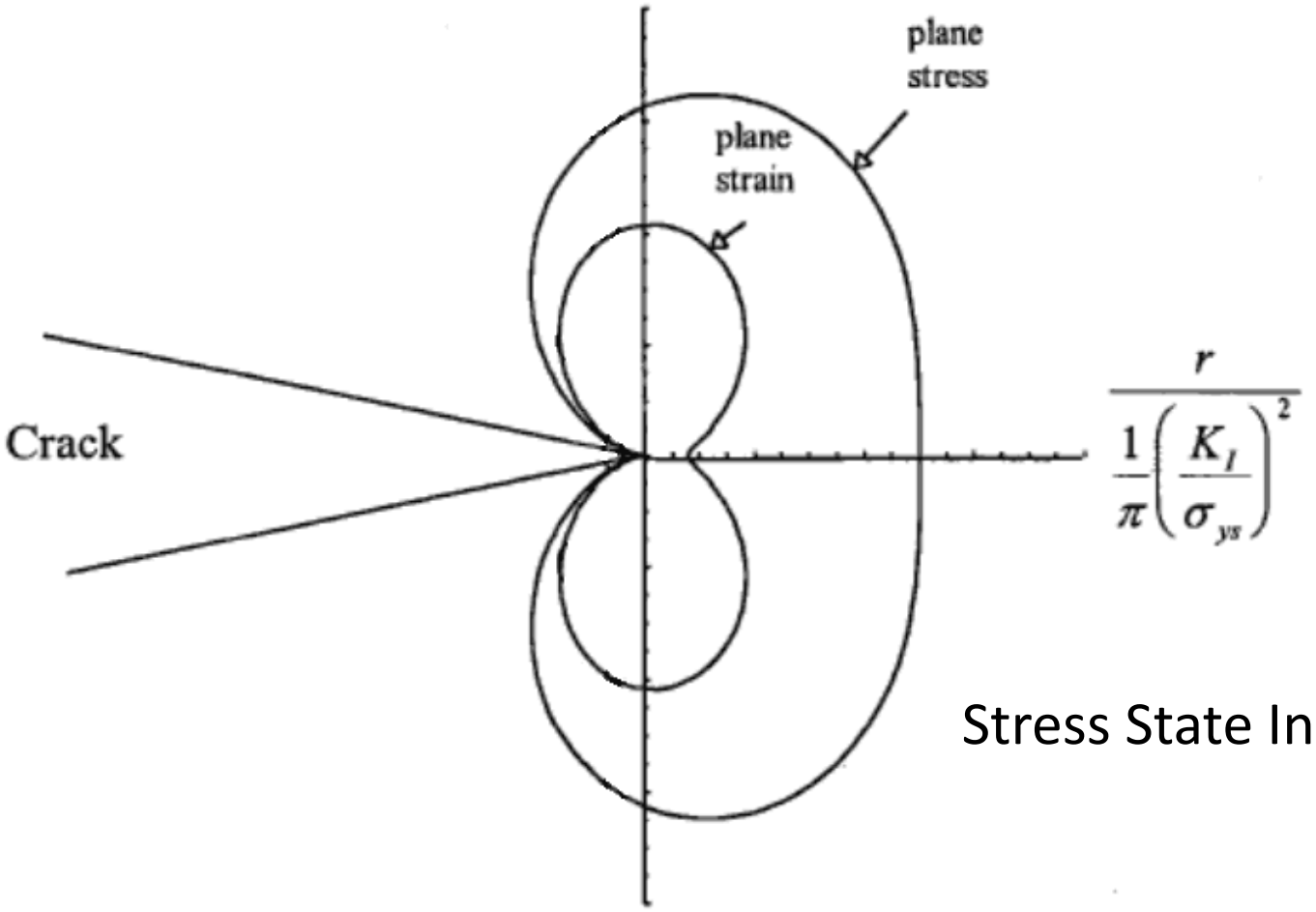
Material Parameter: Yield Strength

Physical Parameters: Crack Length
Thickness
Distance to a Free Edge

(local constraint)

Mechanical Parameter: Stress Intensity (K)

Constraint and Yield Zone Size



Irwin Yield Zone Size

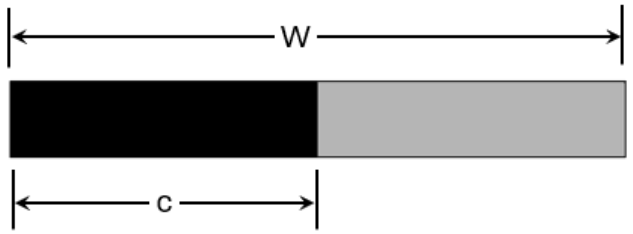
Plane Stress (2): $\frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$

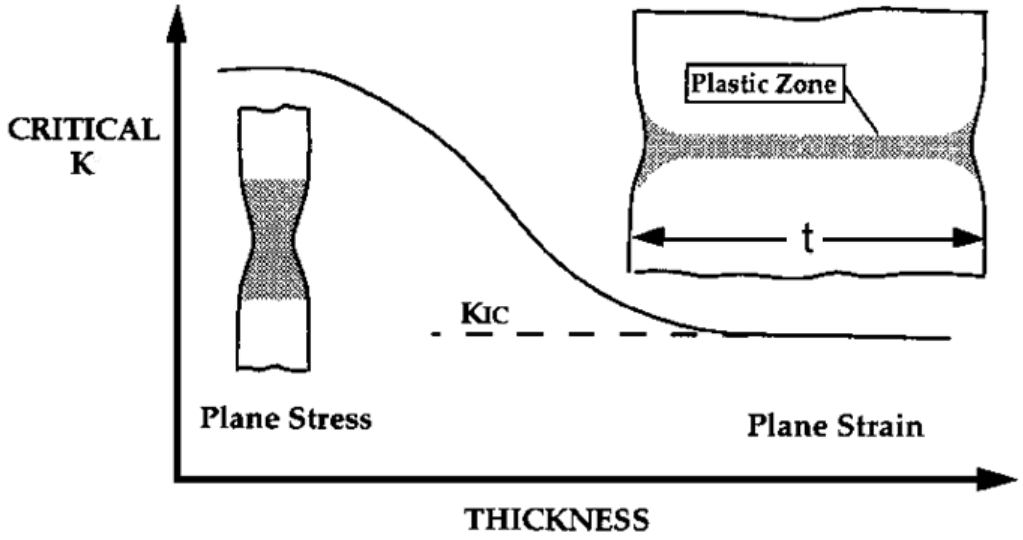
Plane Strain (6): $\frac{1}{6\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$

Stress State Index: Plane Stress (2), Plane Strain (6)

Index Limit [2 – 6]

ASTM Plane Strain Standard





Each parameter must meet the requirement to ensure local constraint surrounds the crack tip

$$c, t, (W - c) \geq 2.5 \left(\frac{K_I}{\sigma_{yield}} \right)^2 \rightarrow \textit{Plane Strain}$$

(ASTM Standard E-399)

Simplified Stress State Criteria Using Plate Thickness

Plane Strain options

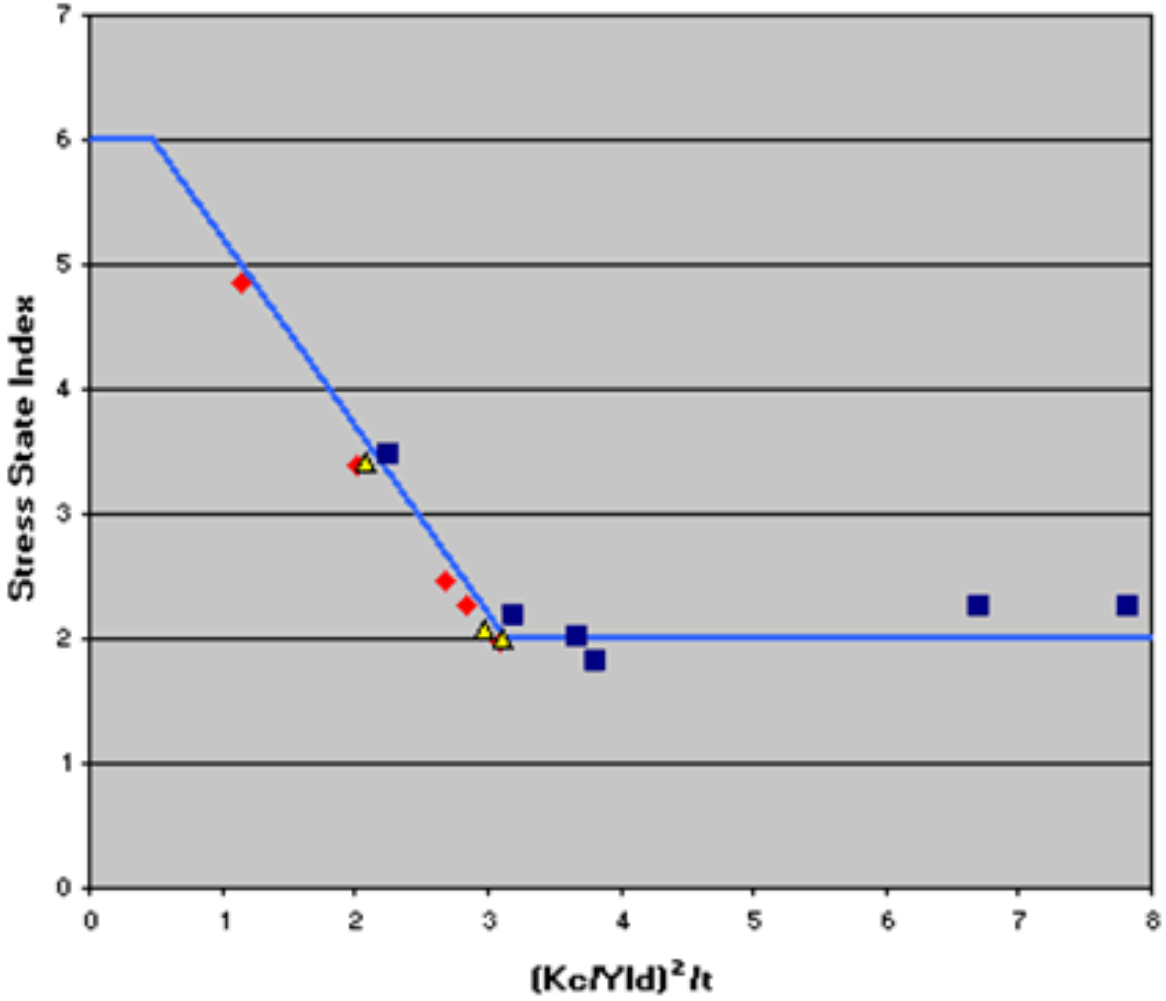
$$\left(\frac{K_I}{Yield}\right)^2 / t \leq 0.4 \quad (ASTM)$$ Conservative

$$\left(\frac{K_I}{Yield}\right)^2 / t \leq 0.47 \quad (David\ Broek)$$

Static fracture testing was done at WPAFB in the early 1990's to establish a Plane Stress criterion for AFGROW

$$\left(\frac{K_I}{Yield}\right)^2 / t \geq \pi \quad (Test\ Results)$$

Stress State Index



Static fracture Test results for MT specimens at various initial crack lengths

Plane Strain (6) and Plane Stress (2) toughness values were known for each material

Index was linearly interpolated from test result

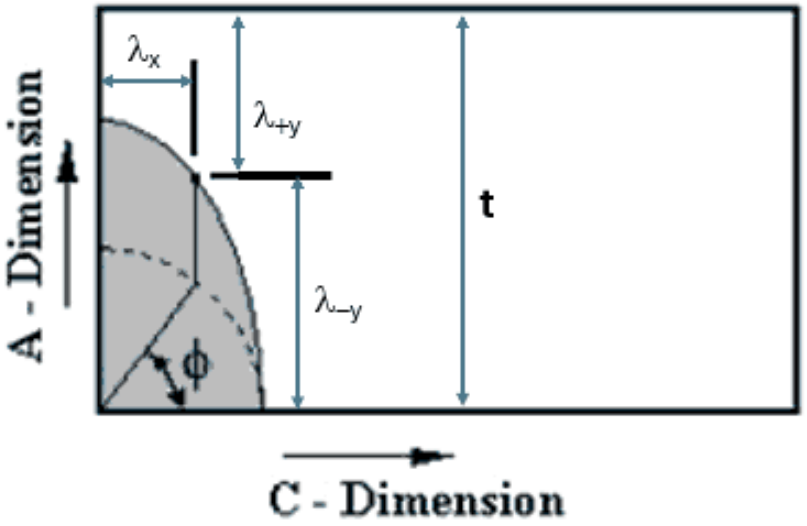
Plane Strain: $\frac{\left({}^{K_{max}}/_{Yield}\right)^2}{t} = 0.47$

Plane Stress: $\frac{\left({}^{K_{max}}/_{Yield}\right)^2}{t} = \pi$

$$\text{Stress State Index} = 6.7037 - \frac{1.4972}{t}\left(\frac{K_{\max}}{Yield}\right)^2$$

Index Limit [2 – 6]

Proposed Local Stress State Estimate for Part-Through Cracks



$$Stress\ State\ Index = 6.7037 - \frac{1.4972}{Parameter}\left(\frac{K}{Yield}\right)^2$$

Index Limit [2 – 6]

Use minimum [λ] as the Index Parameter

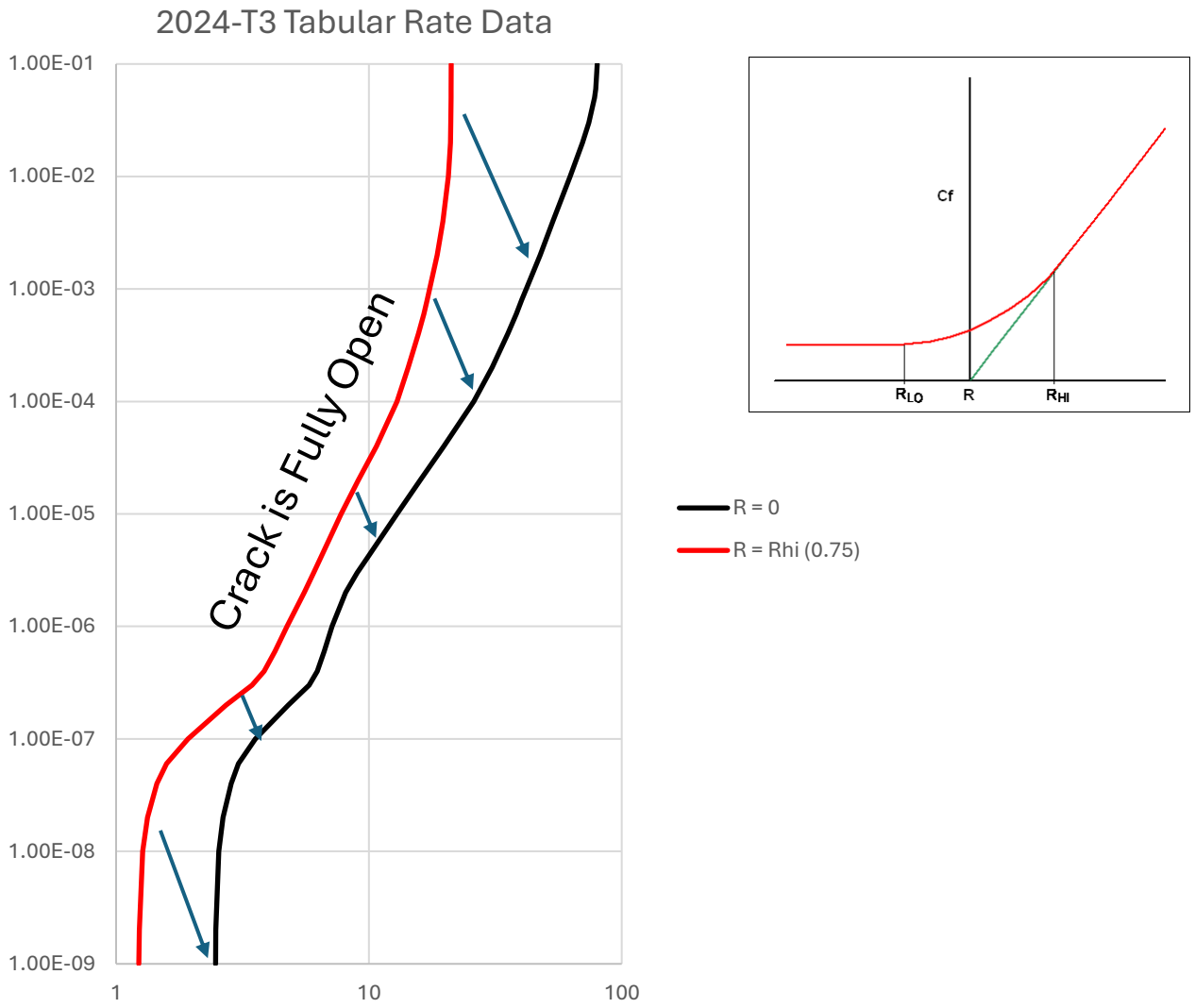
(local constraint)

Proposed Short Crack Growth Rate Approach

Short crack growth starts on the material Rhi curve (fully open)

Crack growth rate transitions (linearly) from Rhi (fully open) to the stress ratio for long cracks (R = 0 in this example) when it reaches a user-defined fraction of the yield zone penetrated

Transition path is a function of the User-Specified yield zone fraction (steeper path as YZ fraction decreases)



Proposed Short Crack Life Prediction Steps

- Set desired yield zone fraction to define short/long crack transition
- Calculate Kmax and R for the current spectrum cycle
- Determine yield zone index parameter
- Calculate stress state index
- Calculate yield zone size
- Calculate yield zone fraction penetrated (delta length or length)
- Interpolate the effective stress ratio between Rhi and R using the YZ fraction
- Delta K (or Kmax if Reff < 0) is not changed (only the effective R)
- Calculate the crack growth rate
- Increment the crack length

Options to Calculate the YZ Fraction Penetrated

(Crack Length/YZ Size)

Measure crack growth from the initial crack length (delta length)

- Current option coded in AFGROW for testing purposes
- Assumes no plastic wake effect on the initial crack
- For multiple crack models, the short crack option must be assigned to each crack to allow combinations of short and long cracks in a life prediction

Use the crack length itself

- The plastic wake effect is assumed to exist from crack length = 0
- For multiple crack models, each crack would transition to long crack data based on the user-defined YZ fraction for that crack
- It would not be necessary to assign the short crack option to each crack in given life prediction

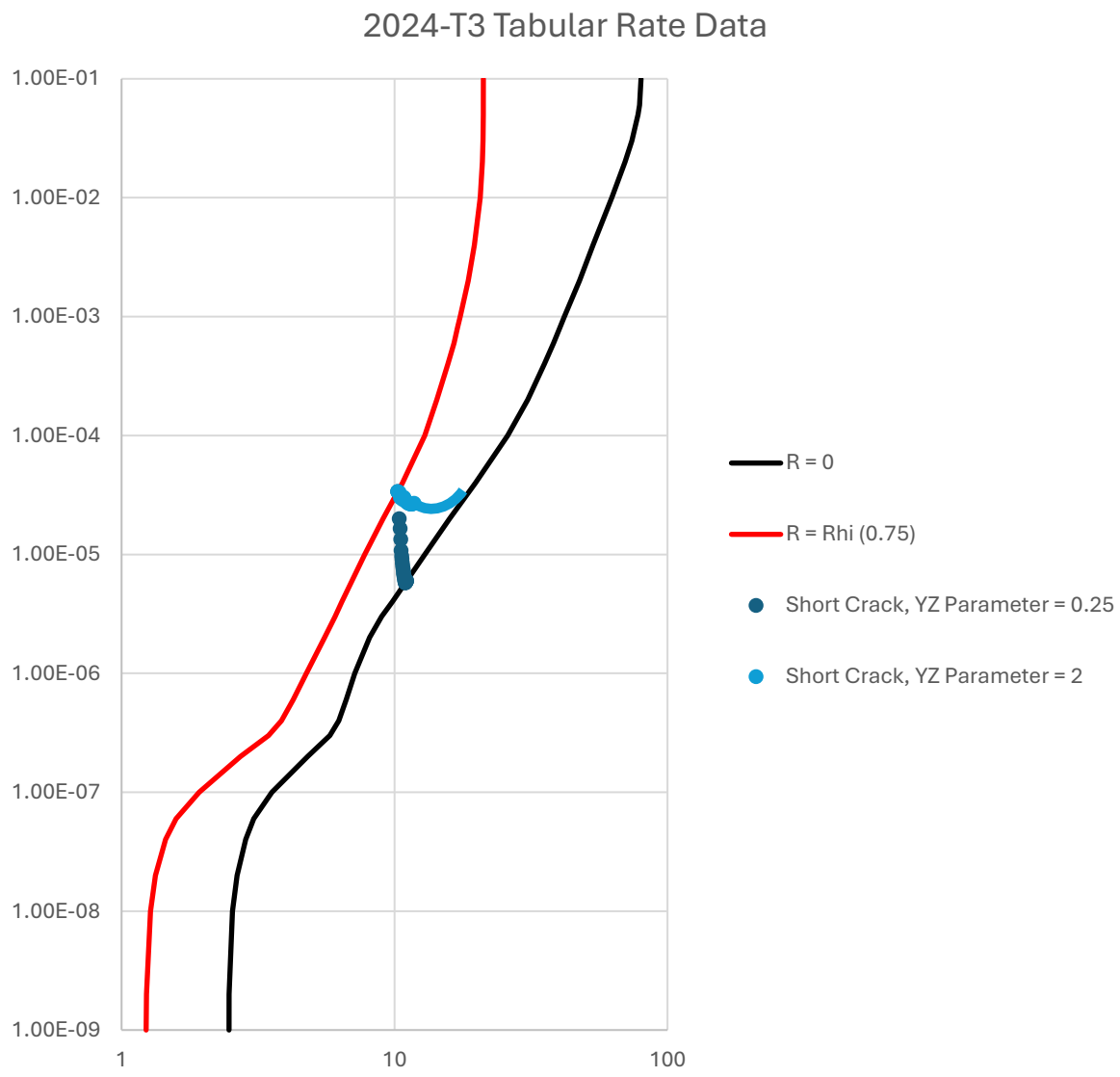
Example Calculation (YZ Parameter = 0.25)

Classic Centered Hole			Initial	Material	Tabular 2024-T3		Constant Amplitude Loading				Yield Zone	
Hole Dia	Width	Thickness	C	Yield	Rhi		Max Stress	R			Parameter	
0.25	4	0.25	0.01	49	0.75		20	0			0.25	
Using Delta C this time			Index						Equiv	Growth	Applied	
C	Beta	Kmax	Parameter	Index	Yld Zone	Yld Z Fraction	Delta K	Rfinal	Smax	Rate	Cycles	Delta C
0.01000	2.8989	10.2763	0.01000	2.00	7.000E-03	0.000	10.2763	0.75	80.000	3.37E-05	10	3.37E-04
0.01034	2.8850	10.3981	0.01034	2.00	7.167E-03	0.047	10.3981	0.61	51.127	1.99E-05	10	1.99E-04
0.01054	2.8769	10.4685	0.01054	2.00	7.264E-03	0.074	10.4685	0.53	42.404	1.65E-05	10	1.65E-04
0.01070	2.8705	10.5265	0.01070	2.00	7.345E-03	0.096	10.5265	0.46	37.276	1.34E-05	10	1.34E-04
0.01084	2.8649	10.5715	0.01084	2.00	7.408E-03	0.113	10.5715	0.41	33.996	1.08E-05	10	1.08E-04
0.01094	2.8609	10.6092	0.01094	2.00	7.461E-03	0.126	10.6092	0.37	31.781	9.69E-06	10	9.69E-05
0.01104	2.8569	10.6412	0.01104	2.00	7.506E-03	0.139	10.6412	0.33	30.040	9.12E-06	10	9.12E-05
0.01113	2.8534	10.6720	0.01113	2.00	7.549E-03	0.150	10.6720	0.30	28.586	8.48E-06	10	8.48E-05
0.01122	2.8498	10.6990	0.01122	2.00	7.588E-03	0.160	10.6990	0.27	27.363	8.03E-06	10	8.03E-05
0.01130	2.8466	10.7252	0.01130	2.00	7.625E-03	0.170	10.7252	0.24	26.311	7.69E-06	10	7.69E-05
0.01137	2.8439	10.7514	0.01137	2.00	7.662E-03	0.179	10.7514	0.21	25.389	7.43E-06	10	7.43E-05
0.01145	2.8407	10.7743	0.01145	2.00	7.695E-03	0.188	10.7743	0.19	24.557	7.03E-06	10	7.03E-05
0.01152	2.8380	10.7971	0.01152	2.00	7.728E-03	0.196	10.7971	0.16	23.828	6.90E-06	10	6.90E-05
0.01159	2.8353	10.8191	0.01159	2.00	7.759E-03	0.205	10.8191	0.14	23.158	6.62E-06	10	6.62E-05
0.01165	2.8329	10.8408	0.01165	2.00	7.790E-03	0.212	10.8408	0.11	22.556	6.55E-06	10	6.55E-05

Example Calculation (YZ Parameter = 2)

Classic Centered Hole			Initial	Material	Tabular 2024-T3		Constant Amplitude Loading				Yield Zone	
Hole Dia	Width	Thickness	C	Yield	Rhi		Max Stress	R			Parameter	
0.25	4	0.25	0.01	49	0.75		20	0			2	
Using Delta C this time			Index						Equiv	Growth	Applied	
C	Beta	Kmax	Parameter	Index	Yld Zone	Yld Z Fraction	Delta K	Rfinal	Smax	Rate	Cycles	Delta C
0.01000	2.8989	10.2763	0.01000	2.00	7.000E-03	0.000	10.2763	0.75	80.000	3.37E-05	10	3.37E-04
0.01034	2.8850	10.3981	0.01034	2.00	7.167E-03	0.047	10.3981	0.73	74.725	3.30E-05	10	3.30E-04
0.01067	2.8717	10.5142	0.01067	2.00	7.328E-03	0.091	10.5142	0.72	70.383	2.99E-05	10	2.99E-04
0.01097	2.8597	10.6158	0.01097	2.00	7.470E-03	0.129	10.6158	0.70	67.001	3.07E-05	10	3.07E-04
0.01127	2.8478	10.7187	0.01127	2.00	7.616E-03	0.167	10.7187	0.69	63.959	2.89E-05	10	2.89E-04
0.01156	2.8364	10.8118	0.01156	2.00	7.749E-03	0.202	10.8118	0.67	61.422	3.01E-05	10	3.01E-04
0.01186	2.8247	10.9066	0.01186	2.00	7.885E-03	0.236	10.9066	0.66	59.060	2.89E-05	10	2.89E-04
0.01215	2.8136	10.9954	0.01215	2.00	8.014E-03	0.269	10.9954	0.65	57.021	2.81E-05	10	2.81E-04
0.01243	2.8028	11.0791	0.01243	2.00	8.136E-03	0.299	11.0791	0.64	55.220	2.75E-05	10	2.75E-04
0.01271	2.7922	11.1587	0.01271	2.00	8.254E-03	0.328	11.1587	0.63	53.605	2.71E-05	10	2.71E-04
0.01298	2.7821	11.2364	0.01298	2.00	8.369E-03	0.356	11.2364	0.62	52.143	2.69E-05	10	2.69E-04
0.01325	2.7720	11.3109	0.01325	2.00	8.481E-03	0.383	11.3109	0.61	50.802	2.67E-05	10	2.67E-04
0.01352	2.7620	11.3831	0.01352	2.00	8.589E-03	0.409	11.3831	0.60	49.563	2.66E-05	10	2.66E-04
0.01378	2.7524	11.4546	0.01378	2.00	8.697E-03	0.435	11.4546	0.59	48.416	2.65E-05	10	2.65E-04
0.01405	2.7426	11.5232	0.01405	2.00	8.802E-03	0.460	11.5232	0.58	47.342	2.66E-05	10	2.66E-04

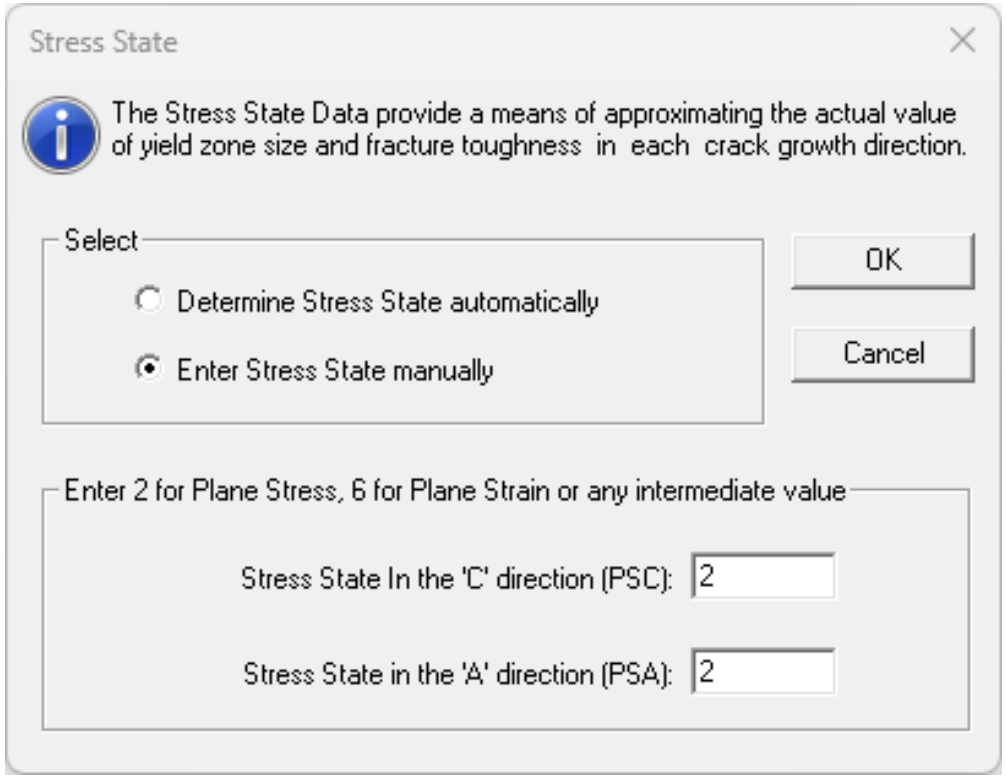
Example Crack Growth Rate Predictions



Example for a Variable Amplitude Spectrum

Classic Centered Hole			Initial	Material	Tabular 2024-T3		Variable Amplitude Loading				Yield Zone				
Hole Dia	Width	Thickness	C	Yield	Rhi	Rlo					Parameter				
0.25	4	0.25	0.01	49	0.75	-0.33					0.25				
Using Delta C this time			Index									Equiv	Growth	Applied	
C	Beta	Smax	Smin	R	Kmax	Parameter	Index	Yld Zone	Yld Z Fraction	Delta K*	Rshort	Smax	Rate	Cycles	Delta C
0.0100000	2.8989	10	0	0.000	5.1382	0.01000	5.06	6.92E-04	0.000	5.1382	0.750	40.000	1.42E-06	10	1.42E-05
0.0100142	2.8983	20	0	0.000	10.2815	0.01001	2.00	7.01E-03	0.002	10.2815	0.744	78.102	3.41E-05	1	3.41E-05
0.0100483	2.8969	18	15	0.833	9.2646	0.01005	2.00	5.69E-03	0.008	1.5441	0.833	18.000	5.33E-08	25	1.33E-06
0.0100496	2.8969	6	2	0.333	3.0884	0.01005	6.00	2.11E-04	0.235	2.0589	0.358	6.227	4.36E-08	20	8.72E-07
0.0100505	2.8968	10	-2	-0.200	5.1474	0.01005	5.06	6.94E-04	0.073	5.1474	0.590	24.391	9.14E-07	10	9.14E-06
0.0100596	2.8965	5	-5	-1.000	2.5746	0.01006	6.00	1.46E-04	0.407	2.5746	-1.000	5.000	6.34E-08	10	6.34E-07
0.0100603															

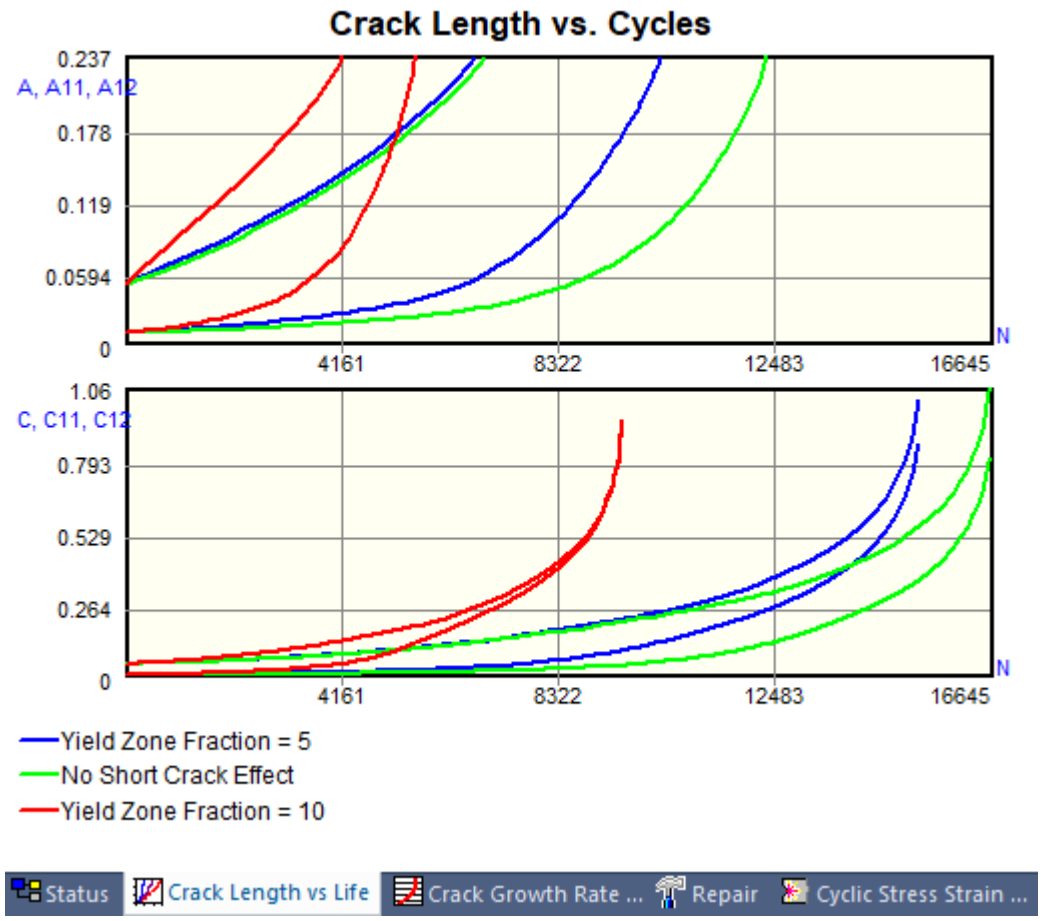
Current AFGROW Beta Testing Under Plane Stress Conditions

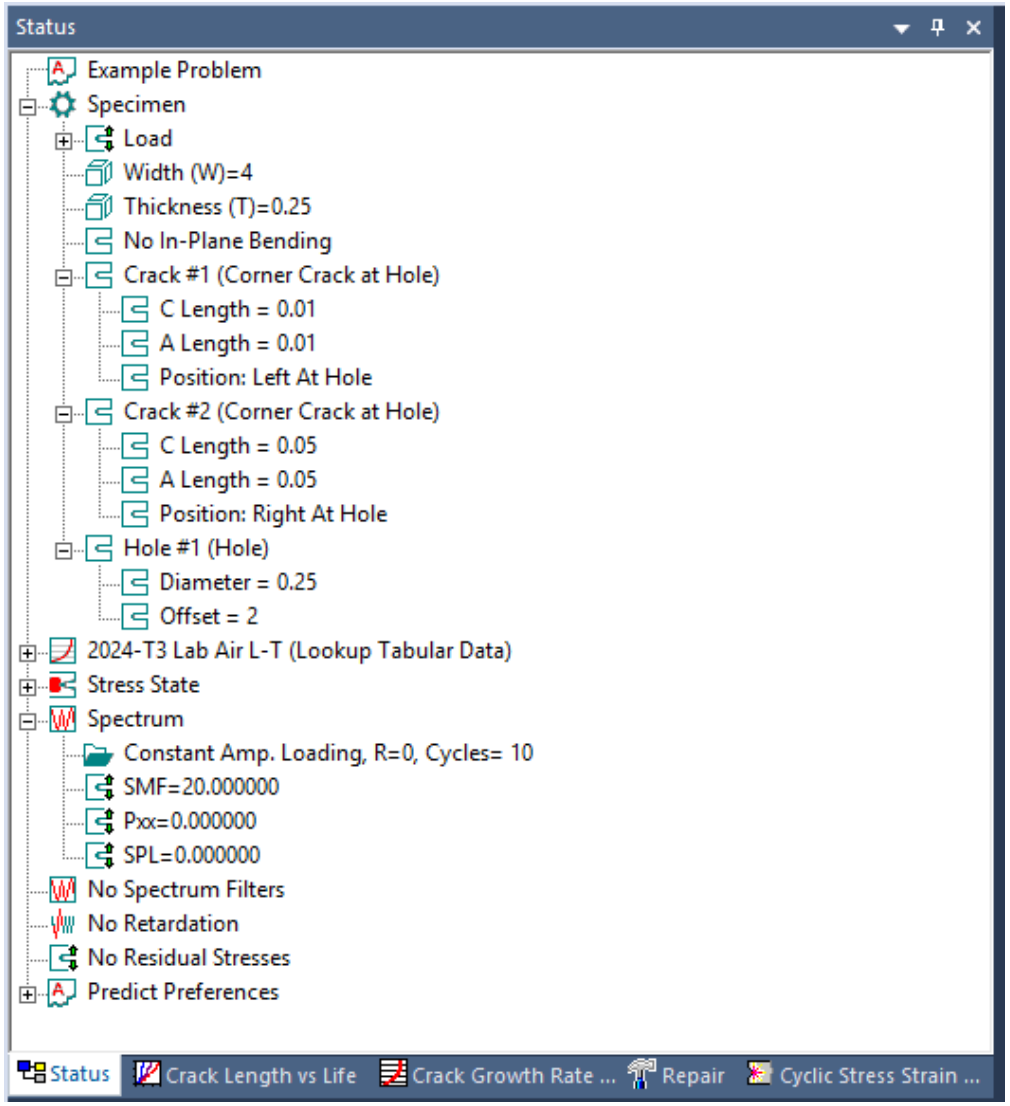


Internal coding complications required this in the short term

This can cause longer cracks to incorrectly operate under plane stress conditions

Continuing Damage Case





Demonstrations

Potential Problem with Yield Zone Approach

- When the crack is short and operating at low K-values, the local yield zone is very small
- In this situation, a larger yield zone fraction is required to obtain the “short” crack effect
- This may trigger the “short” crack effect on “long” crack(s) in the model (this may be improved when automatic stress state calculations are restored in AFGROW)

Another option is a grain size model?

Unfortunately, grain size is not commonly available in standard LEFM material datasets