Modeling Small Crack Growth

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LexTech, Inc.

Motivation

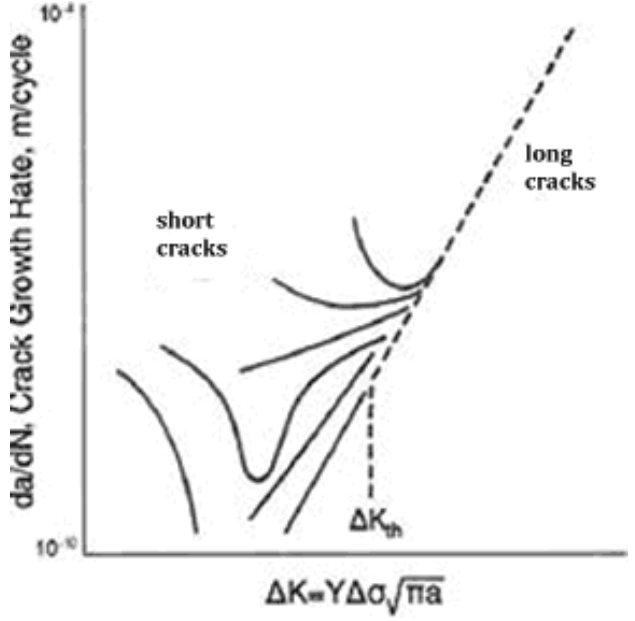
Over the years, many researchers/analysts have proposed extending “long” crack growth rate curves from Region II backwards to resolve the issue where “small” cracks demonstrate crack growth below the long crack threshold.

Potential Shortcomings of this approach:

Similitude? (Can we use LEFM for small cracks?)

Lack of a physical basis?

When K >= short/long crack intersection point, there is no small crack effect?



Are we confident that we know the true threshold behavior of “long” cracks?

Consider a Yield Zone Based Model

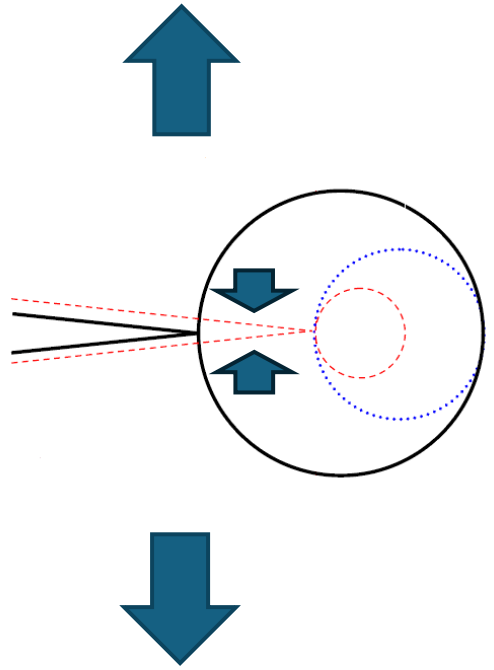
Long cracks have grown into the compressive plastic wake ahead of the crack tip

Constant amplitude long crack growth rate data includes the effect of residual compressive stress (contact behind the crack tip), crack face roughness, and debris (crack closure)

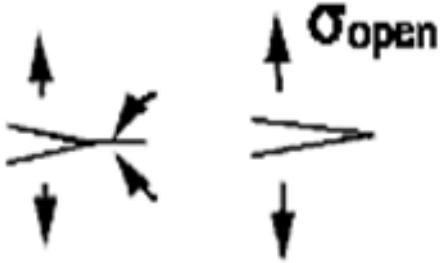
Small cracks may not long enough to be affected by the residual crack closing force to the same extent

We need to know when “small” cracks exhibit the closure effects seen in “long” cracks

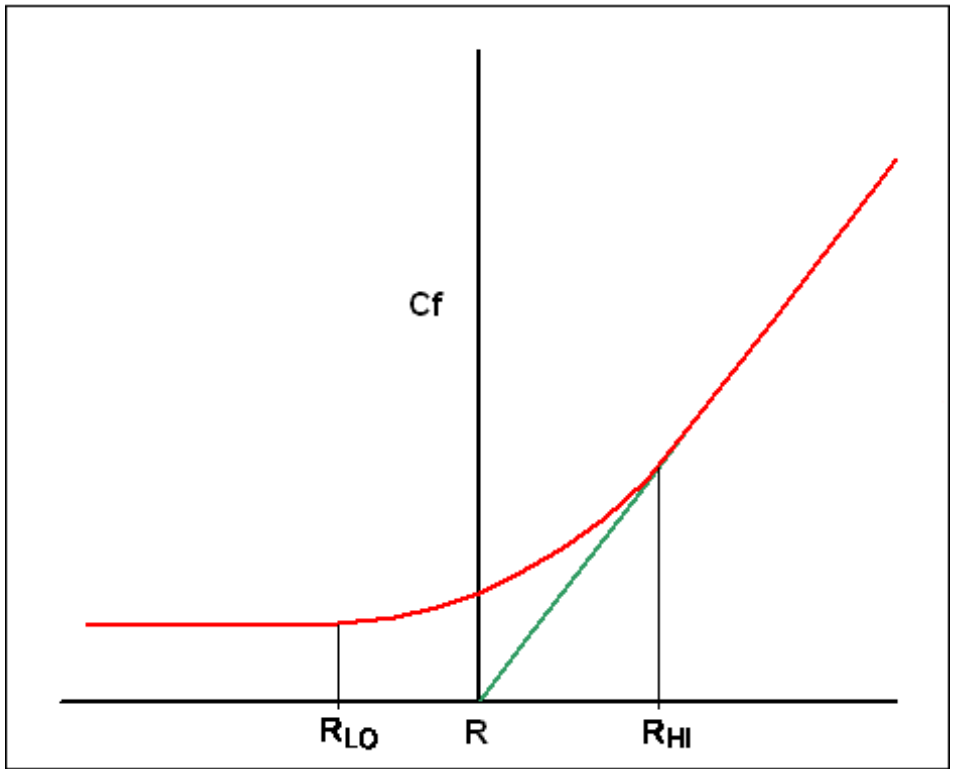
Differences in crack growth rate behavior between small and long cracks may be quantifiable using a yield zone model



Crack Closure Concept



$$C_f = \frac{\sigma_{open}}{\sigma_{max}} \qquad 0 < Cf <= 1$$



When R >= Rhi, the crack is open during the entire loading cycle

We will assume small cracks are open during the entire loading cycle since they have not yet grown into the plastic wake

$$\Delta K_{Applied} = K_{MAX} - K_{MIN} \qquad \Delta K_{Effective} = K_{MAX} - K_{OPEN}$$

Yield zone size is a function of several parameters

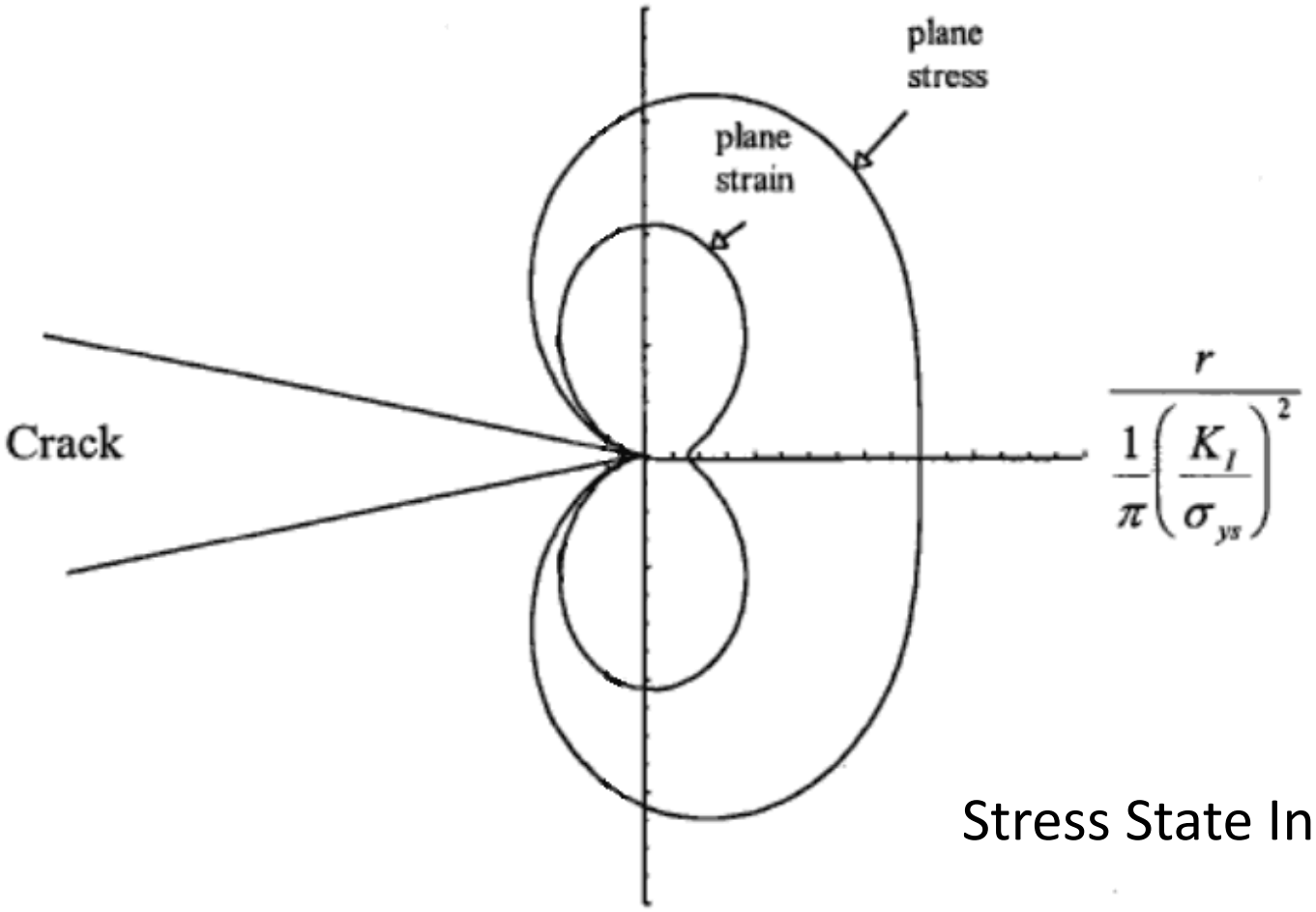
Material Parameter: Yield Strength

Physical Parameters: Crack Length
Thickness
Distance to a Free Edge

(local constraint)

Mechanical Parameter: Stress Intensity (K)

Constraint and Yield Zone Size



Irwin Yield Zone Size

Plane Strain (6): $\frac{1}{6\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$

(Fully Constrained)

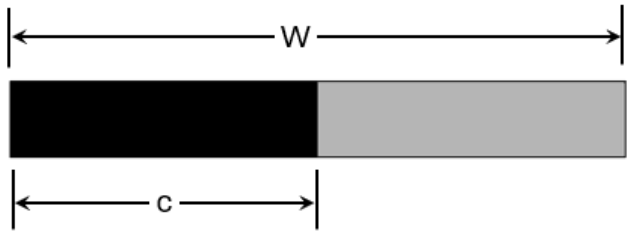
Plane Stress (2): $\frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$

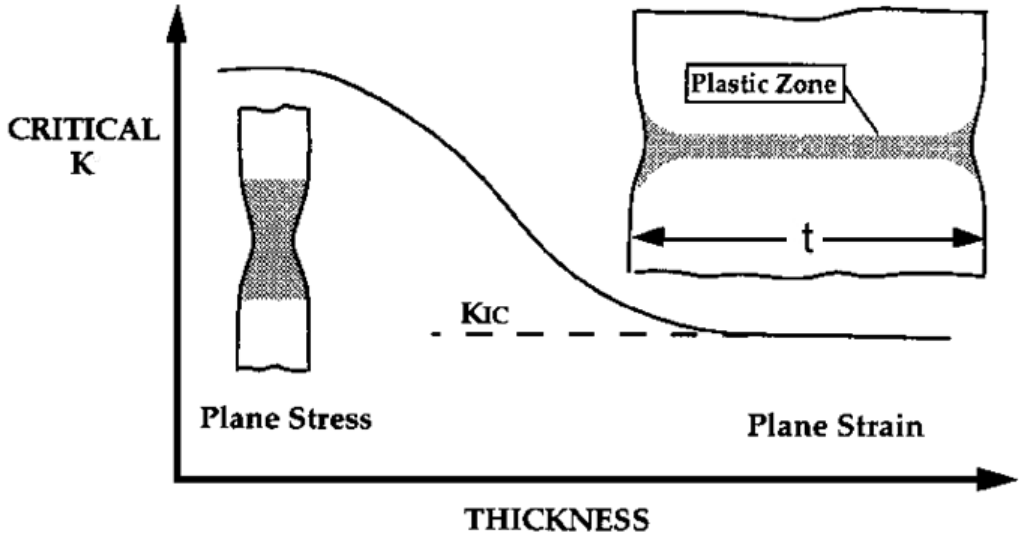
(Fully Unconstrained)

Stress State Index: Plane Stress (2), Plane Strain (6)

Index Limit [2 – 6]

ASTM Plane Strain Standard





Each parameter must meet the requirement to ensure local constraint surrounds the crack tip

$$c, t, (W - c) \geq 2.5 \left(\frac{K_I}{\sigma_{yield}} \right)^2 \rightarrow \textit{Plane Strain}$$

(ASTM Standard E-399)

Simplified Stress State Criteria Using Plate Thickness

Plane Strain criteria

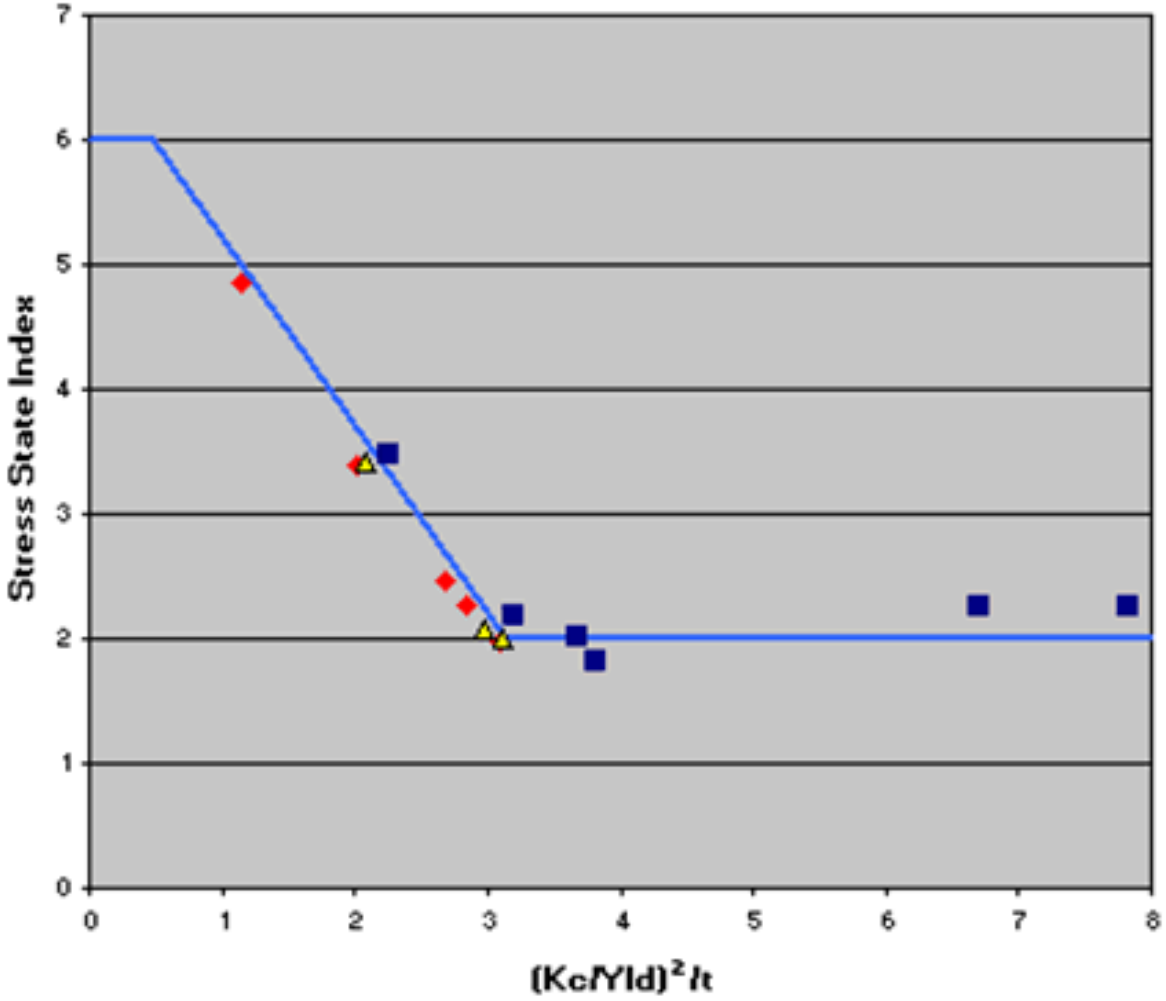
$$\left(\frac{K_I}{Yield}\right)^2 / t \leq 0.4 \quad (ASTM)$$ Conservative

$$\left(\frac{K_I}{Yield}\right)^2 / t \leq 0.47 \quad (David\ Broek)$$

Static fracture testing was done at WPAFB in the early 1990's to establish a Plane Stress criterion for AFGROW

$$\left(\frac{K_I}{Yield}\right)^2 / t \geq \pi \quad (Test\ Results)$$

AFGROW Stress State Index



Static fracture Test results for MT specimens at various initial crack lengths

Plane Strain (6) and Plane Stress (2) toughness values were known for each material

Index was linearly interpolated from test result

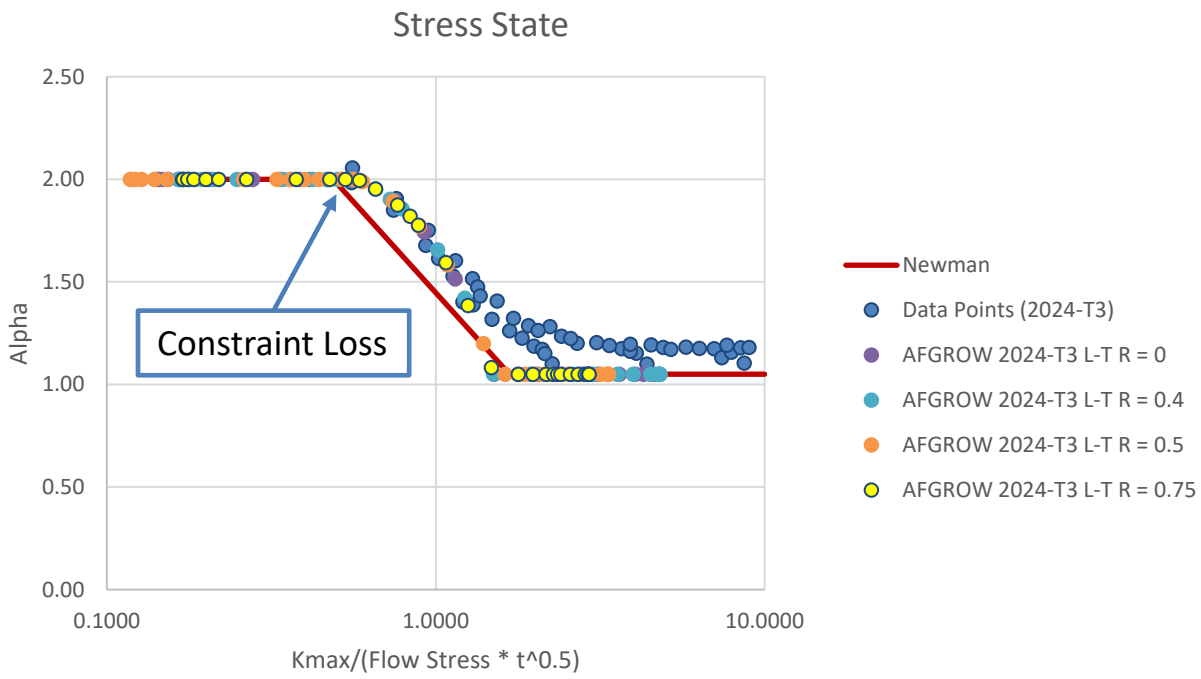
Plane Strain: $\frac{\left({}^{K_{max}}/_{Yield}\right)^2}{t} = 0.47$

Plane Stress: $\frac{\left({}^{K_{max}}/_{Yield}\right)^2}{t} = \pi$

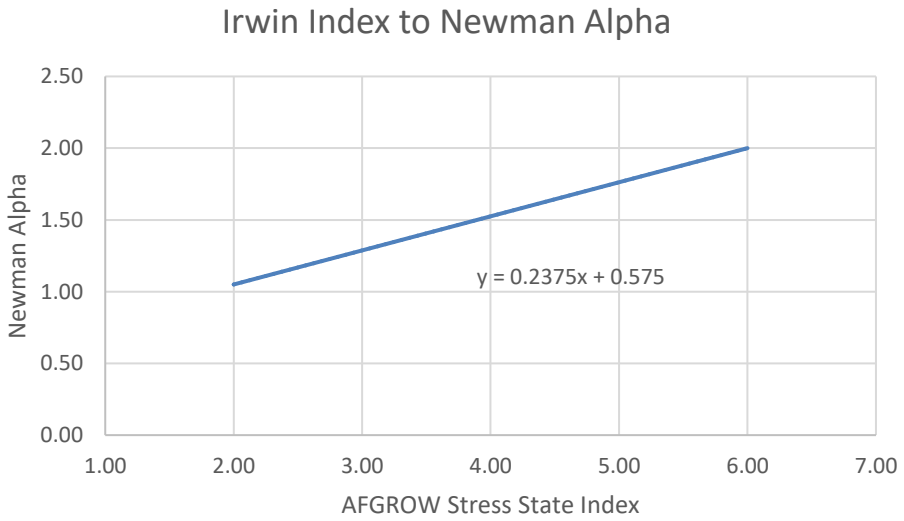
$$\text{Stress State Index} = 6.7037 - \frac{1.4972}{t}\left(\frac{K_{\max}}{Yield}\right)^2$$

Index Limit [2 – 6]

Comparing Newman's Alpha Constraint Parameter and AFGROW's Stress State Index



Assume Linear Conversion



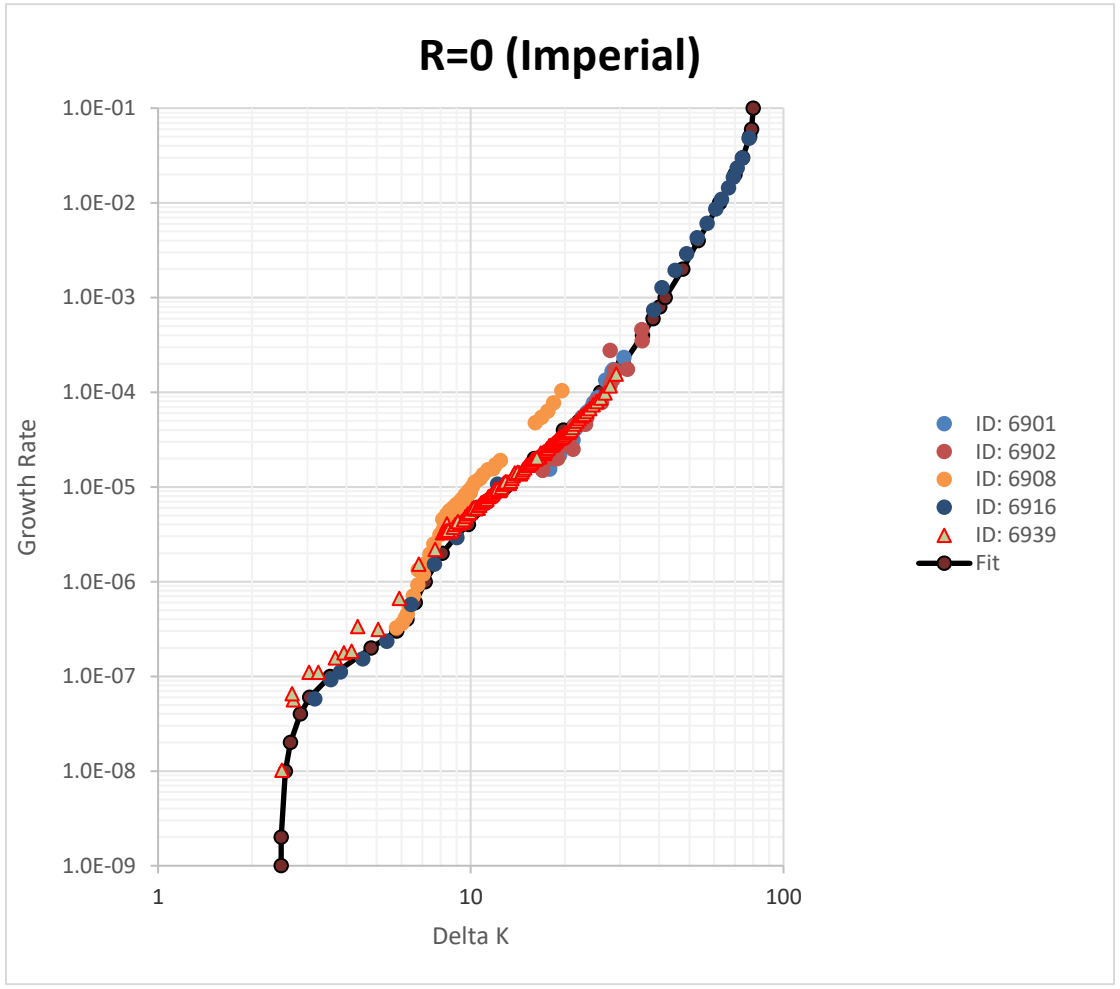
Data points obtained from:

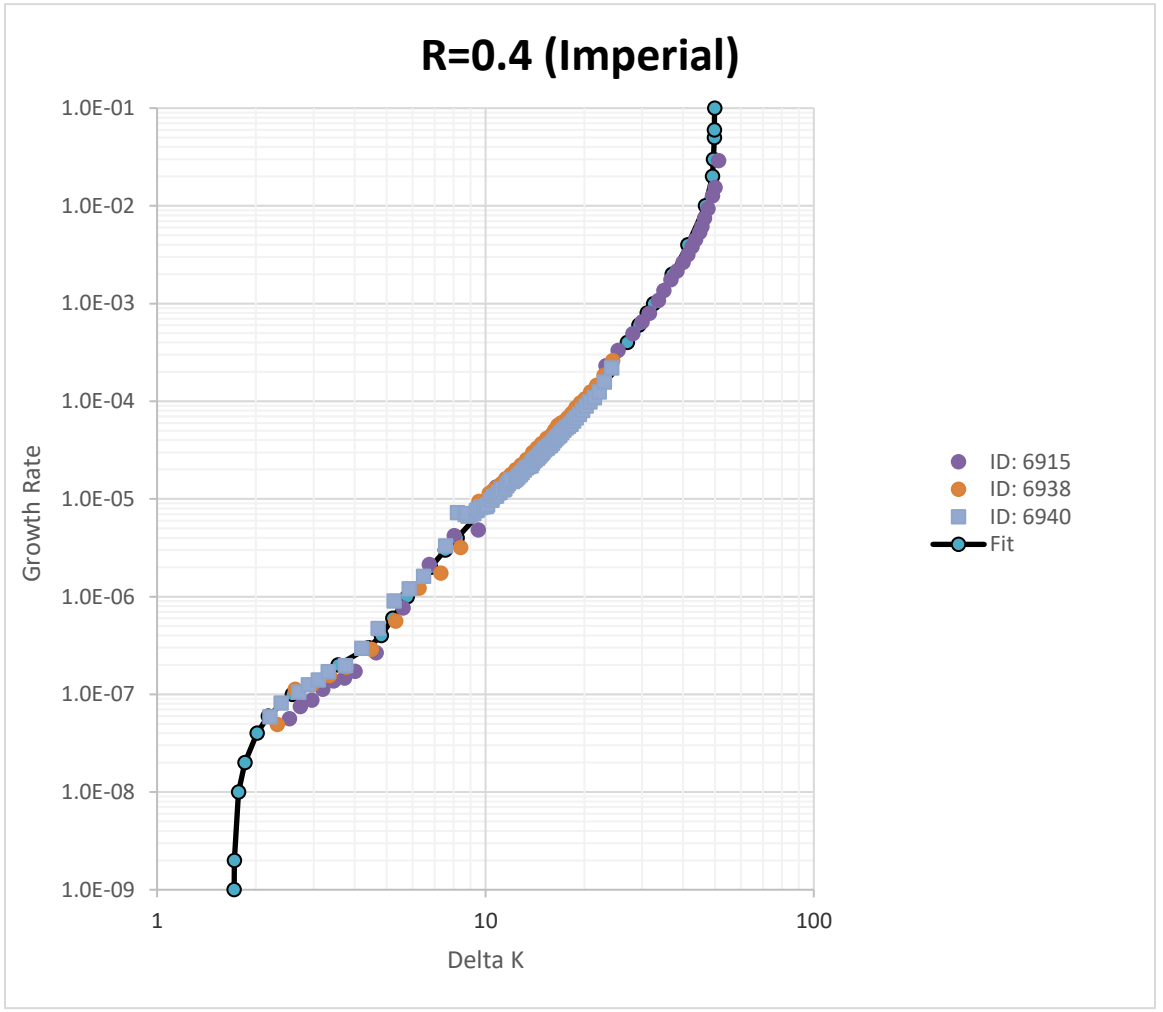
Fatigue crack growth under single-spike overloads-underloads and a simulated aircraft spectrum, EFM 327, 2025

James C. Newman , Kevin F. Walker

AFGROW Tabular Fit

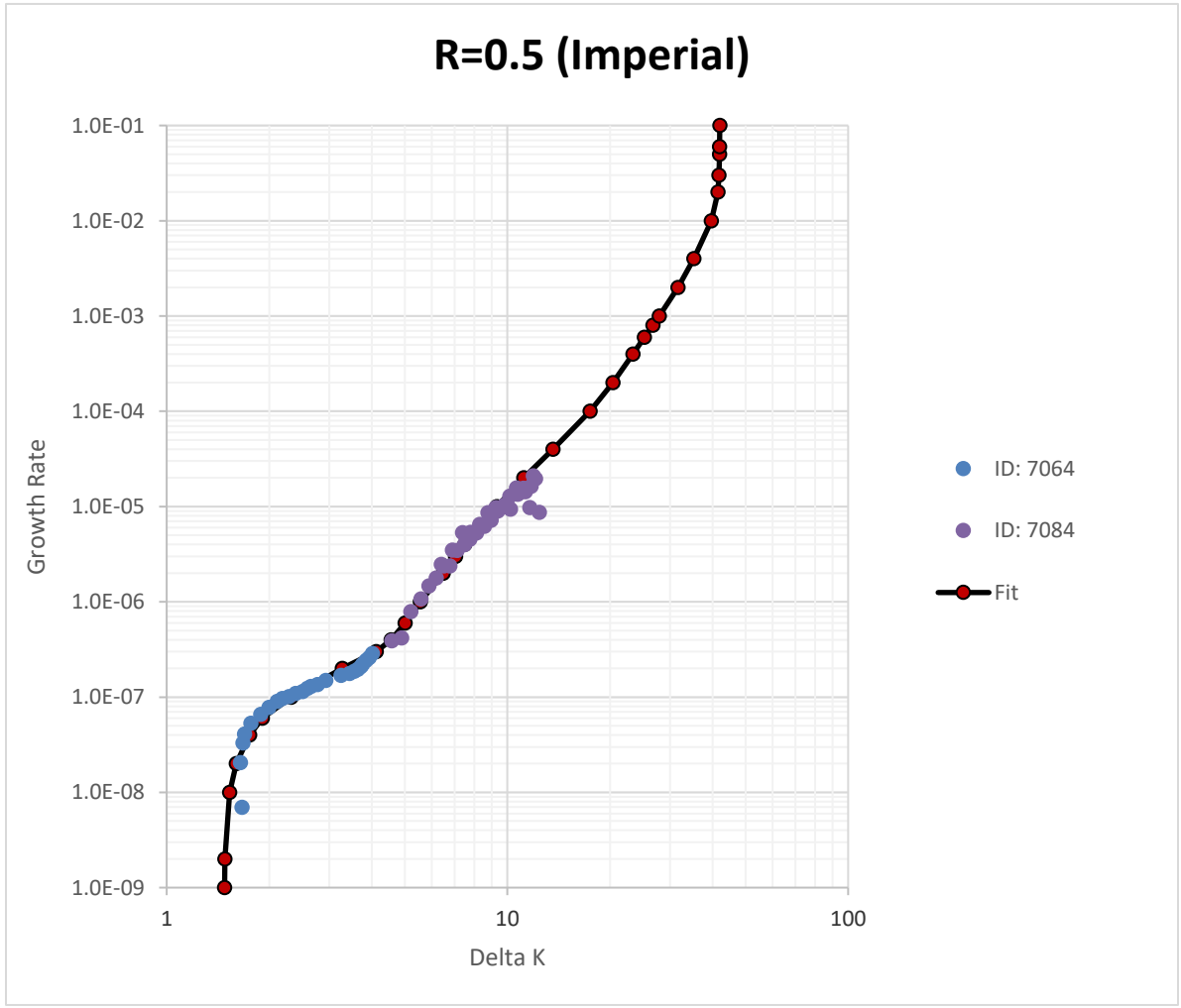
2024-T3 (L-T)

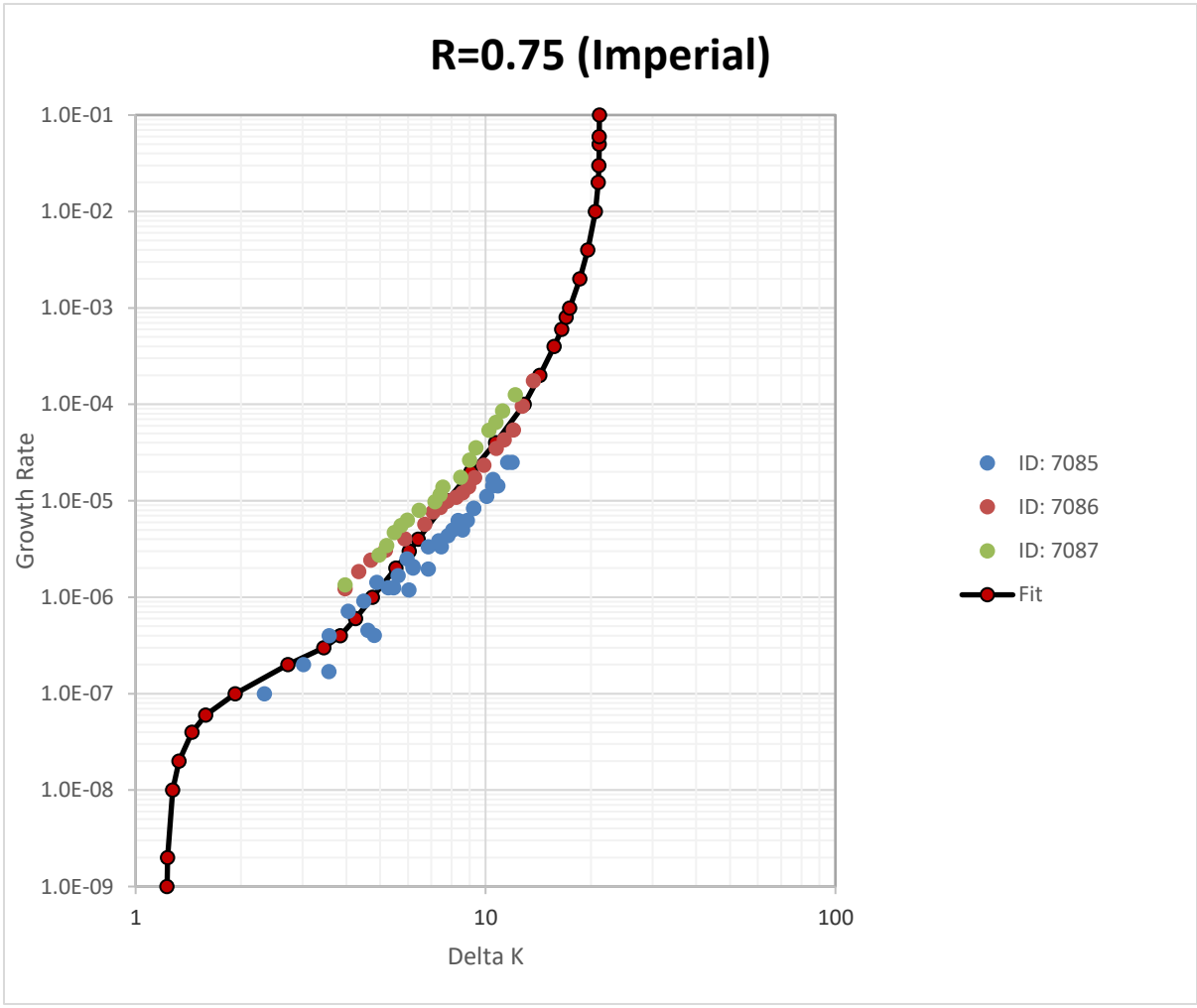




AFGROW Tabular Fit

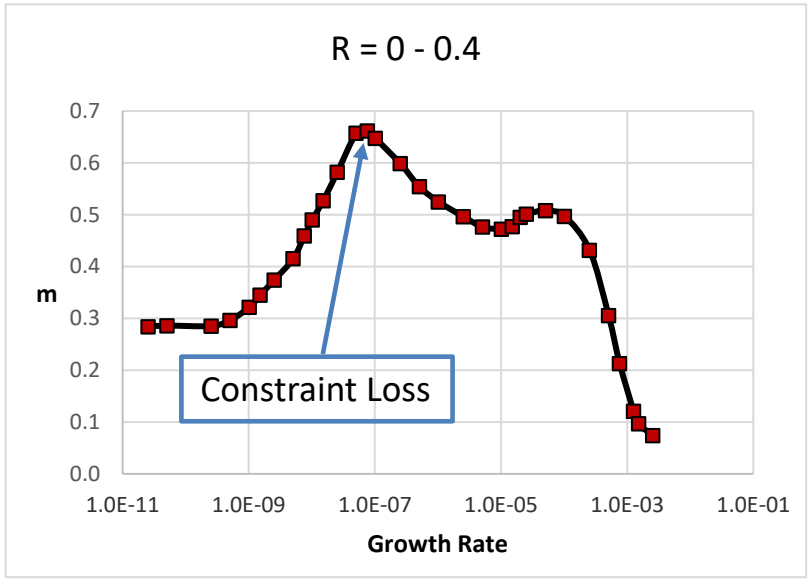
2024-T3 (L-T)

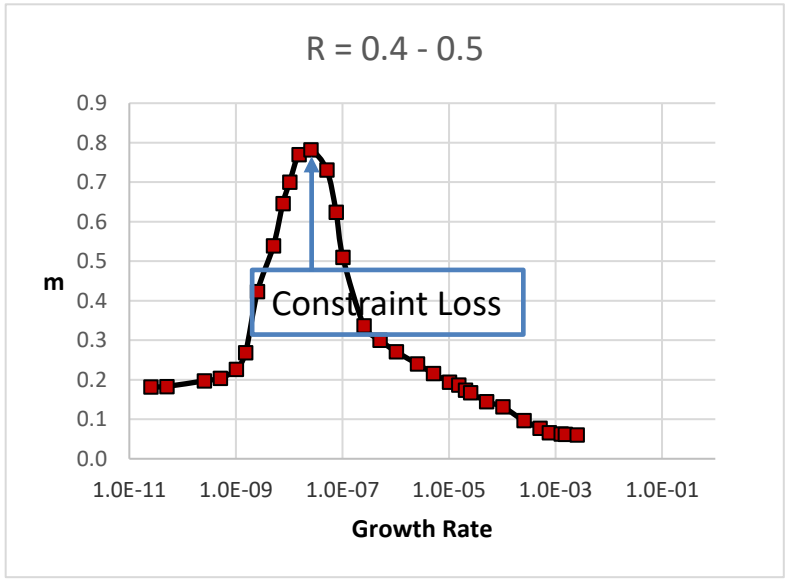


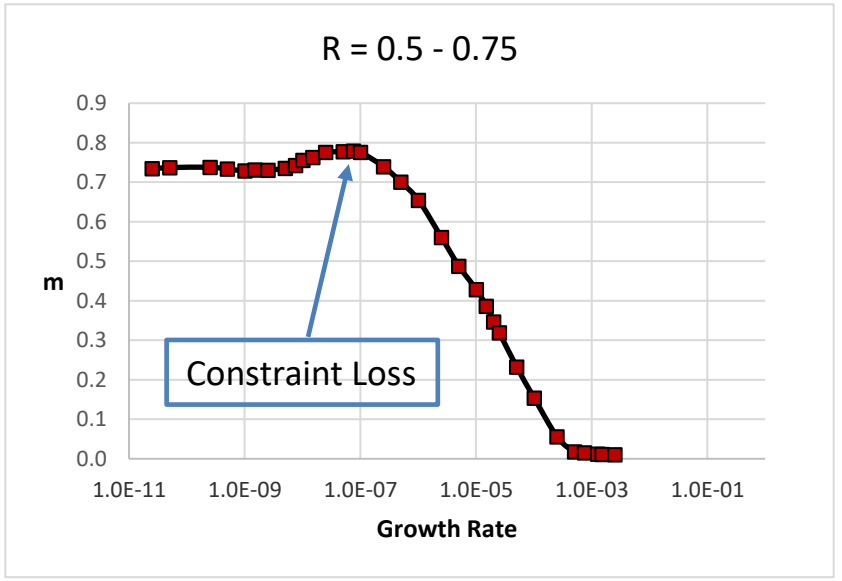


Walker m-values and Constraint Loss

Determined for 2024-T3 Tabular Data in AFGROW

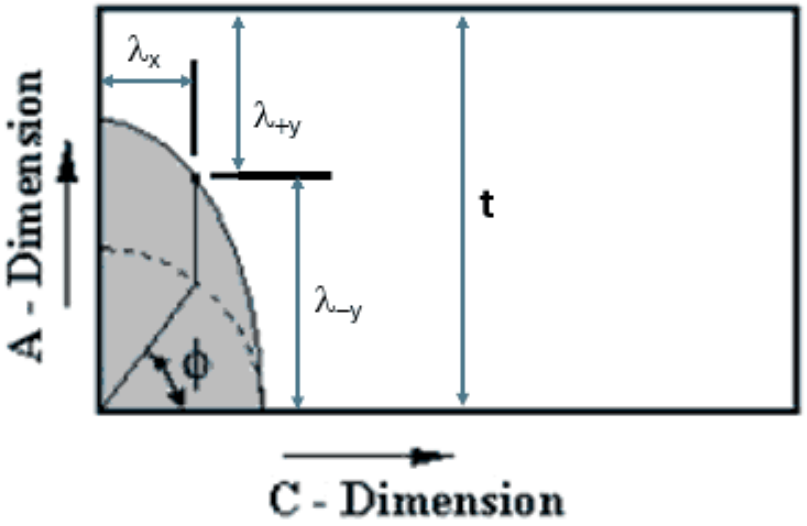






$$\Delta K_1(1-R_1)^{(m-1)} = \Delta K_2(1-R_2)^{(m-1)}$$ (Walker relationship)

Proposed Local Stress State Estimate for Part-Through Cracks



$$Stress\ State\ Index = 6.7037 - \frac{1.4972}{Parameter}\left(\frac{K}{Yield}\right)^2$$

Index Limit [2 – 6]

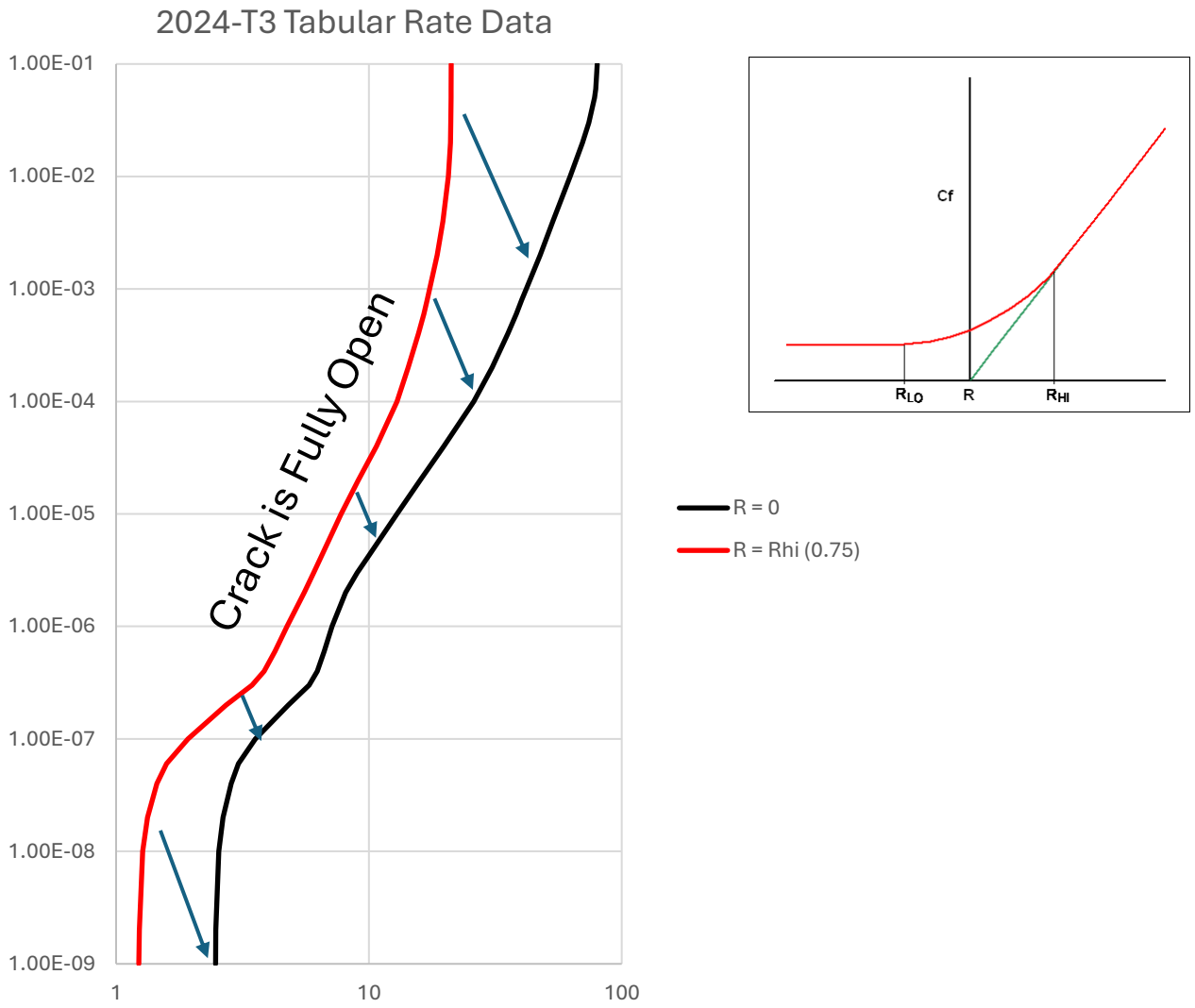
Use minimum [λ] or thickness as the Index Parameter (local constraint)

Closure Based Short Crack Growth Rate Approach

Assume short crack growth starts on the material Rhi curve (fully open)

Crack growth rate transitions (linearly) from Rhi (fully open) to the stress ratio for long cracks (R = 0 in this example) when it reaches a user-defined fraction of the yield zone penetrated

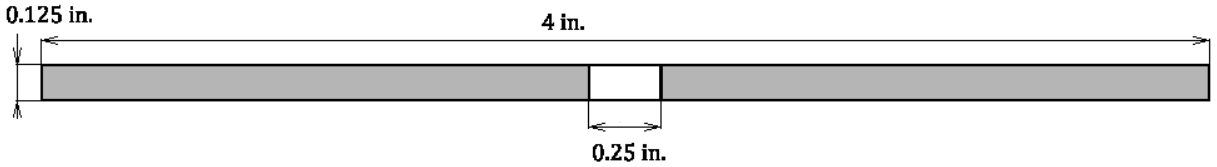
Transition path is a function of the User-Specified yield zone fraction (steeper path as YZ fraction decreases)



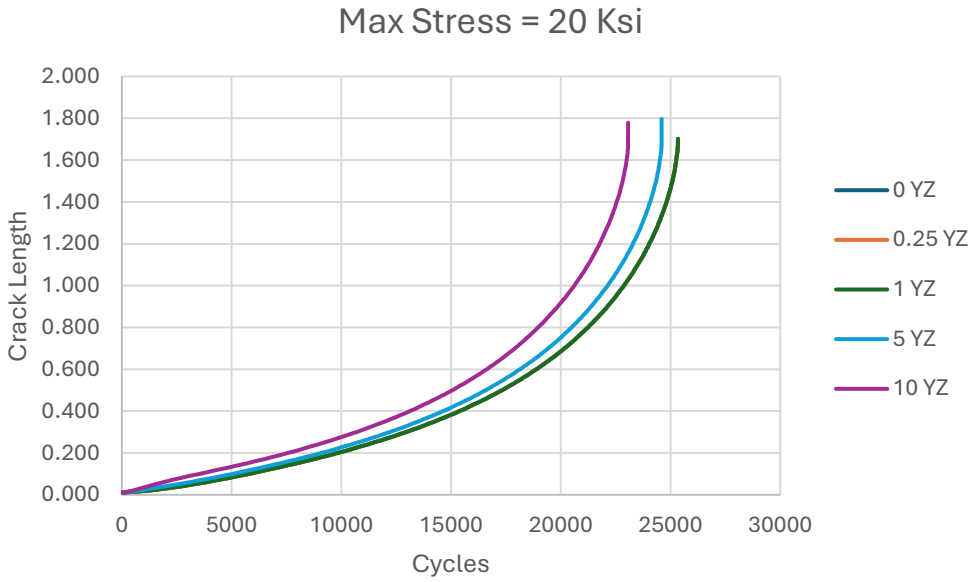
Proposed Small Crack Life Prediction Steps

- Set desired yield zone fraction to define small/long crack transition
- Calculate Kmax and R for the current spectrum cycle
- Determine yield zone index parameter
- Calculate stress state index
- Calculate yield zone size
- Calculate yield zone fraction penetrated (delta length or length)
- Interpolate the effective stress ratio between Rhi and R using the YZ fraction
- Delta K (or Kmax if Reff < 0) is not changed (only the effective R)
- Calculate the crack growth rate
- Increment the crack length

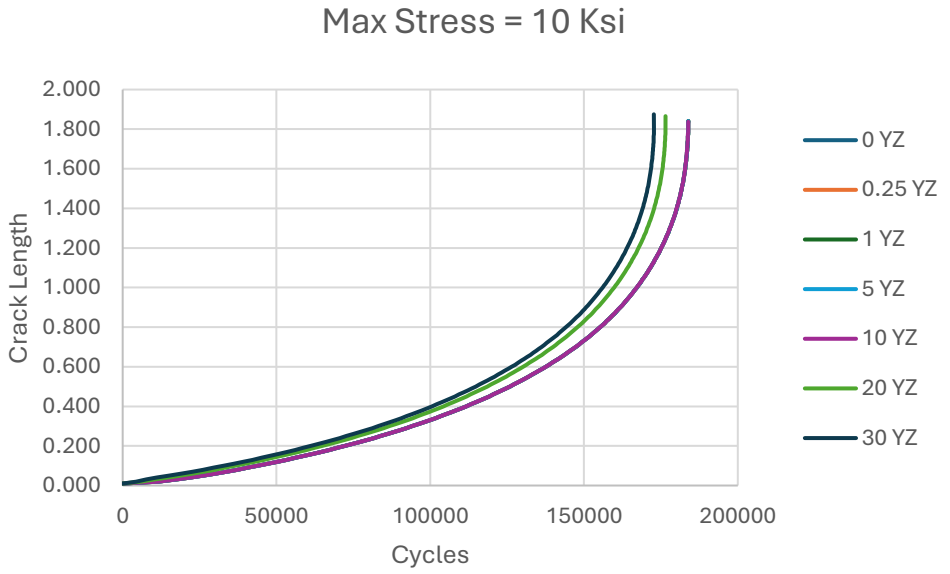
Example Crack Growth Rate Predictions



Initial crack length: 0.01 in (2.54x10-4 m)



Initial yield zone: 0.00289 in (7.341x10-5 m)



Initial yield zone: 0.00036 in (9.144x10-6 m)

Problems with the Yield Zone Approach

- Crack opening tests by R. Sunder have shown that “long cracks” establish closure after growing ¼ yield zone distance into their plastic wake
- Small cracks close very much like large cracks *(Lee JJ, Sharpe WN. Short Fatigue Cracks in Notched Aluminum Specimens. Small Fatigue Cracks: The Metallurgical Society Inc; 1986)*
- An extremely large yield zone fraction (>10) is required to obtain significantly shorter lives when compared to predictions made using “long crack” data
- When a crack is small and operating at low K-values, the local yield zone is very small
- Therefore, small cracks should transition to “long crack” behavior in much less than 10 yield zones and over a much shorter distance than large cracks

Another option is a grain size model?

A physical mechanism for this is not apparent, and grain size is not commonly available in standard LEFM material datasets

Threshold and Small Crack (TASC) Research Program

Defence Science and Technology (DST) Group, Fishermans Bend, VIC, Australia

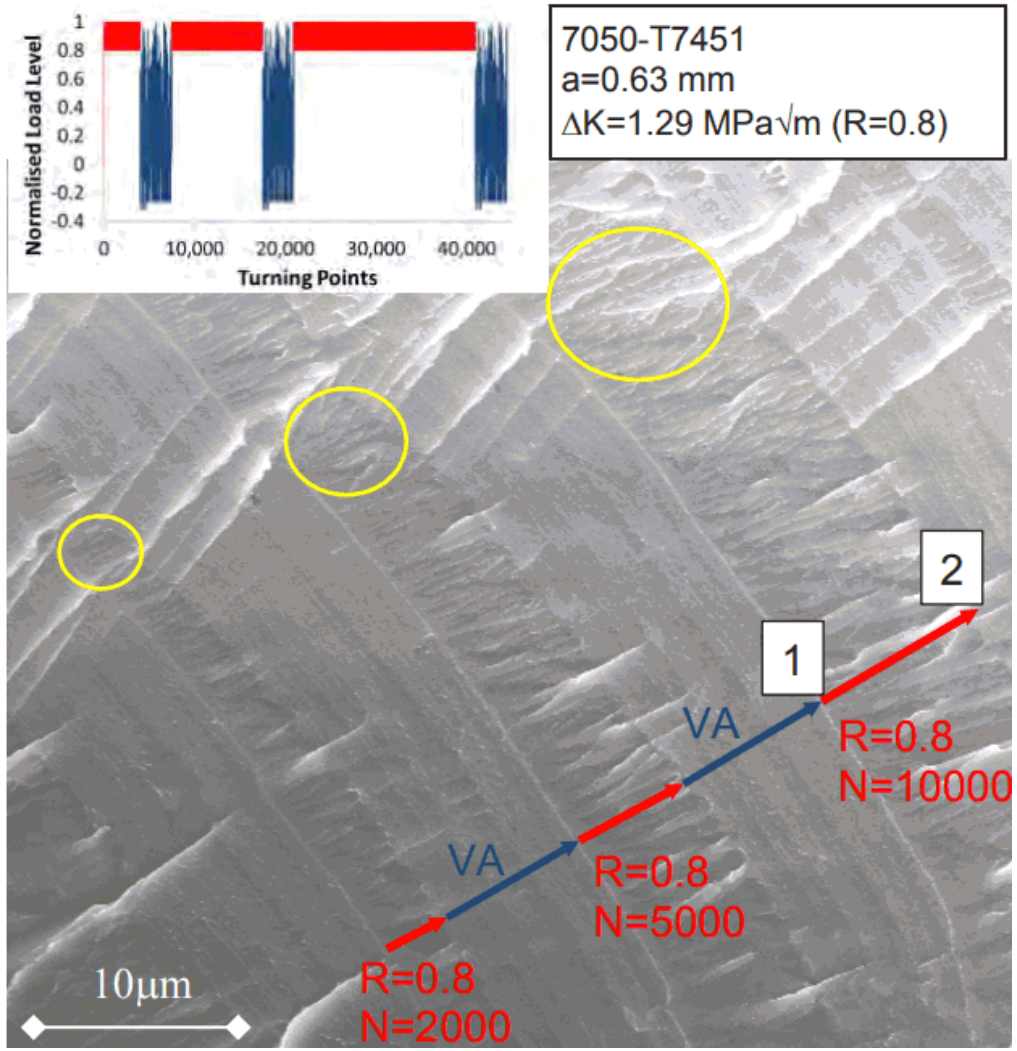
TASC Testing Protocol

Alternating blocks of constant and variable amplitude (wing root bending type) loading were applied to large radius, hourglass shaped test specimens

This was done to facilitate fractographic analysis to measure crack growth for each CA loading block

Naturally occurring cracks originated in the test section by etching and shot peening outside of the test area, or at a laser slot in the test area

Crack growth rate data were extracted using fractography for R = 0.1, 0.5, and 0.8



Small Crack Growth Data for 7050-T7451

Microstructurally small fatigue crack growth rates in aluminium alloys for developing improved predictive models

Madeleine Burchill, Simon Barter, Lok Hin Chan, Michael Jones

MATEC Web of Conferences 165, 13004 (2018)

FATIGUE 2018

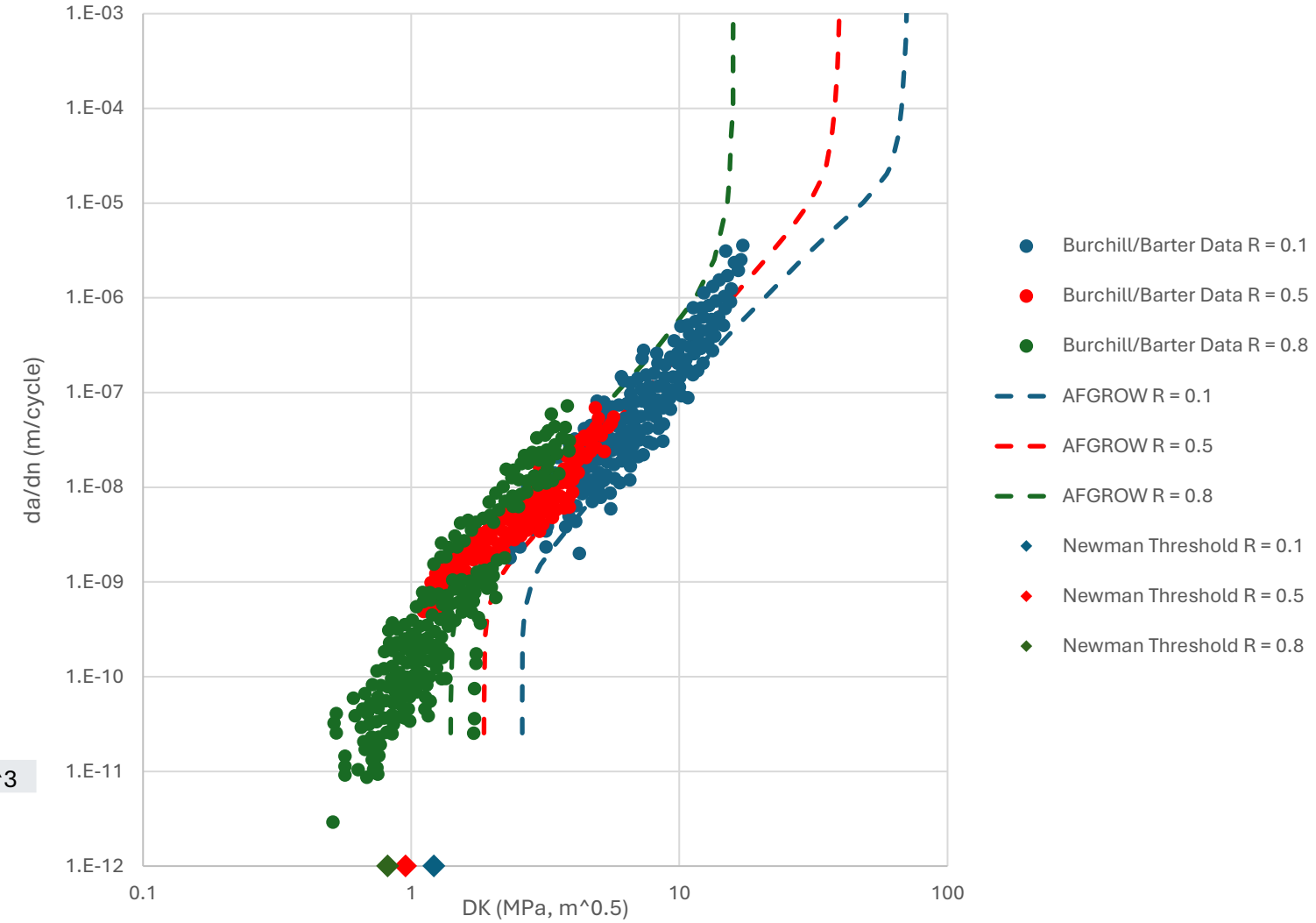
Newman Threshold Equation

$$\Delta K_{th} = 1.2x10^{-5}\ \mathrm{E}\left[\frac{(1-R)}{(1-P_0)}\right]$$

For R > 0, Po = 0.3513 + 0.0303 * R + 0.8607 * R^2 - 0.2672 * R^3

For R <= 0, Po = 0.3513+0.0303 * R

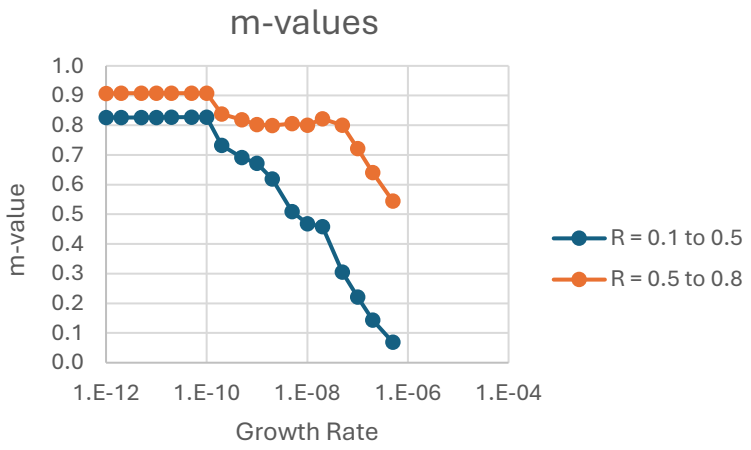
Provided by Dr. J.C. Newman, Jr. 8/2026



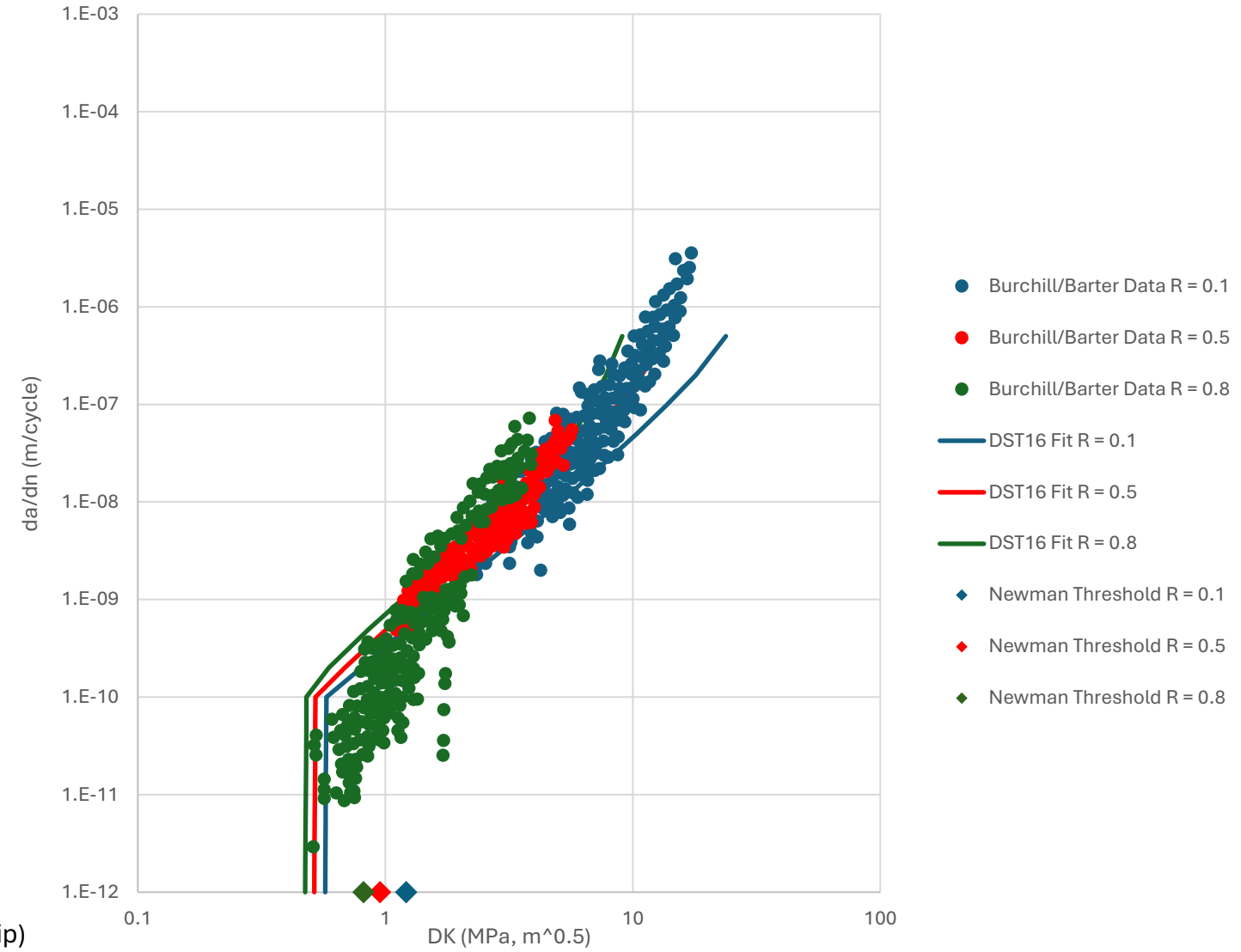
DST16 Tabular Fit for Small Cracks

DST16 tabular fit (Burchill/Barter) was “tuned” to predict life from a small crack with a fighter type spectrum (moderate stress ratio content)

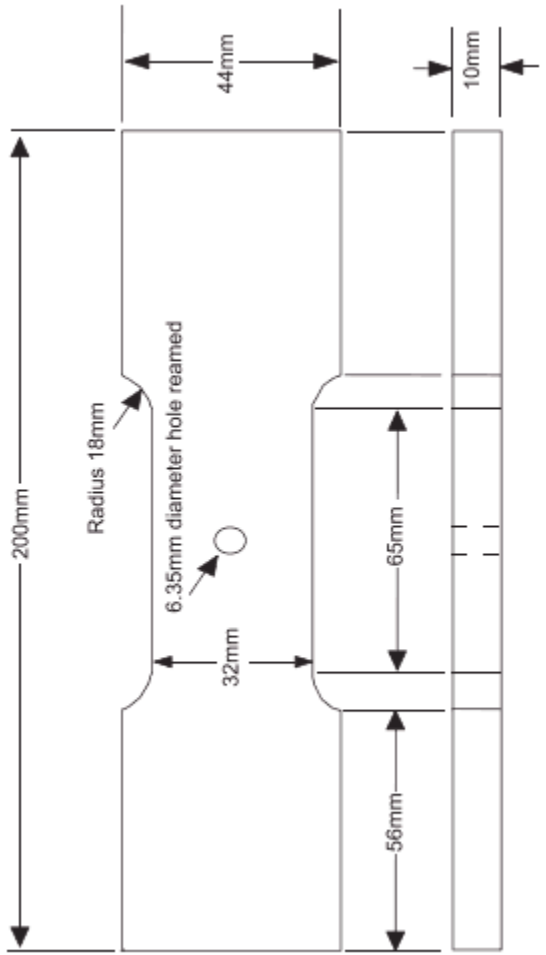
m-plots for the tabular fit do not follow usual trends seen in actual test data

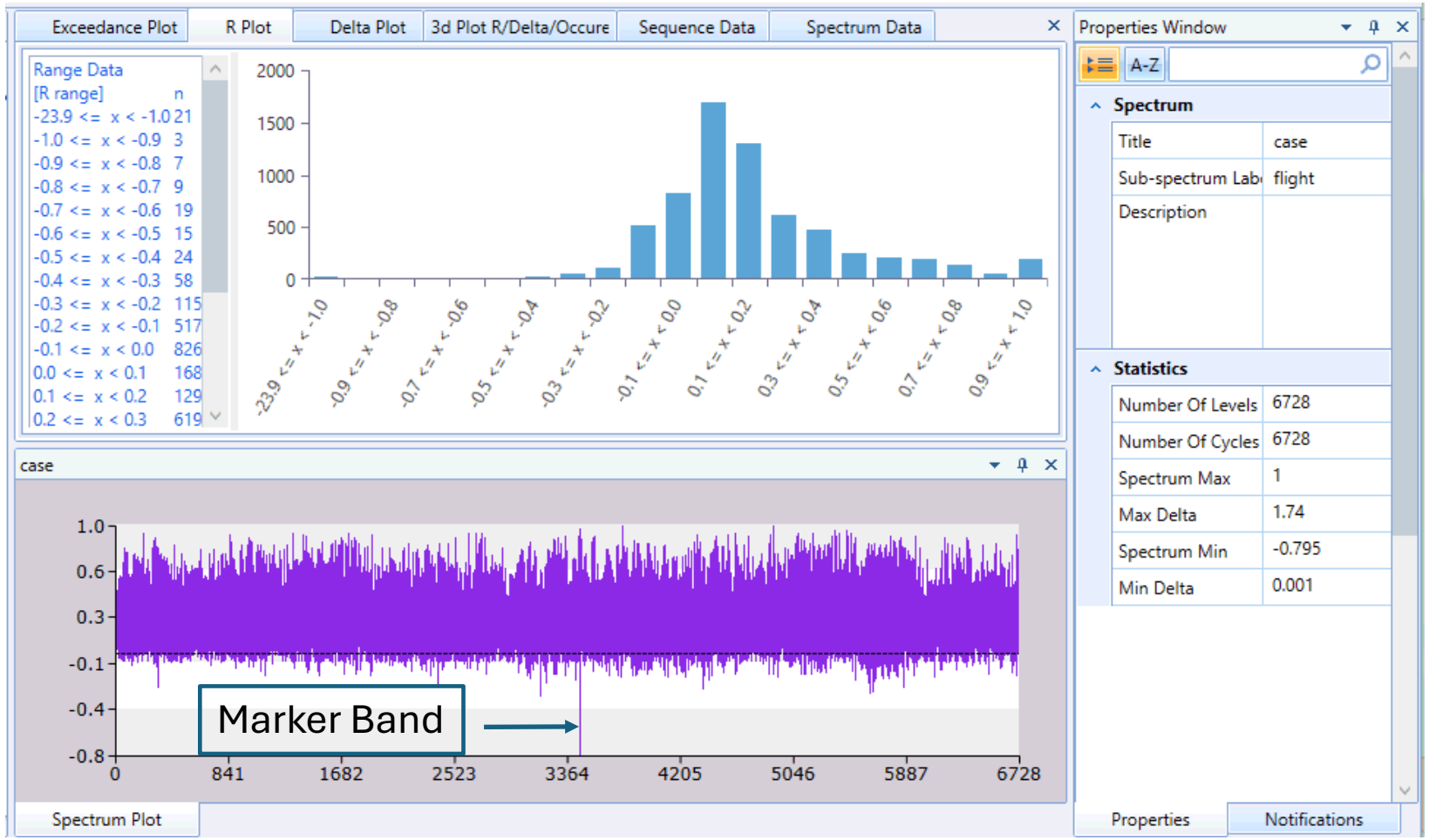


$\Delta K_1(1-R_1)^{(m-1)} = \Delta K_2(1-R_2)^{(m-1)}$ (Walker relationship)



Validation Testing (with and w/o CX)





A fractographic study of fatigue crack growth from a cold expanded fastener hole at an engineering “crack initiation” scale

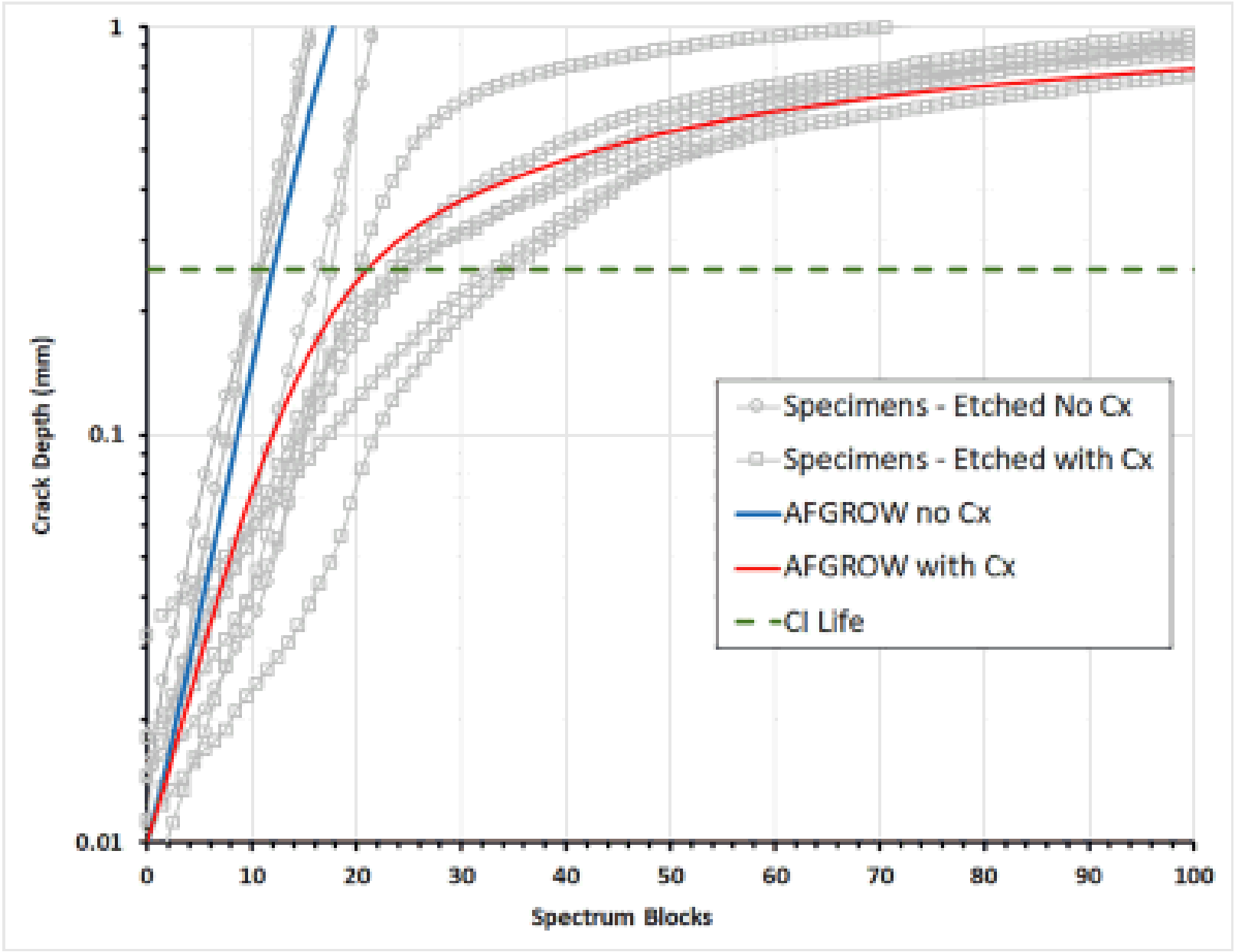
Ben Main | David Russell | Simon Barter
Fatigue Fract Eng Mater Struct. 2024;1–14.

Results

w/o Retardation Model

Spectrum Designed to Limit Retardation Effects

AFGROW life predictions were made using a double, symmetric surface crack at a hole

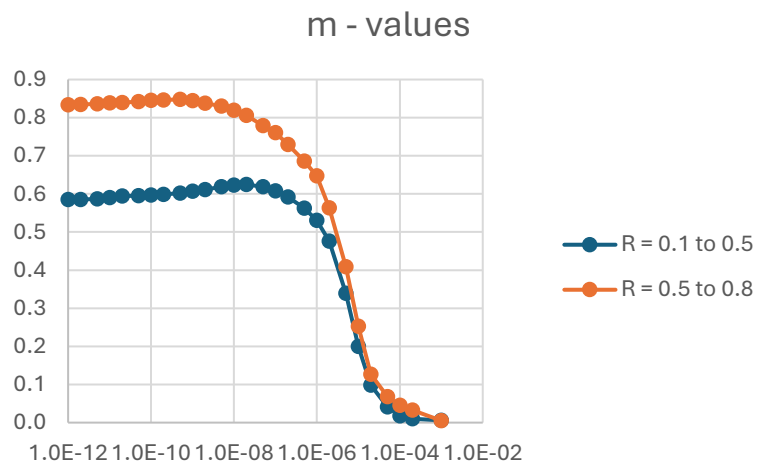


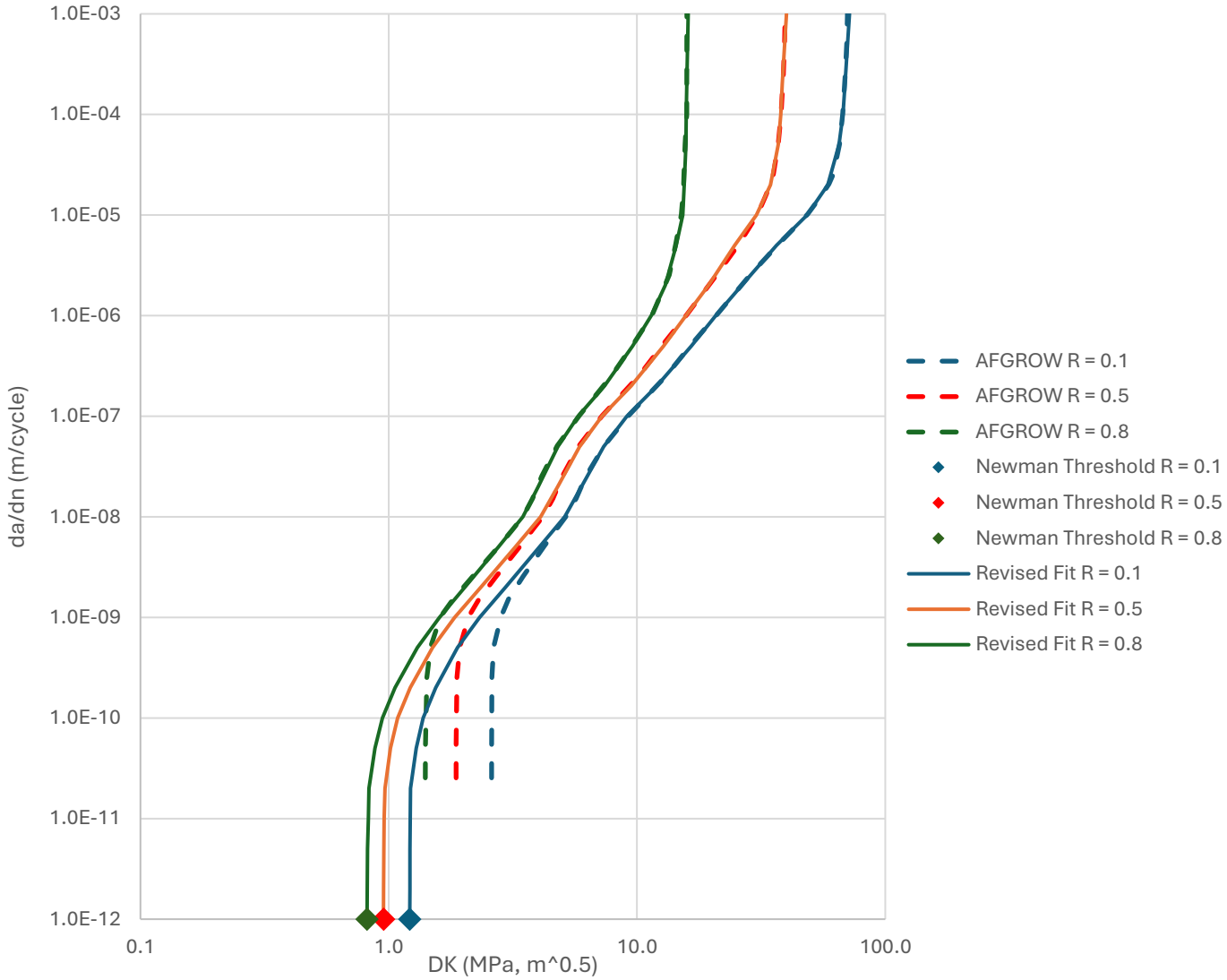
Revised Tabular Fit Using the Newman Threshold Equation

The revised fit was merged to the existing AFGROW tabular fit for 7050-T451 L-T

The merge was made at the point of constraint loss (~1E-8 m/cycle)

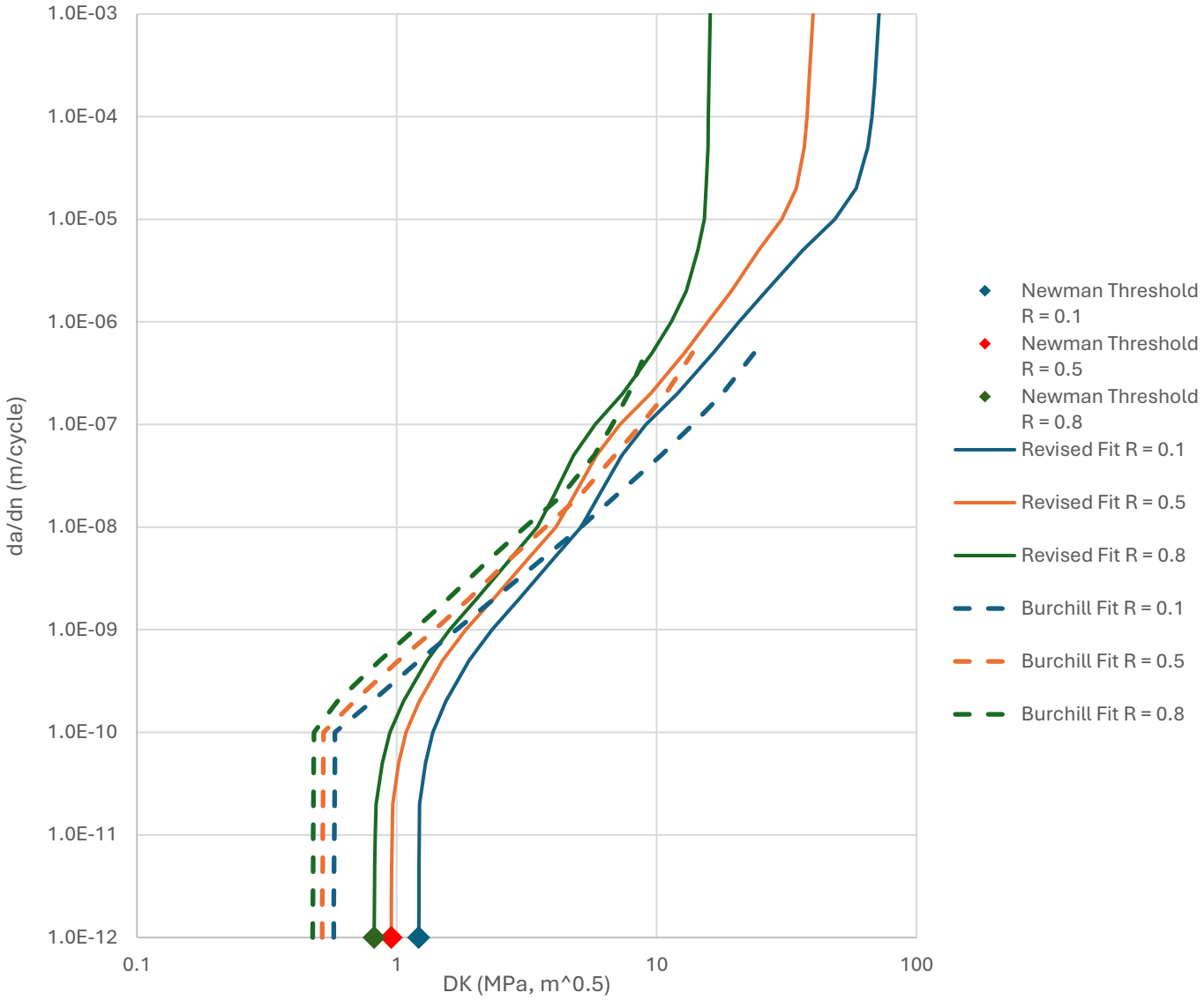
m-curves show little/no “fanning” in region I





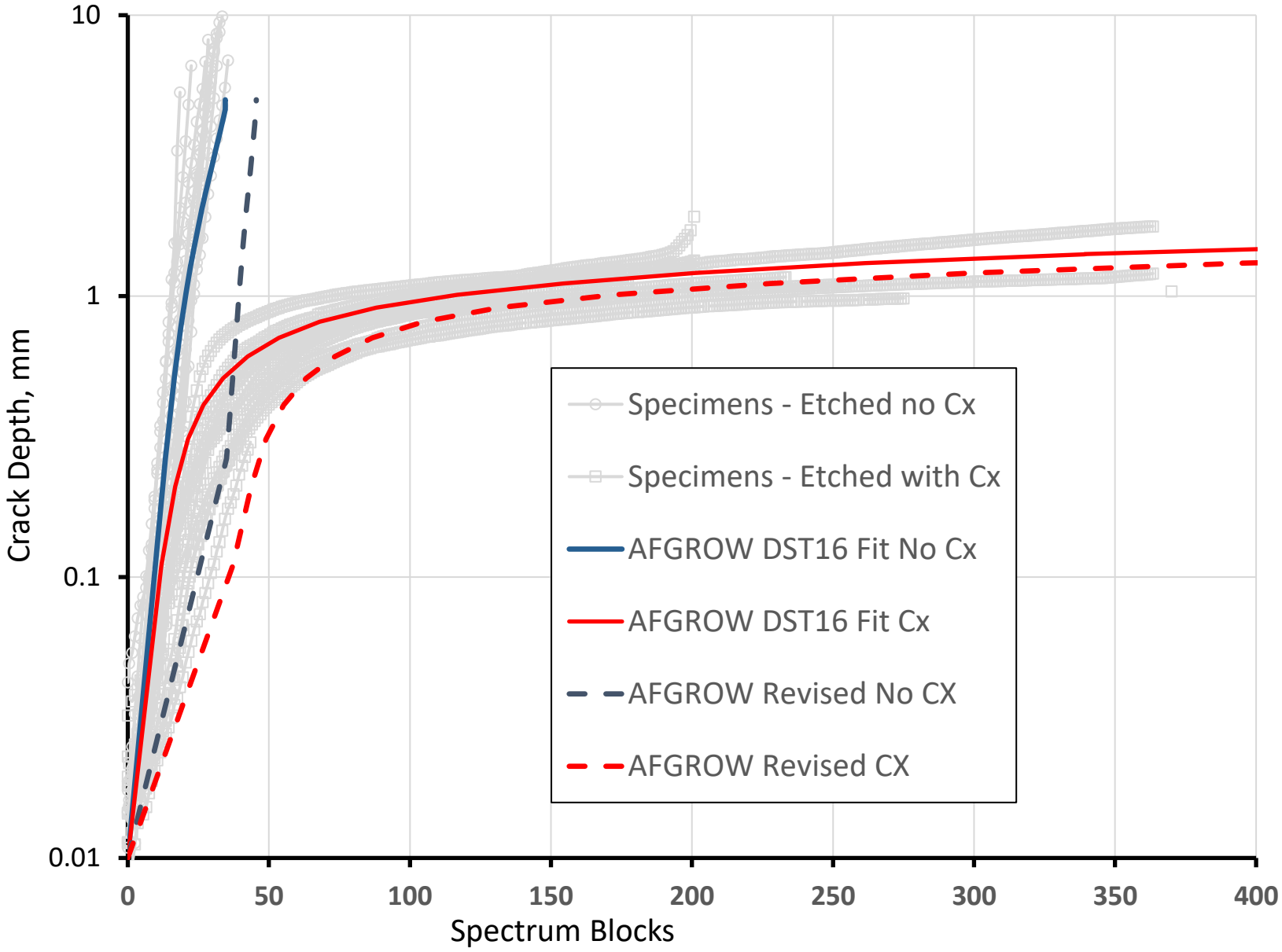
Revised vs. DST16 Fit

The Newman equation threshold is significantly higher than the Burchill fit



Revised Results

Non-Conservative
(as expected)



Crack Face Surface Roughness Effect

Crack growth rate tests of 7050-T7451 (using compression pre-cracking methods) required a low alpha value (1.3) to collapse the data to ΔKeff due to relatively high crack face surface roughness.

da/dN	ΔKeff
1.0E-10	0.570
1.3E-09	1.000
6.0E-09	2.900
2.0E-08	4.200
8.0E-08	6.000
1.0E-06	12.500
1.0E-05	22.000

"Crack-Closure Behavior of 7050 Aluminum Alloy near Threshold Conditions for Wide Range in Load Ratios and Constant Kmax Tests"

J. C. Newman, Jr. and Y. Yamada, Department of Aerospace Engineering, Mississippi State University, Mississippi State, MS 39762 and J. A. Newman NASA Langley Research Center, Hampton, VA 23681

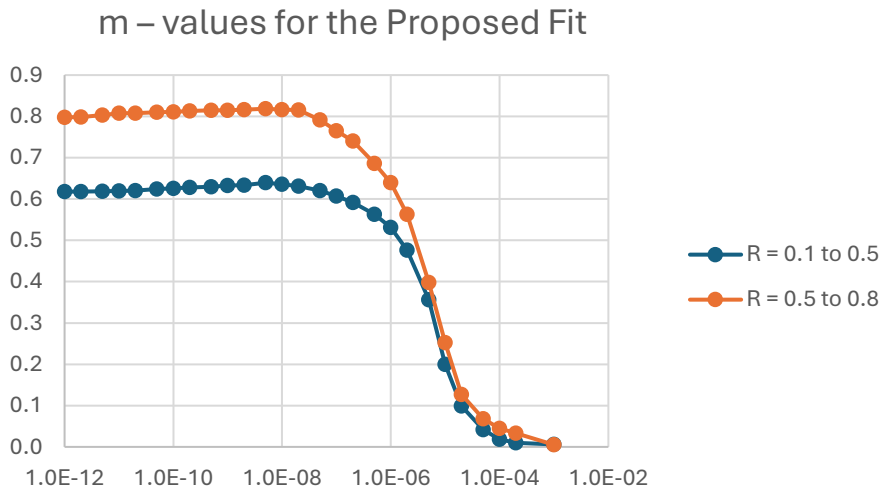
ASTM/ESIS Symposium on Fatigue & Fracture Mechanics, May 20-22, 2009

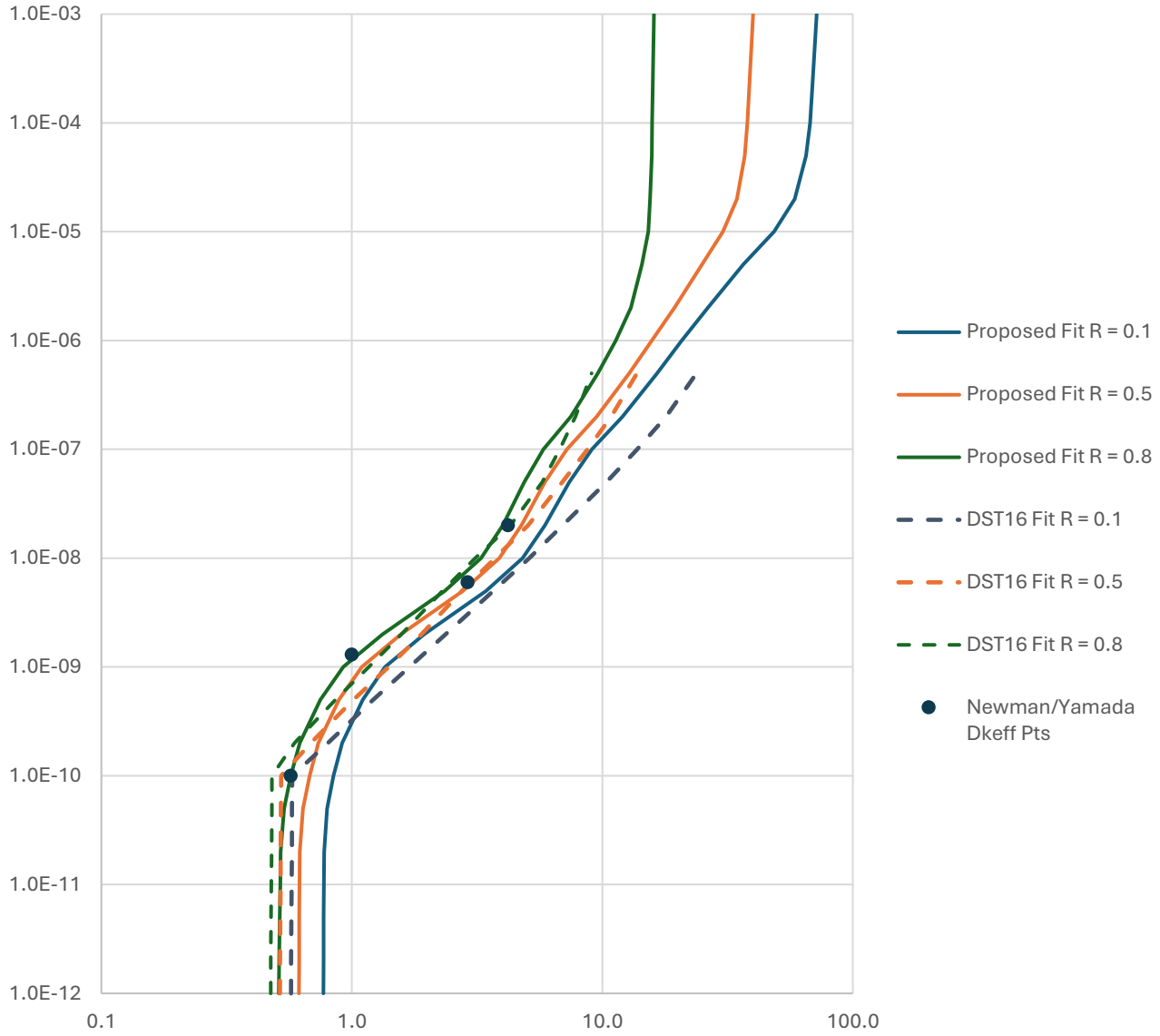
Second Revision of the 7050-T7451 Tabular Fit

The new curve for R = 0.8 was assumed to be equivalent to ΔKeff below 1E-8 m/cycle

The curves for R = 0.5 and 0.1 were estimated using the same initial m-values calculated using the Newman Threshold equation

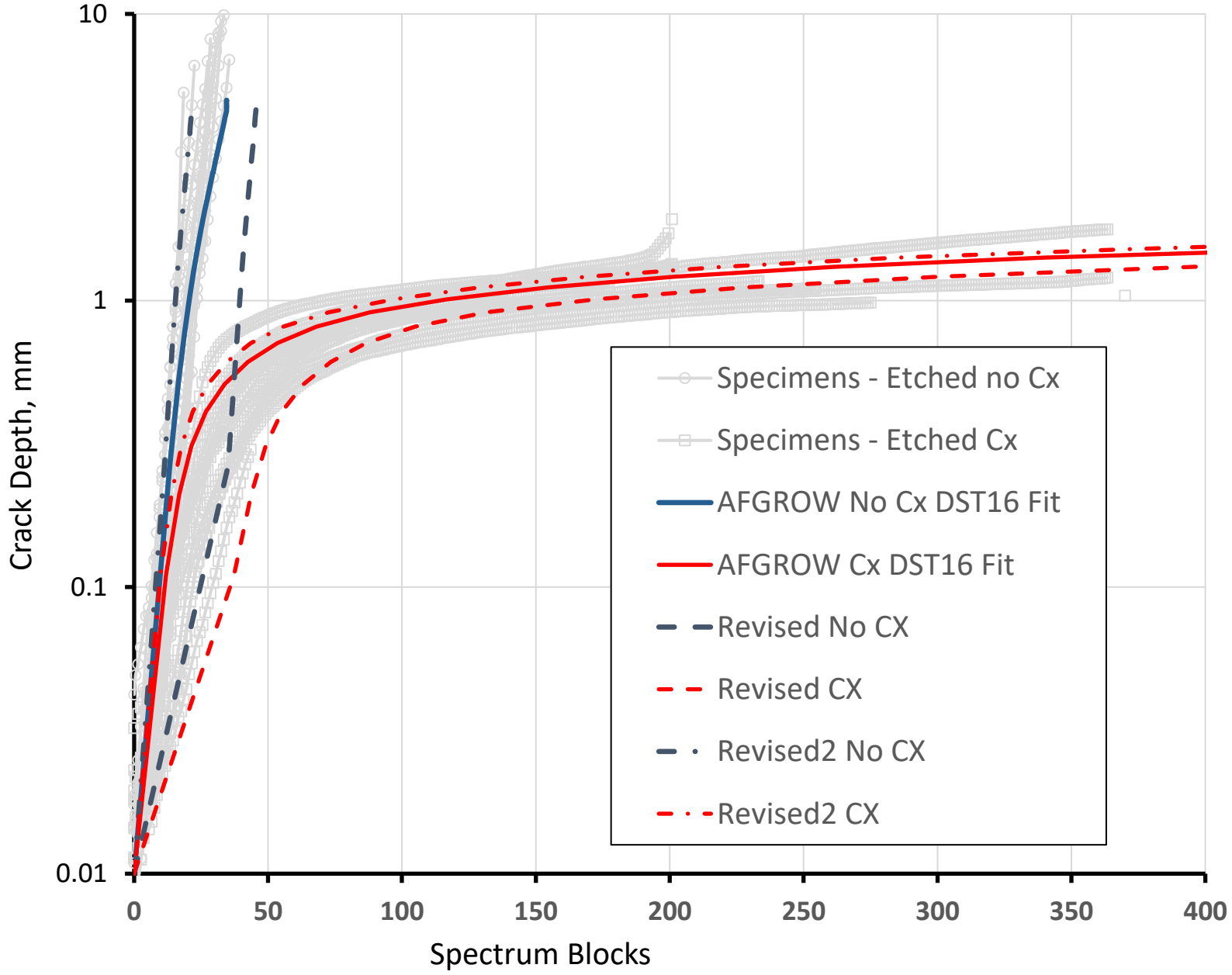
Each curve was smoothly transitioned to the existing AFGROW tabular fit above 1E-8 m/cycle



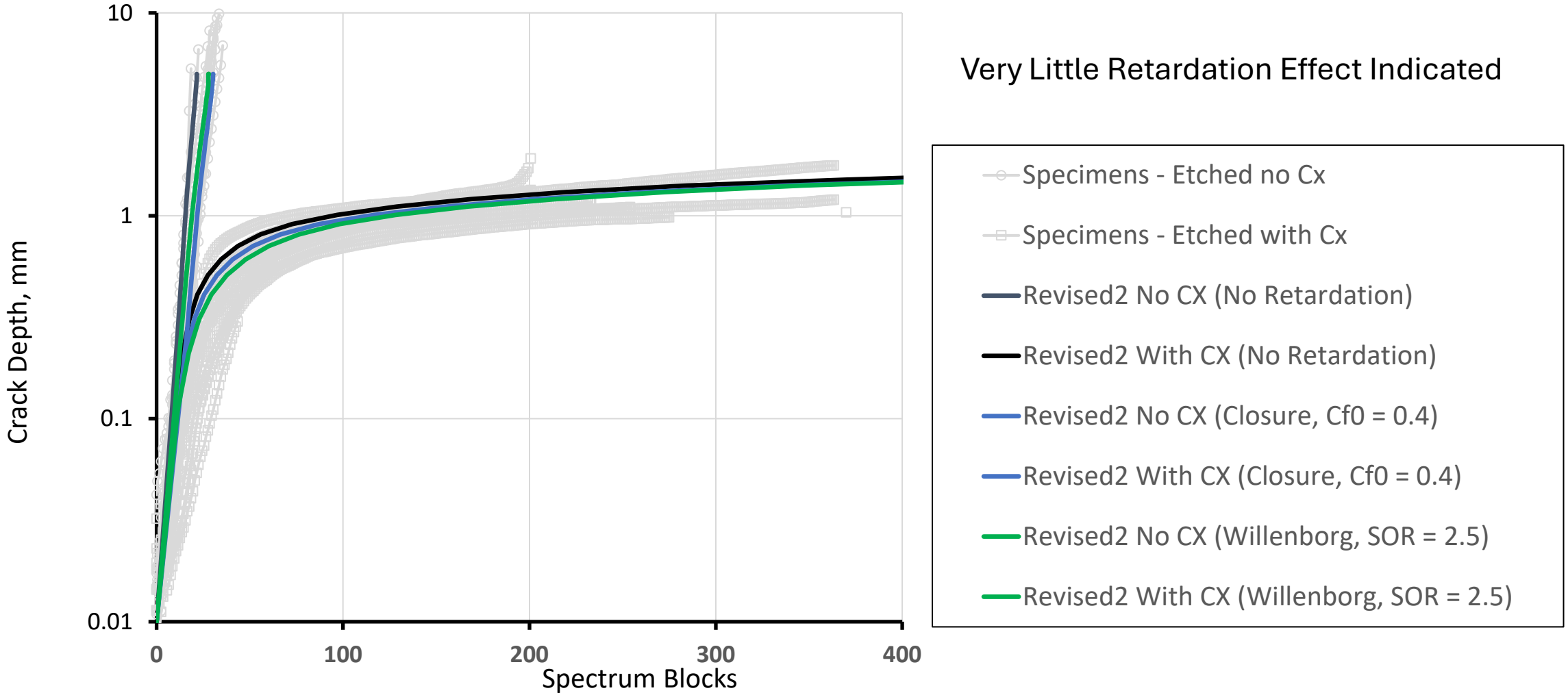


Second Revision Results

Retardation effects are minimal for this loading spectrum, but including retardation effects would increase the predicted lives ~10 – 15% depending on the model



Predictions with Retardation Effects



Updating Tabular Growth Rate Data

- More validation testing is clearly necessary, but it would be possible to use the Newman threshold equation to modify existing tabular crack growth rate data in AFGROW (eliminate the need to use different crack growth rate curves for small and long cracks)
- Is there a demand for this?
- How to manage repeatability for predictions made with current crack growth rate curves?
- Materials that present significant crack face surface roughness are likely to have lower threshold values than calculated using the Newman threshold equation, but the prediction results would be conservative

Discussion