STRESS INTENSITY FACTORS AND FATIGUE CRACK GROWTH OF IRREGULAR PLANAR CRACKS SUBJECTED TO ARBITRARY MODE I STRESS FIELDS

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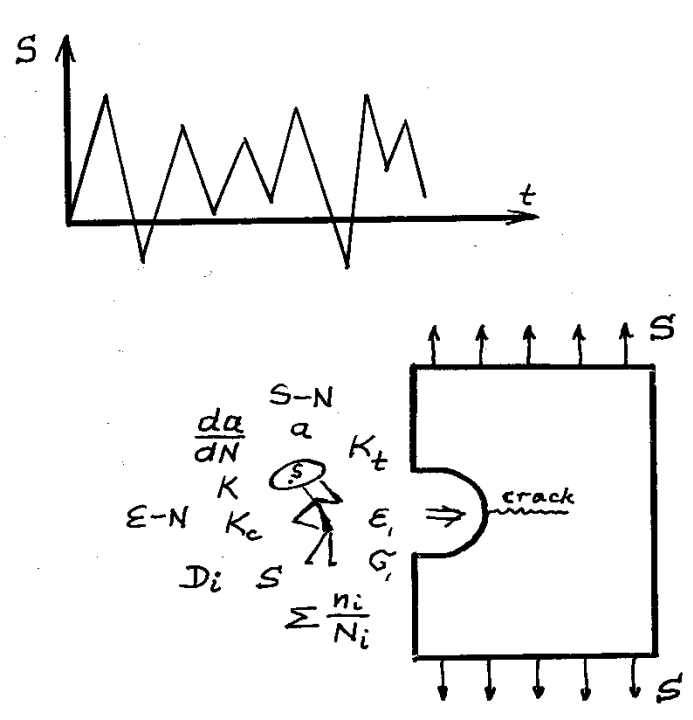
AFGROW Workshop
Salt Lake City, Utah
September 10-11, 2013

FATIGUE !!!!!

$N_i = ?$ $N_p = ?$ $N_T = ?$



Fatigued kitten….

Metal Fatigue is a process which causes premature irreversible damage or failure of a component subjected to repeated loading.

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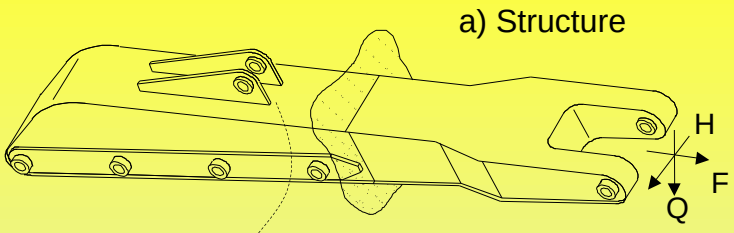
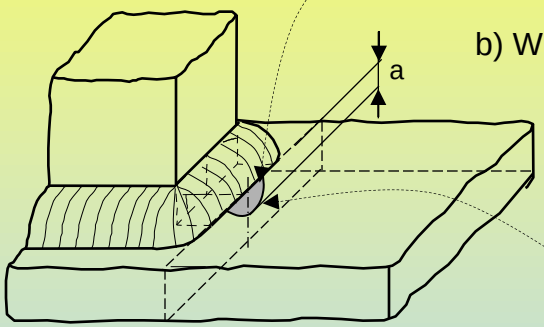
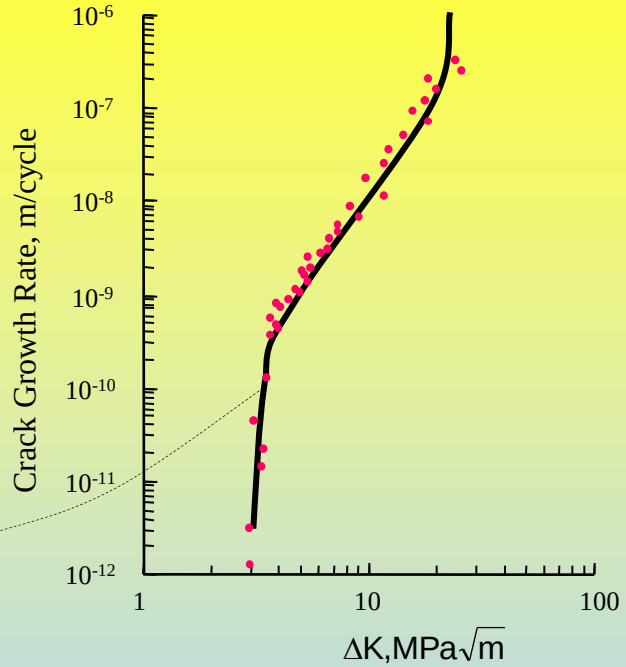
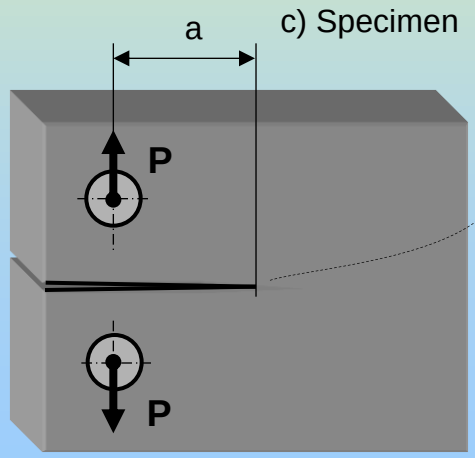
The Similitude Concept in the da/dN – ΔK Method





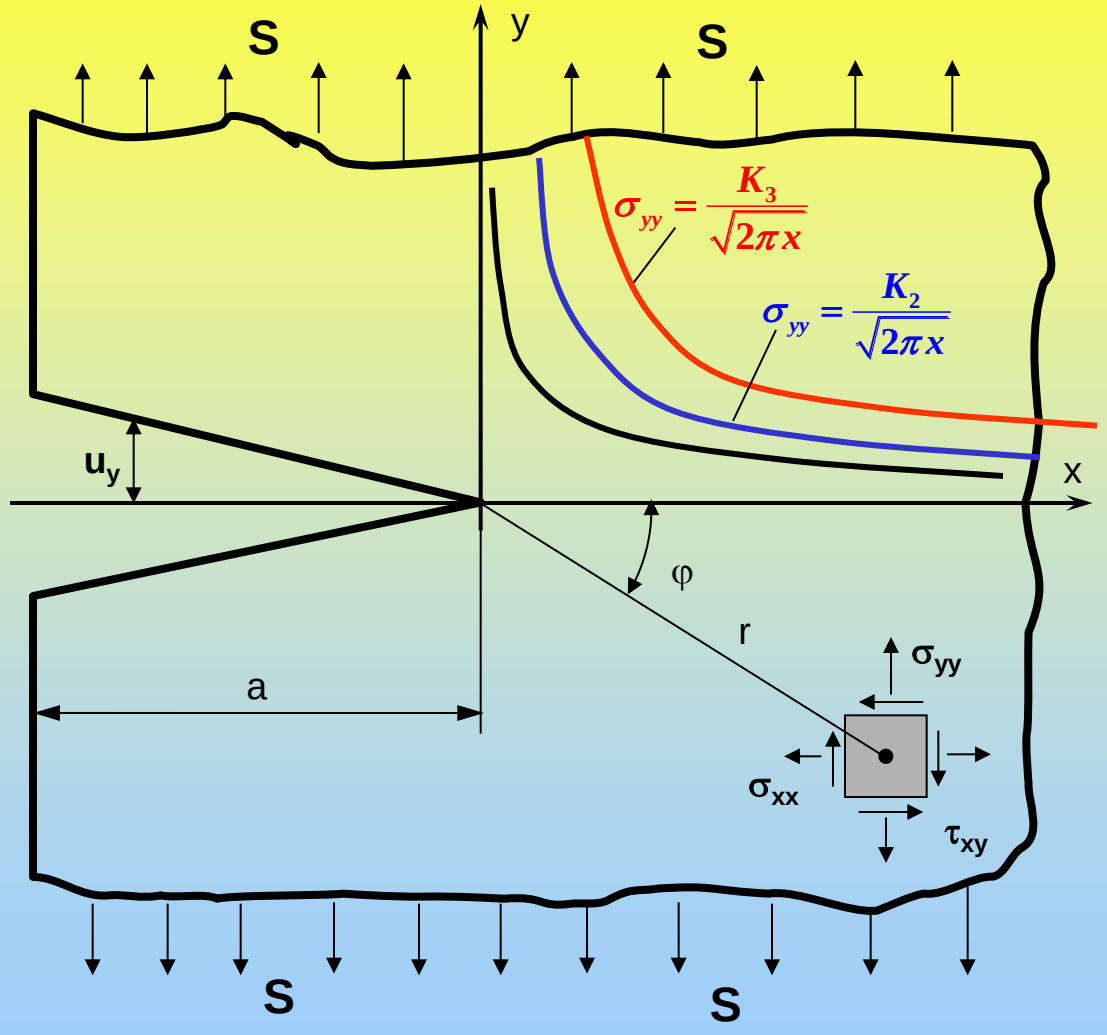


ΔK



The Similitude Concept states that if the stress intensity K for a crack in the actual component and in the test specimen are the same, then the fatigue crack growth response in the component and in the specimen will also be the same and can be described by the material fatigue crack growth curve da/dN - ΔK.

Crack tip stress dependence on the stress intensity factor K



$K_1 < K_2 < K_3$

Stress components, σ_{ij}, at the crack tip depend on the stress intensity factors K_I which is influenced by:

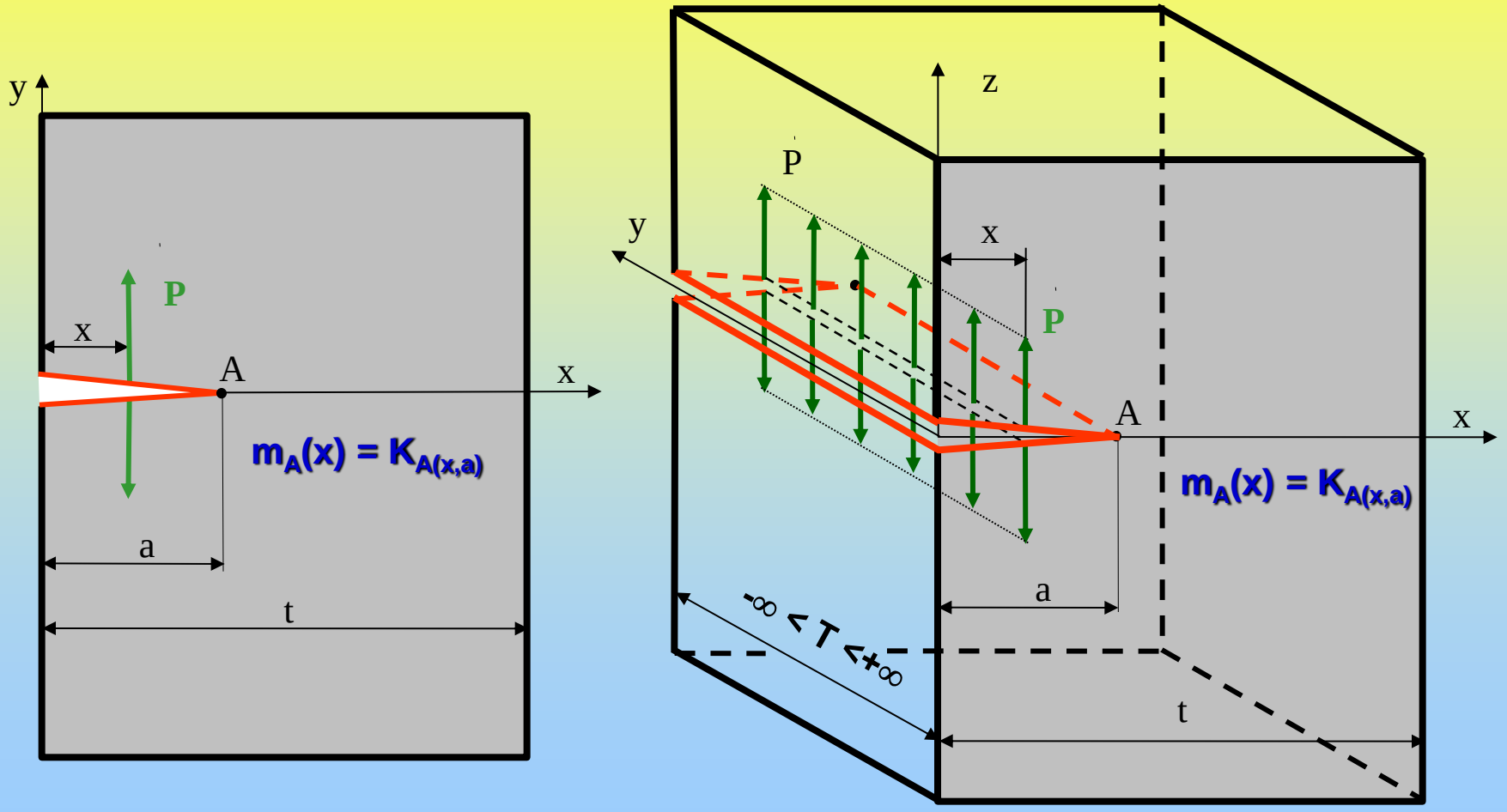
- the load, S

-crack dimensions, a

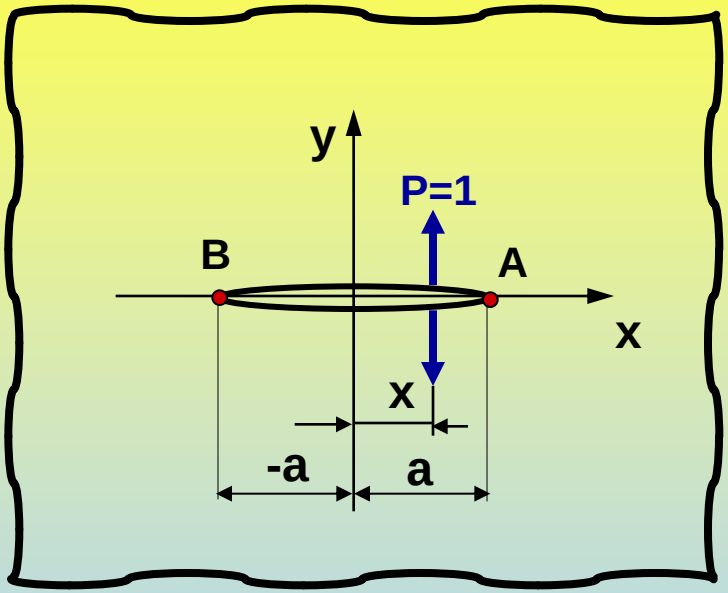
-geometry, Y

$$K = S\sqrt{\pi a}Y$$

The Weight Function WF=m(x) is equal to the SIF K(x,a) induced by unit point load P (2 – D) or unit line load (3 – D) applied to the crack surface at position ‘x’.



The Weight Functions, m_A (m_B), and corresponding Stress Intensity Factors, K_A (K_B), for a pair of splitting forces applied to the surfaces of a through crack in an infinite plate



Through crack in an infinite plate:

$$m_A(x,a) = \frac{1}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}$$

$$m_B(x,a) = \frac{1}{\sqrt{\pi a}}\sqrt{\frac{a-x}{a+x}}$$

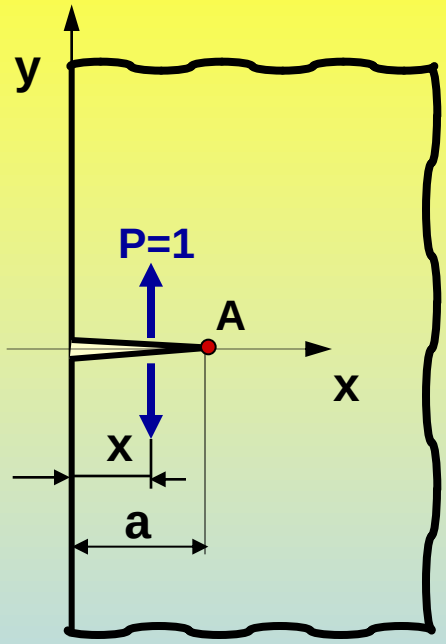
Stress intensity factors for crack crack tips A and B respectively:

$$K_A = m_A(x,a,P) = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}} \qquad K_B = m_B(x,a,P) = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a-x}{a+x}}$$

Green's Functions

F. Erdogan, G.Sih 1962

The Weight Function, m_A, and corresponding Stress Intensity Factor, K_A, for a pair of splitting forces applied to the surfaces of an edge crack in semi-infinite plate



Edge crack in semi-infinite plate:

$$m_A(x,a) = \frac{2P}{\sqrt{\pi a}} \cdot \frac{1.3 - 3\left(\frac{x}{a}\right)^{\frac{5}{2}}}{\sqrt{1-\left(\frac{x}{a}\right)^2}}$$

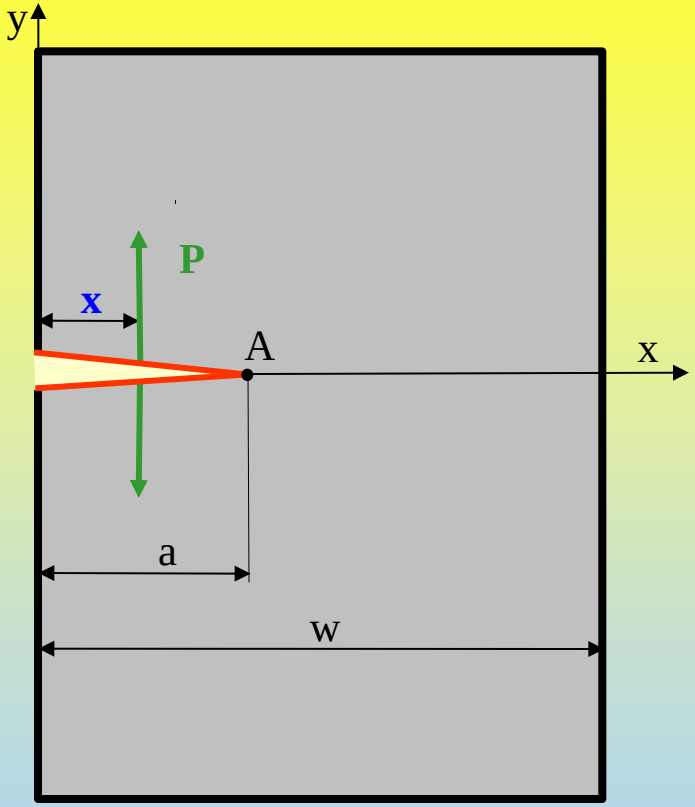
Stress intensity factor for the crack tip A

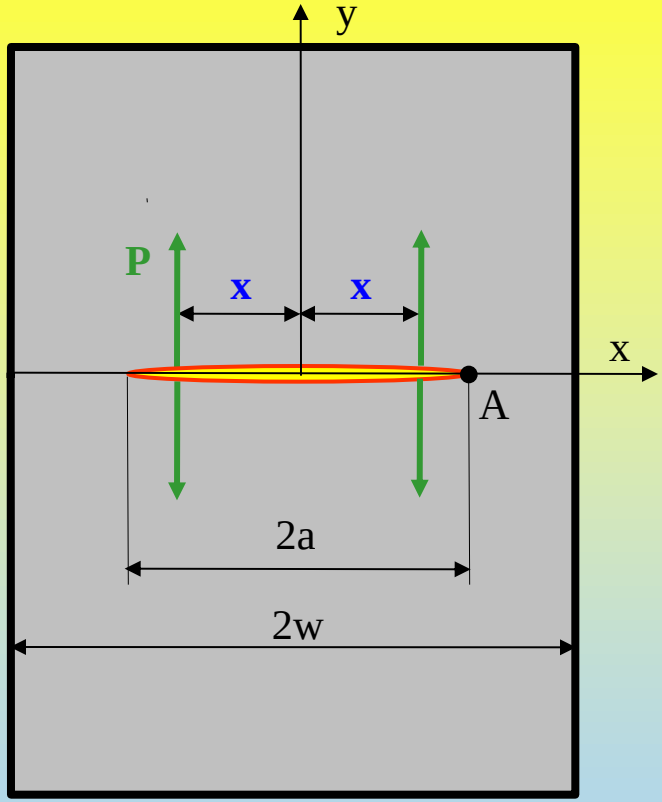
$$K_A = m_A(x,a,P) = \frac{2P}{\sqrt{\pi a}} \cdot \frac{1.3 - 3\left(\frac{x}{a}\right)^{\frac{5}{2}}}{\sqrt{1-\left(\frac{x}{a}\right)^2}}$$

Green's Function

H.Tada, 1971

Geometrical Parameters and Notation for the Universal Weight Function





$$K_A^{P_1} = m\left(x, a, P\right) = \frac{2}{\sqrt{2\pi\left(a-x\right)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

Central through crack in a finite width plate subjected to symmetric loading

$$M_1 = 0.06987 + 0.40117\left(\frac{a}{w}\right) - 5.5407\left(\frac{a}{w}\right)^2 + 50.0886\left(\frac{a}{w}\right)^3 - 200.699\left(\frac{a}{w}\right)^4 + 395.552\left(\frac{a}{w}\right)^5 - 377.939\left(\frac{a}{w}\right)^6 + 140.218\left(\frac{a}{w}\right)^7$$

$$M_2 = -0.09049 - 2.14886\left(\frac{a}{w}\right) + 22.5325\left(\frac{a}{w}\right)^2 - 89.6553\left(\frac{a}{w}\right)^3 + 210.599\left(\frac{a}{w}\right)^4 - 239.445\left(\frac{a}{w}\right)^5 + 111.128\left(\frac{a}{w}\right)^6$$

$$M_3 = 0.427216 + 2.56001\left(\frac{a}{w}\right) - 29.6349\left(\frac{a}{w}\right)^2 + 138.40\left(\frac{a}{w}\right)^3 - 347.255\left(\frac{a}{w}\right)^4 + 457.128\left(\frac{a}{w}\right)^5 - 295.882\left(\frac{a}{w}\right)^6 + 68.1575\left(\frac{a}{w}\right)^7$$

Edge crack in a finite width plate

$$M_1 = 0.0719768 - 1.51346\left(\frac{a}{w}\right) - 61.1001\left(\frac{a}{w}\right)^2 + 1554.95\left(\frac{a}{w}\right)^3 - 14583.8\left(\frac{a}{w}\right)^4 + 71590.7\left(\frac{a}{w}\right)^5$$
$$-205384\left(\frac{a}{w}\right)^6 + 356469\left(\frac{a}{w}\right)^7 - 368270\left(\frac{a}{w}\right)^8 + 208233\left(\frac{a}{w}\right)^9 - 49544\left(\frac{a}{w}\right)^{10}$$

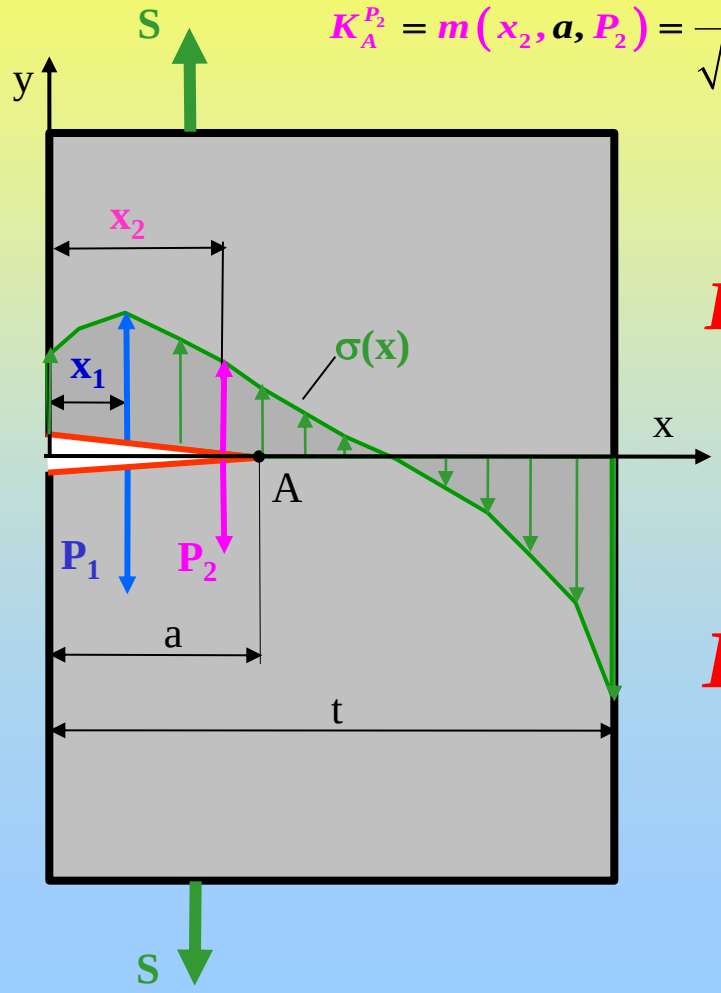
$$M_2 = 0.246984 + 6.47543\left(\frac{a}{w}\right) + 176.457\left(\frac{a}{w}\right)^2 - 4058.76\left(\frac{a}{w}\right)^3 + 37303.8\left(\frac{a}{w}\right)^4 - 181755\left(\frac{a}{w}\right)^5$$
$$+520551\left(\frac{a}{w}\right)^6 - 904370\left(\frac{a}{w}\right)^7 + 936863\left(\frac{a}{w}\right)^8 - 531940\left(\frac{a}{w}\right)^9 + 127291\left(\frac{a}{w}\right)^{10}$$

$$M_3 = 0.529659 - 22.3235\left(\frac{a}{w}\right) + 532.074\left(\frac{a}{w}\right)^2 - 5479.53\left(\frac{a}{w}\right)^3 + 28592.2\left(\frac{a}{w}\right)^4 - 81388.6\left(\frac{a}{w}\right)^5$$
$$+128746\left(\frac{a}{w}\right)^6 - 106246\left(\frac{a}{w}\right)^7 + 35780.7\left(\frac{a}{w}\right)^8$$

The use of Weight Functions for calculating Stress Intensity Factors

$$K_A^{P_1} = m(x_1, a, P_1) = \frac{2P_1}{\sqrt{2\pi(a - x_1)}}\left[1 + M_1\left(1 - \frac{x_1}{a}\right)^{\frac{1}{2}} + M_2\left(1 - \frac{x_1}{a}\right)^{1} + M_3\left(1 - \frac{x_1}{a}\right)^{\frac{3}{2}}\right]$$

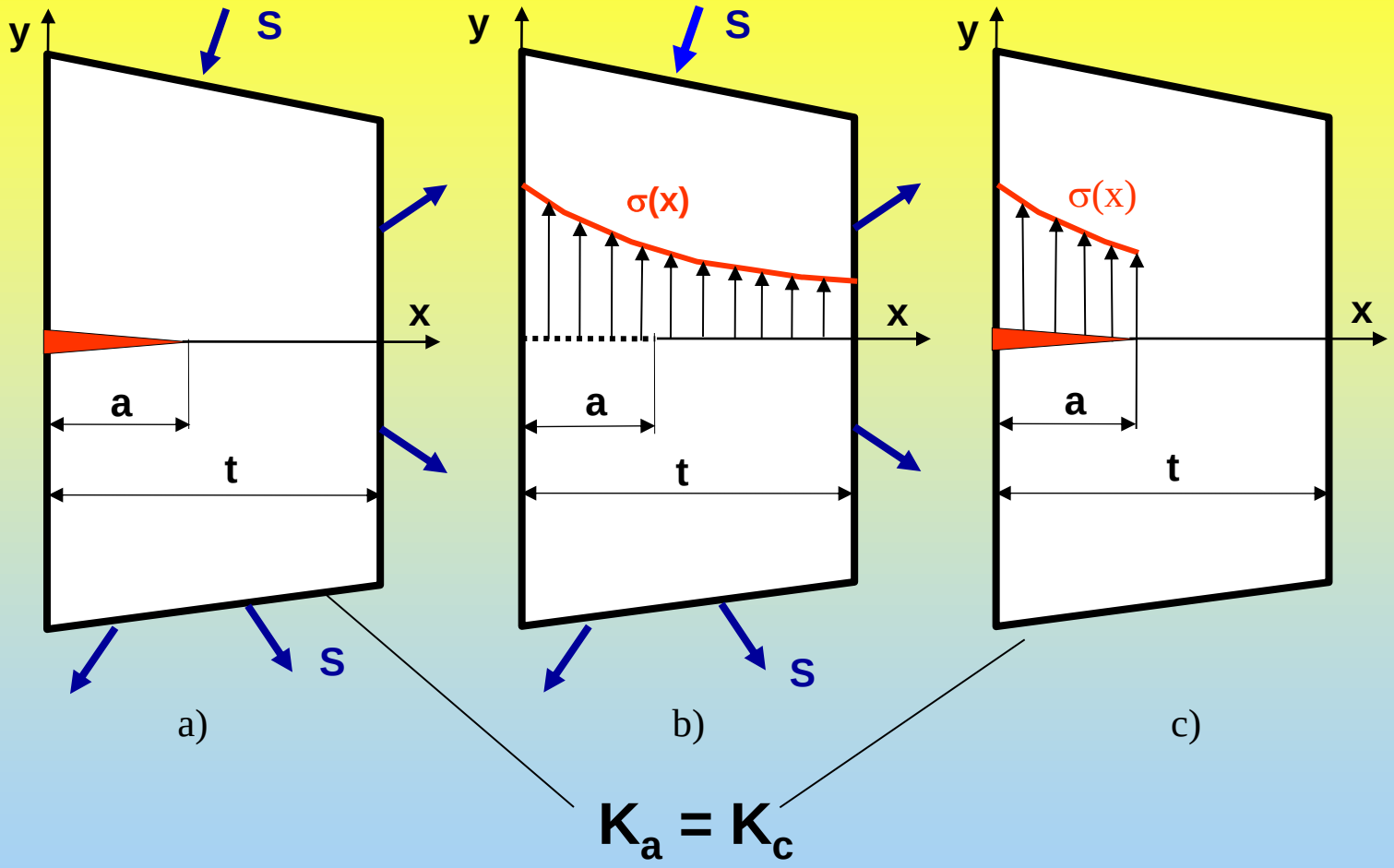
$$K_A^{P_2} = m(x_2, a, P_2) = \frac{2P_2}{\sqrt{2\pi(a - x_2)}}\left[1 + M_1\left(1 - \frac{x_2}{a}\right)^{\frac{1}{2}} + M_2\left(1 - \frac{x_2}{a}\right)^{1} + M_3\left(1 - \frac{x_2}{a}\right)^{\frac{3}{2}}\right]$$



$$K_A^{P_1+P_2} = m(x_1, a, P_1) + m(x_2, a, P_2)$$

$$K_A^{\sigma(x)} = \int_0^a \left[\sigma(x)\, m(x, a)\right] dx$$

The Superposition Principle for Cracks and Stress Intensity Factors



$$K_a = K_c$$

The **Stress Intensity Factor** for any loading case is equal to the stress intensity factor obtained by applying to the crack faces the stress ditribution that used to be there when there was no crack.

Stepwise Procedure for Calculating Stress Intensity Factors by the Weight Function Method

1. Calculate stress distribution σ(x) in the prospective crack plane in the absence of the crack (un-cracked body, linear elastic analysis).

$$\sigma(x) = f(\sigma_0, x)$$

2. Apply the stress distribution σ(x) to the crack surface as tractions.

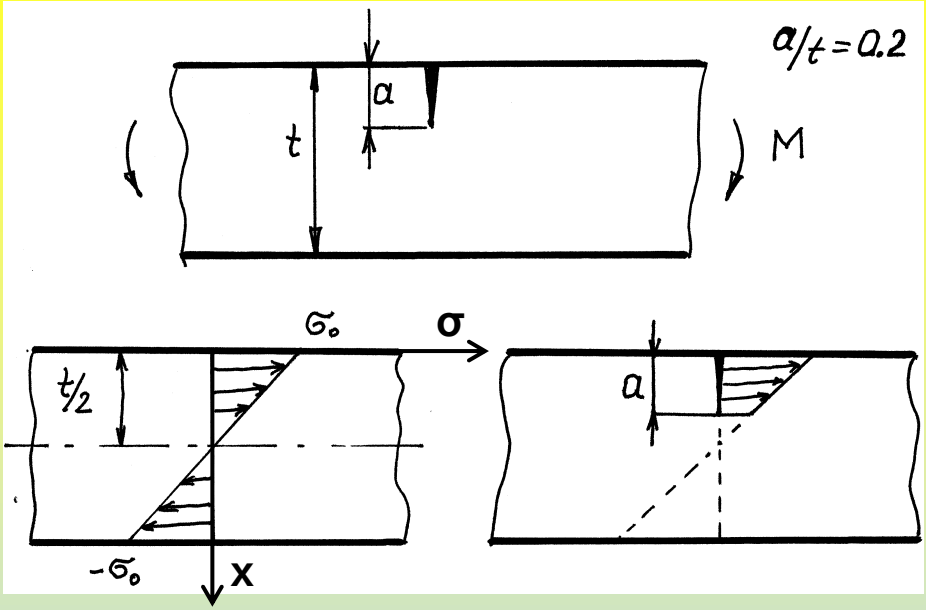
3. Choose appropriate weight function, i.e. parameters M_1, M_2 and M_3.

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{1/2} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{3/2}\right]$$

4. Integrate the product of the stress distribution σ(x) and the weight function m(x,a).

$$K = \int_{-a}^{+a} \sigma(x)\, m(x,a)\, dx$$

An Edge Crack in a Finite Thickness Plate under Bending Load - *Example*



Step 1

Step 2

Step 1 $\sigma(x)=\sigma_0\left(1-\frac{2x}{t}\right);\quad \sigma_0=\frac{6M}{B\cdot t^2};\quad \frac{a}{t}=0.2;\quad B=thickness$

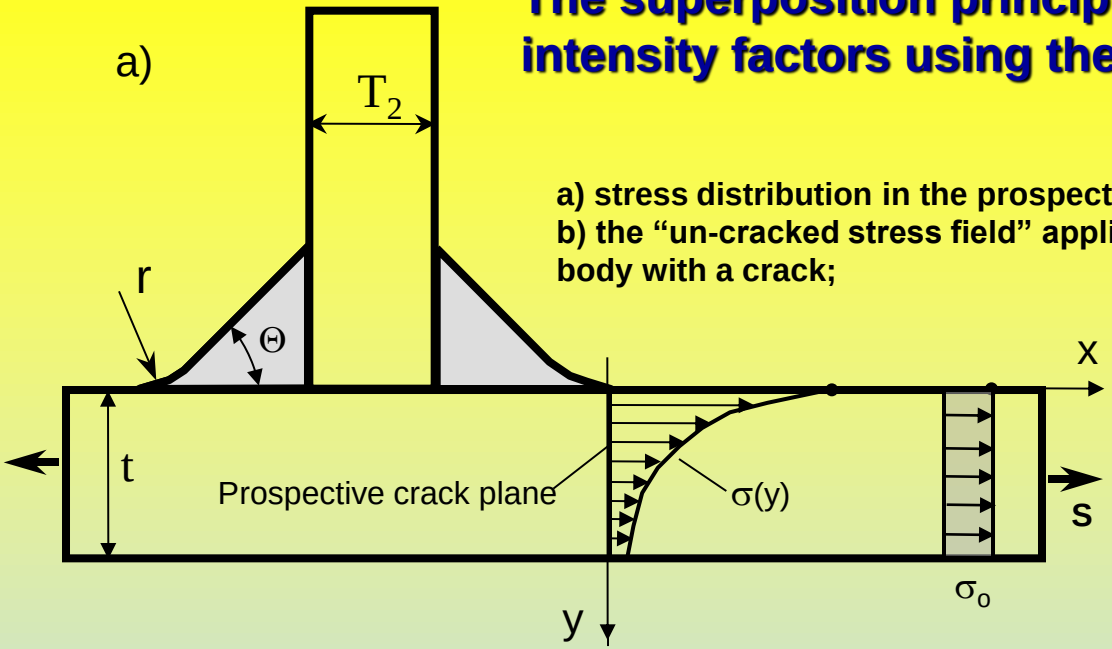
Step 3 $$m(x,a)=\frac{2}{\sqrt{2\pi(a-x)}}\left[1-0.083\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+1.533\left(1-\frac{x}{a}\right)^{1}+0.263\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

Step 4 $$K=\int_0^a \sigma(x)\,m(x,a)\,dx=\int_0^{a=0.2t}\sigma_0\left(1-\frac{2x}{t}\right)\frac{2}{\sqrt{2\pi(a-x)}}\left[1-0.083\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+1.533\left(1-\frac{x}{a}\right)^{1}+0.263\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]dx$$

WF : $K=1.052\sigma_0\sqrt{\pi a}$; *Handbook (Murakami)* : $K=1.035\sigma_0\sqrt{\pi a}$; *Difference* < 2%

The superposition principle for calculation of stress intensity factors using the weight function approach;

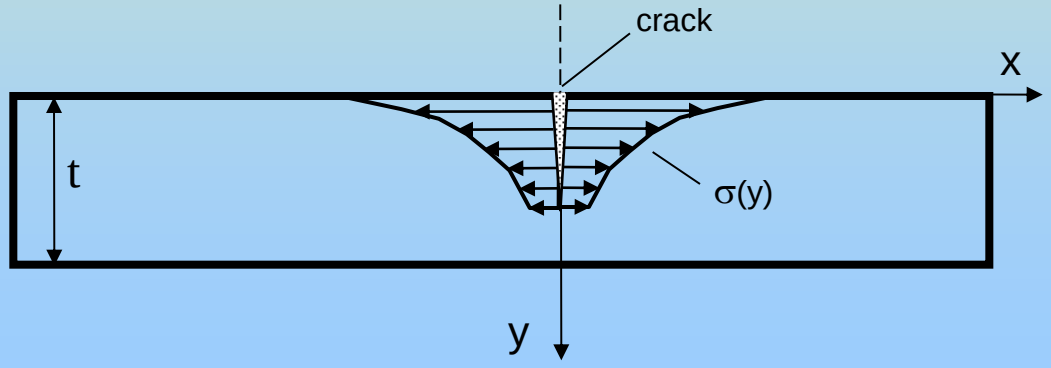
a)



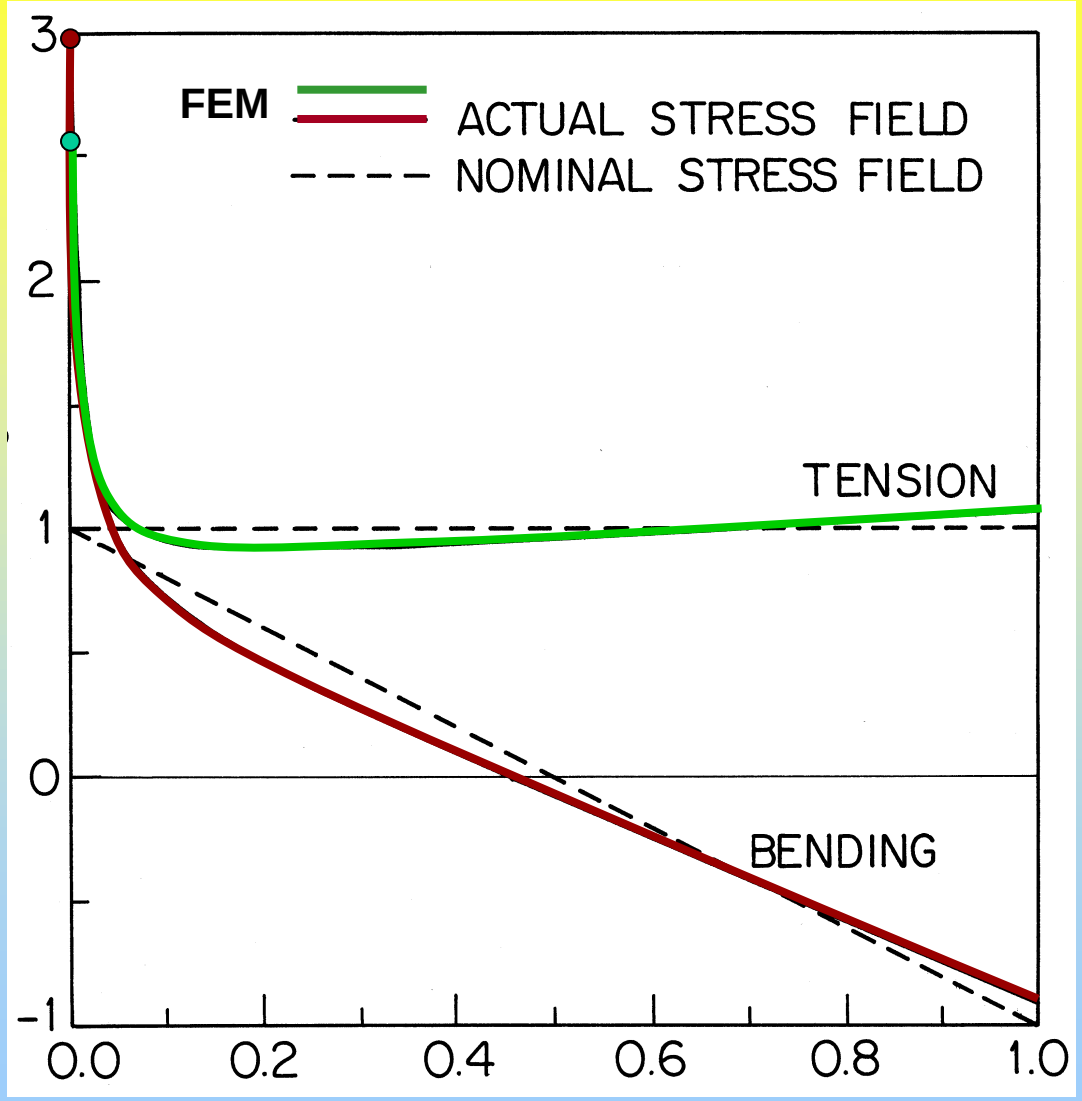
a) stress distribution in the prospective crack plane in the un-cracked body;
b) the "un-cracked stress field" applied to the crack surfaces of identical body with a crack;

b)

$$K_I = \int_0^a m\left(y, \frac{a}{t}\right) \cdot \sigma(y) \cdot dy$$

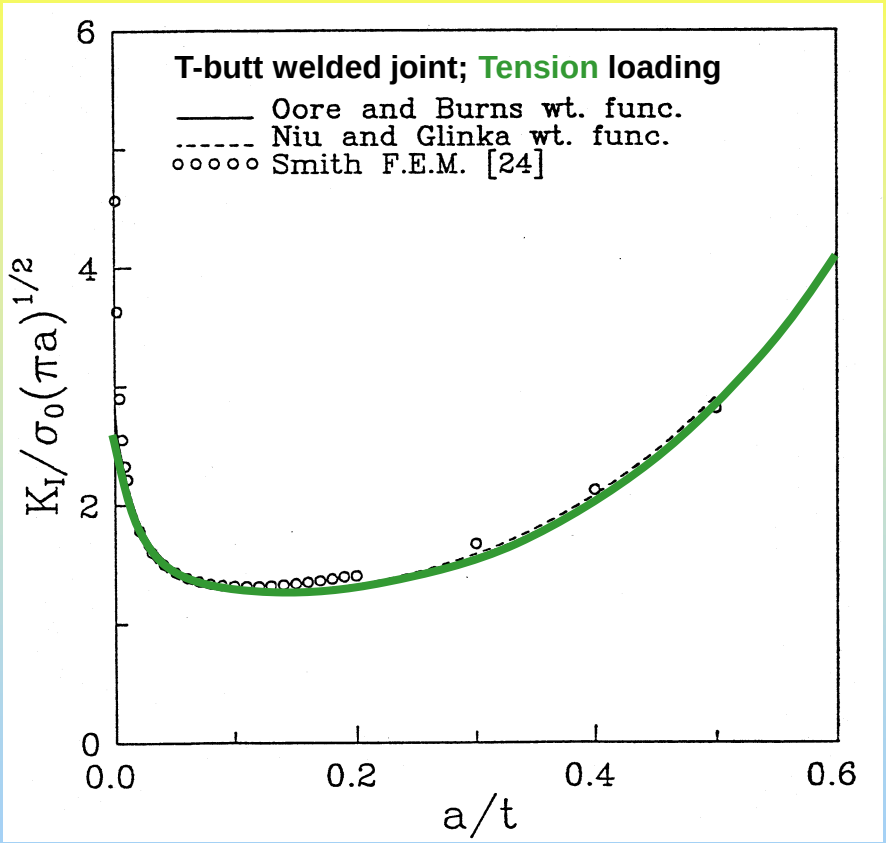


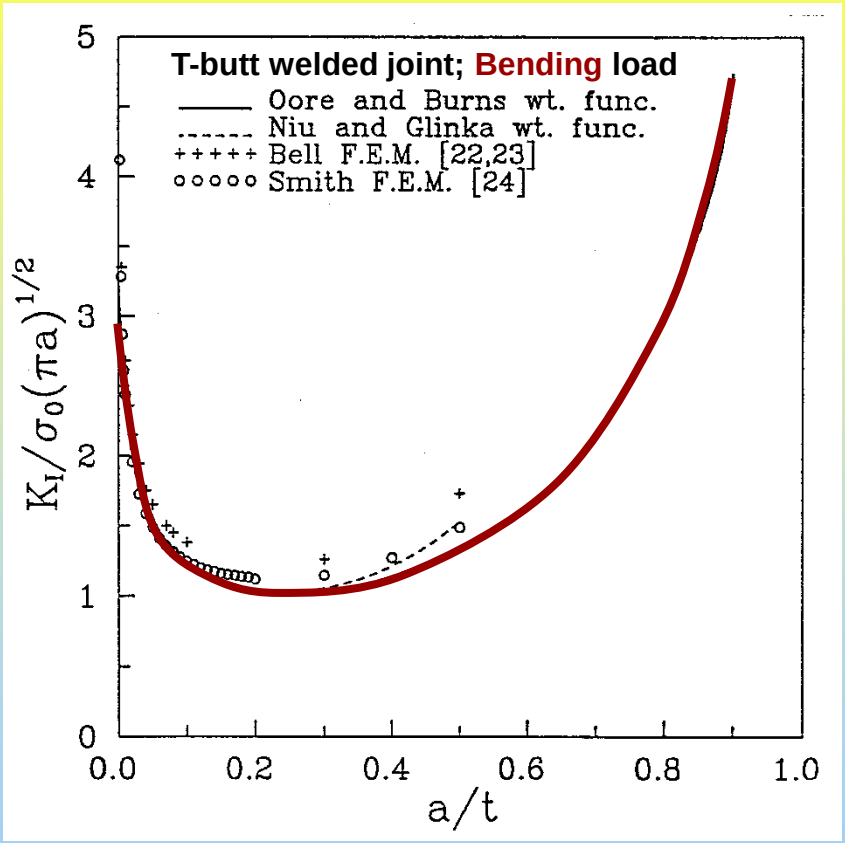
Through the plate thickness stress distributions in a T-butt weldment obtained for r/t = 1/25, Θ = 45° (in the weld toe cross section)



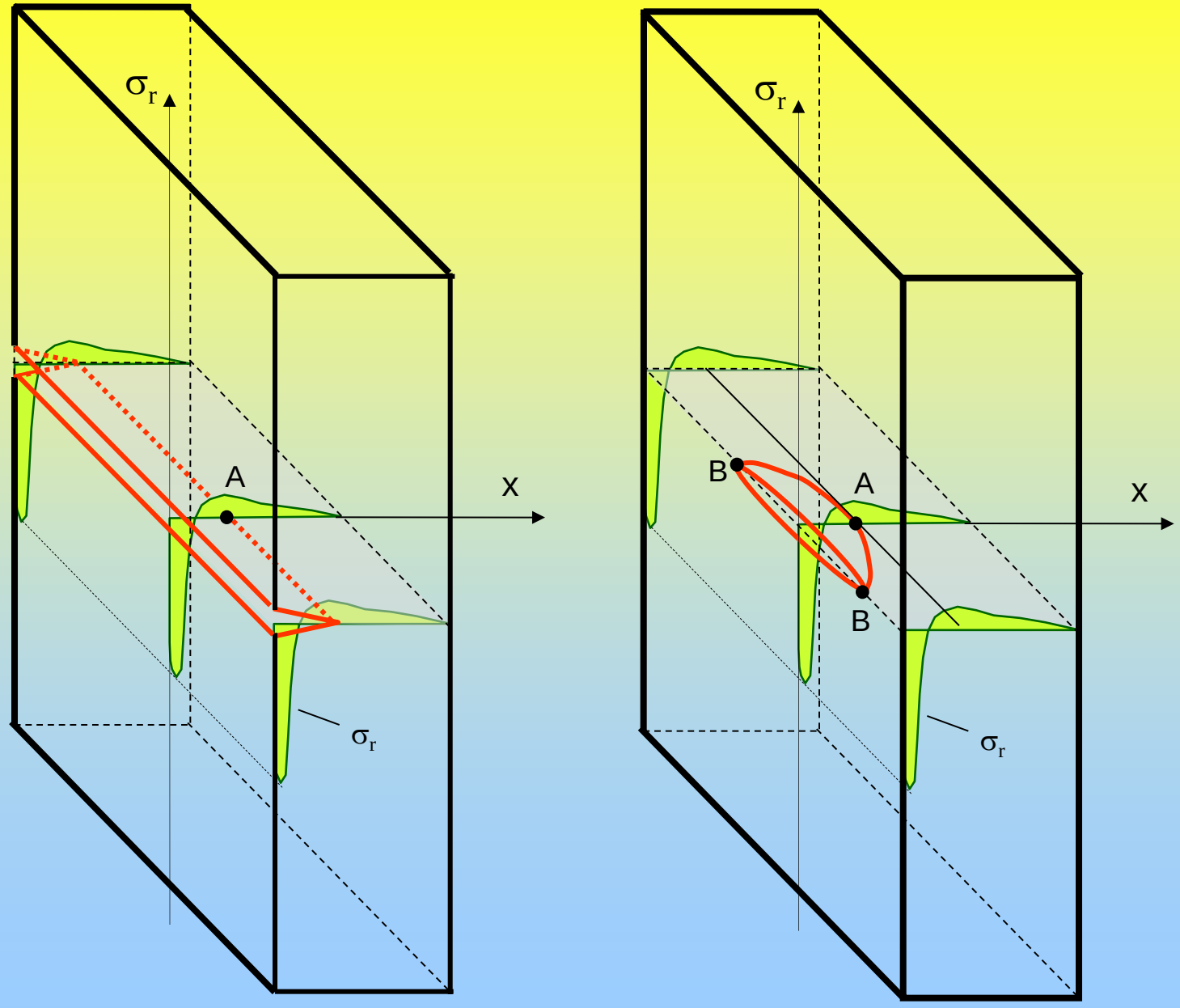
Geometrical Stress Intensity Correction Factor "Y" for an Edge Crack Emanating from the Weld Toe

(Comparison of WF and FEM data)

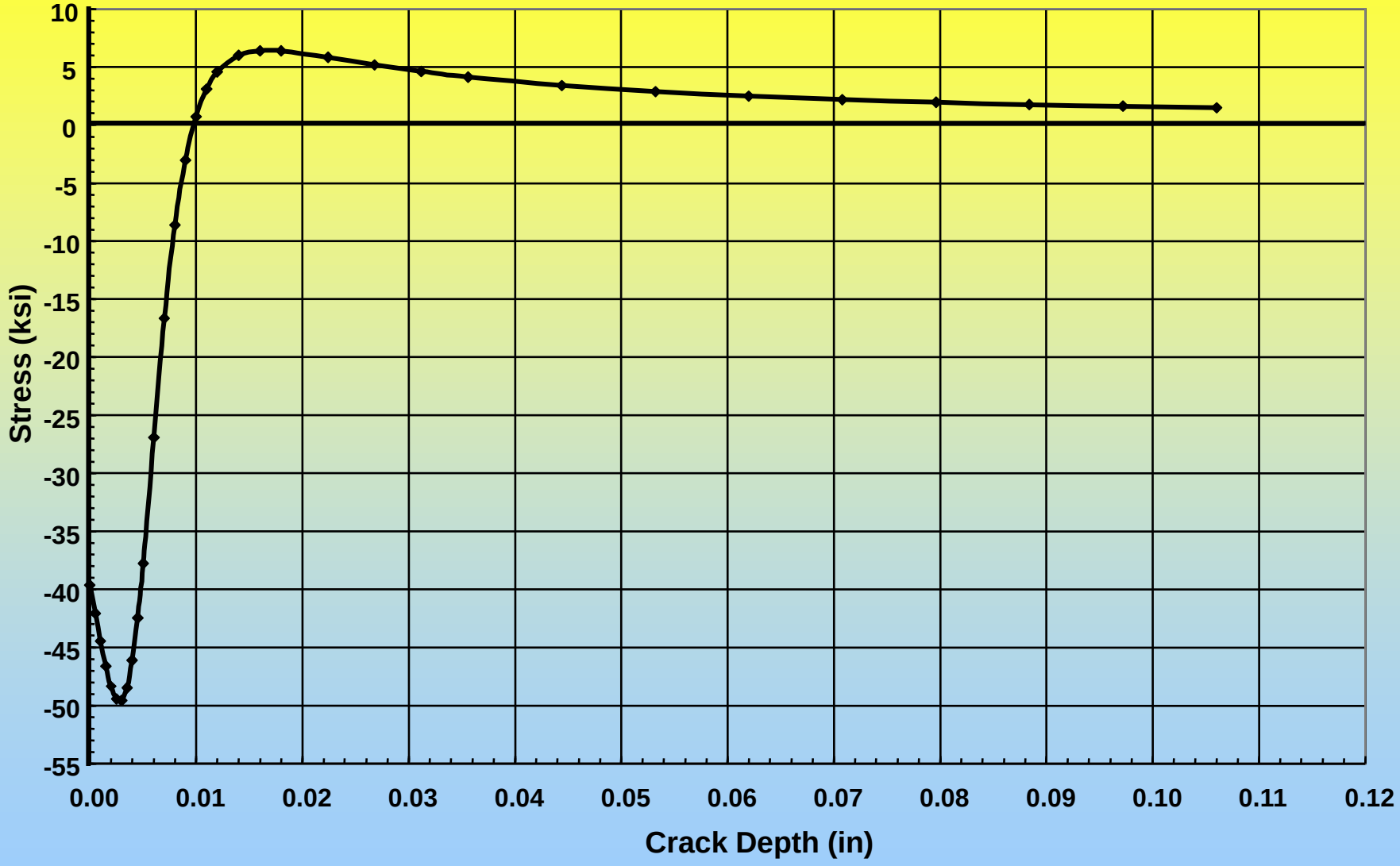




Edge and Semi-Elliptical Surface crack in a plate with post shot-peening residual stressesm (Northroop Grumman)

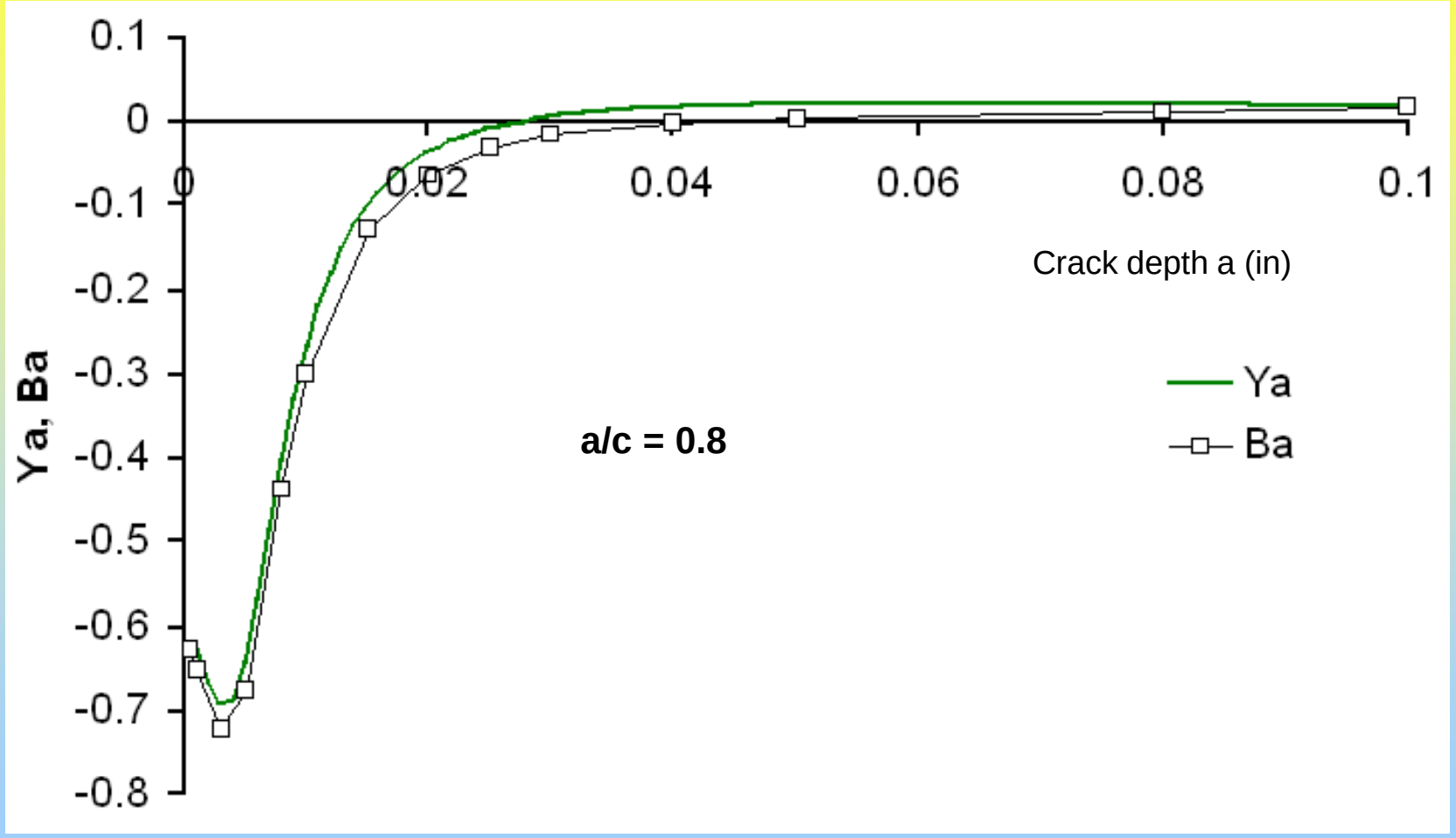


Shot Peen Residual Stress Distribution (1-D)

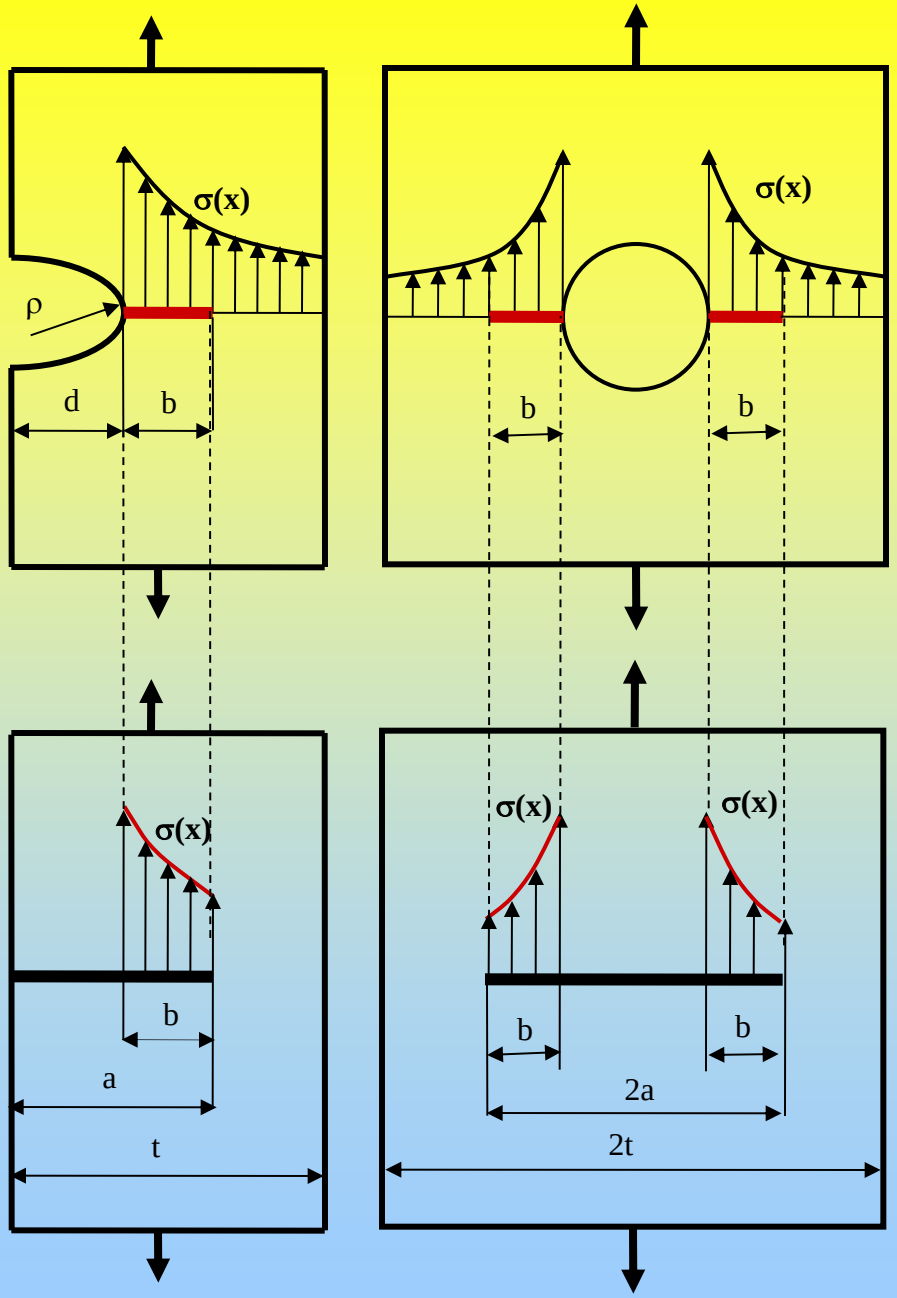


SIF geometric correction factors at the deepest point A:

Ya – WF and Ba - Boundary Element – comparison

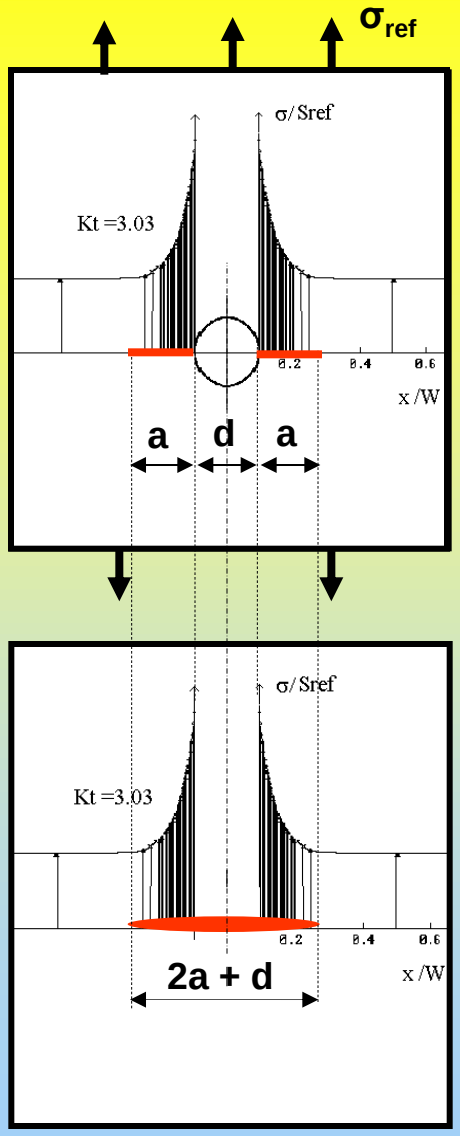


Cracks at Notches

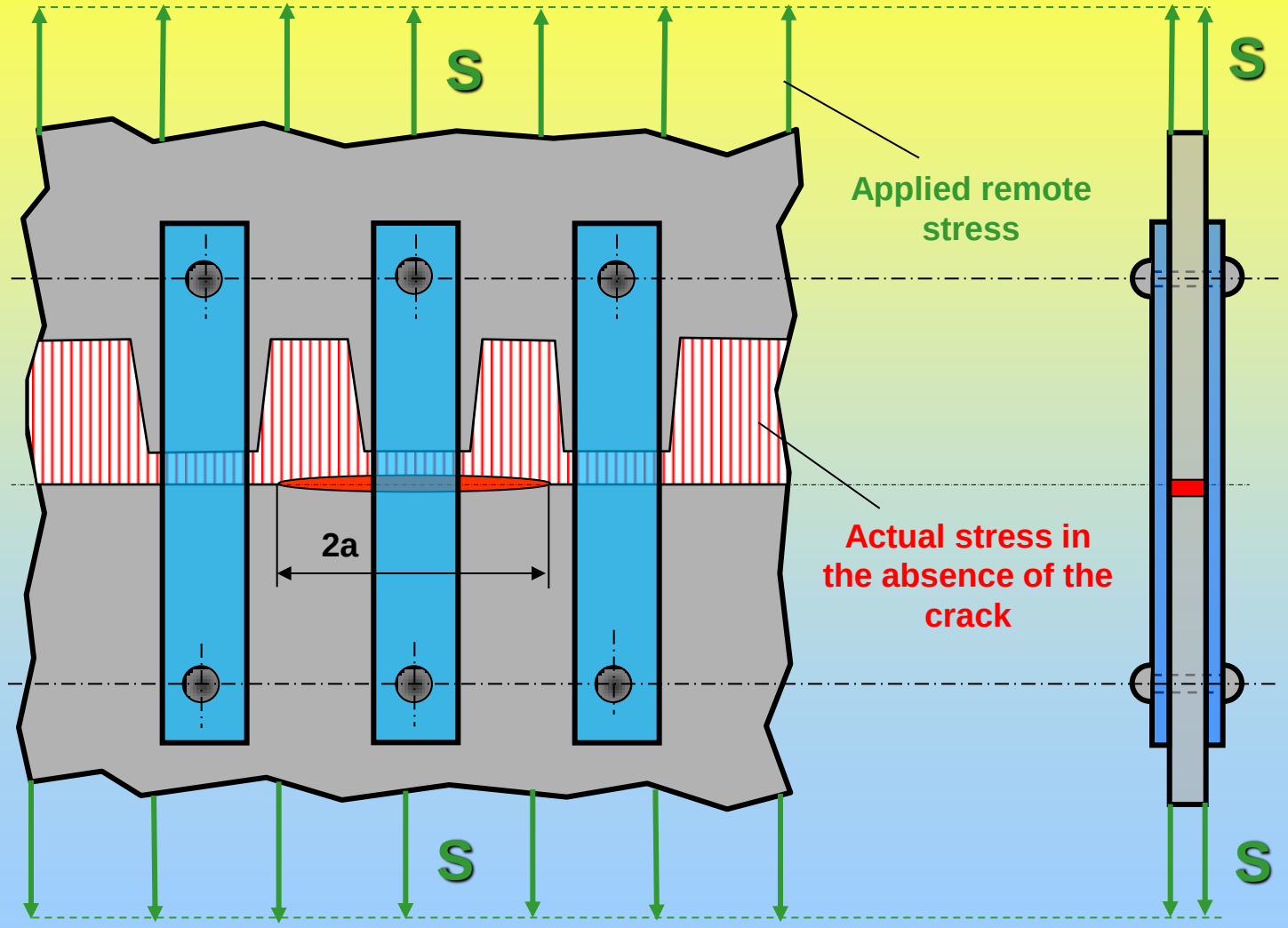


In order to determine the SIF the notch and the crack are modelled as one crack but the stress distribution comes from the un-cracked notched body - resulting in the analysis of partially loaded crack.

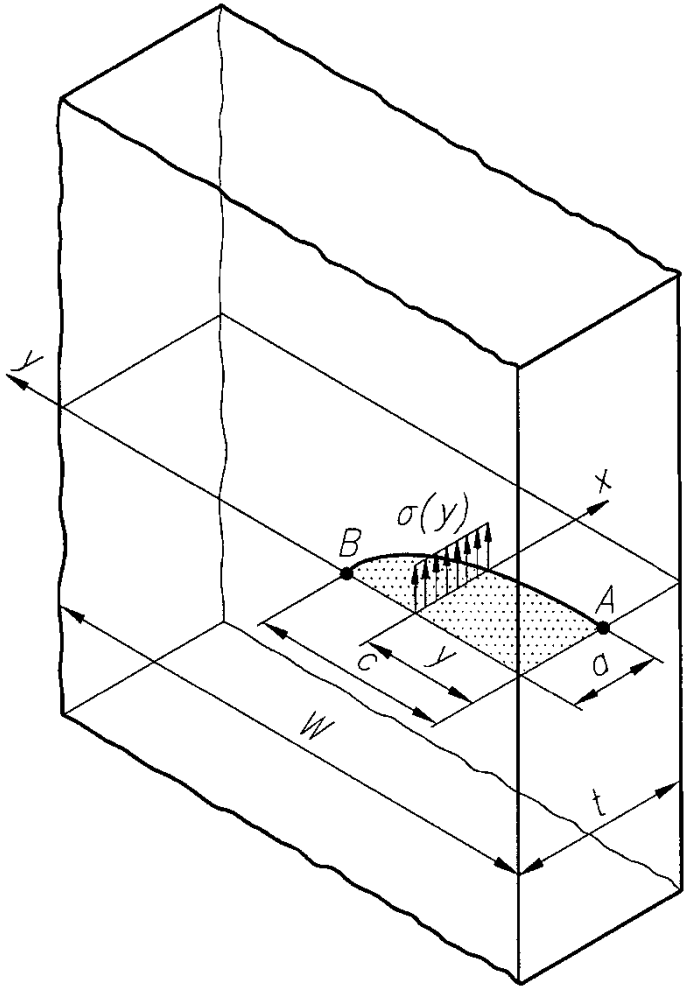
Comparison with Newman/Handbook FEM data



Riveted Stiffeners Effect



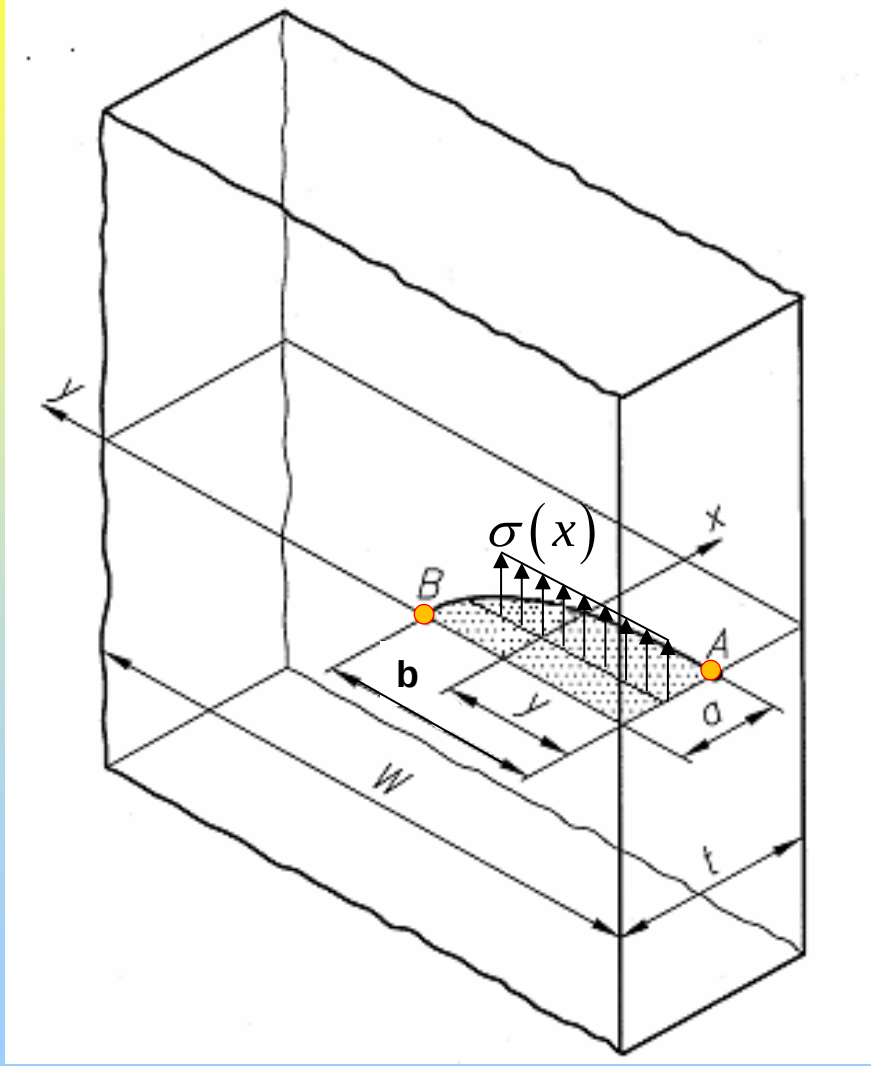
Corner Crack Under In-Plane Stress Distribution

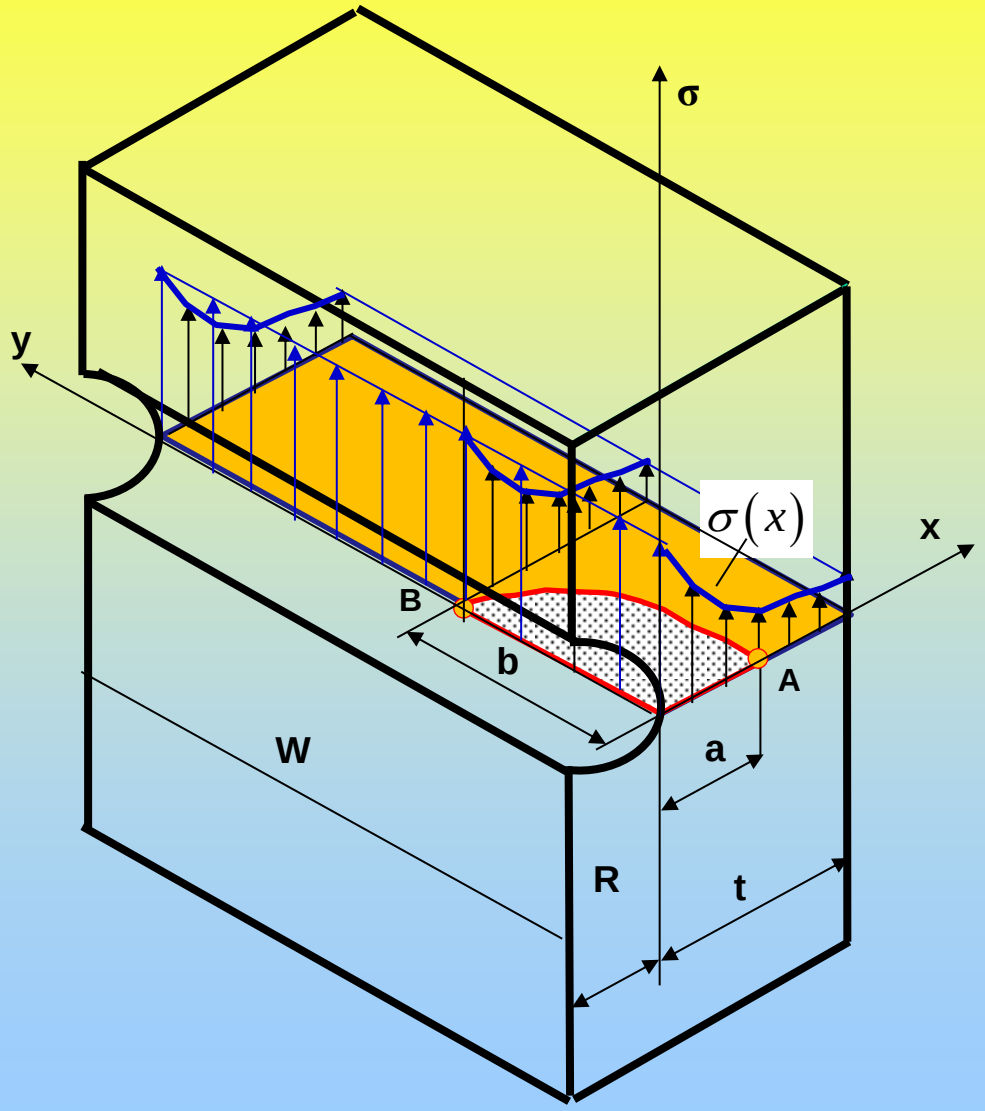


$$m_A(y,c)=\frac{2}{\sqrt{\pi y}}\left[1+M_{1A}\left(\frac{y}{c}\right)^{1/2}+M_{2A}\left(\frac{y}{c}\right)+M_{3A}\left(\frac{y}{c}\right)^{3/2}\right]$$

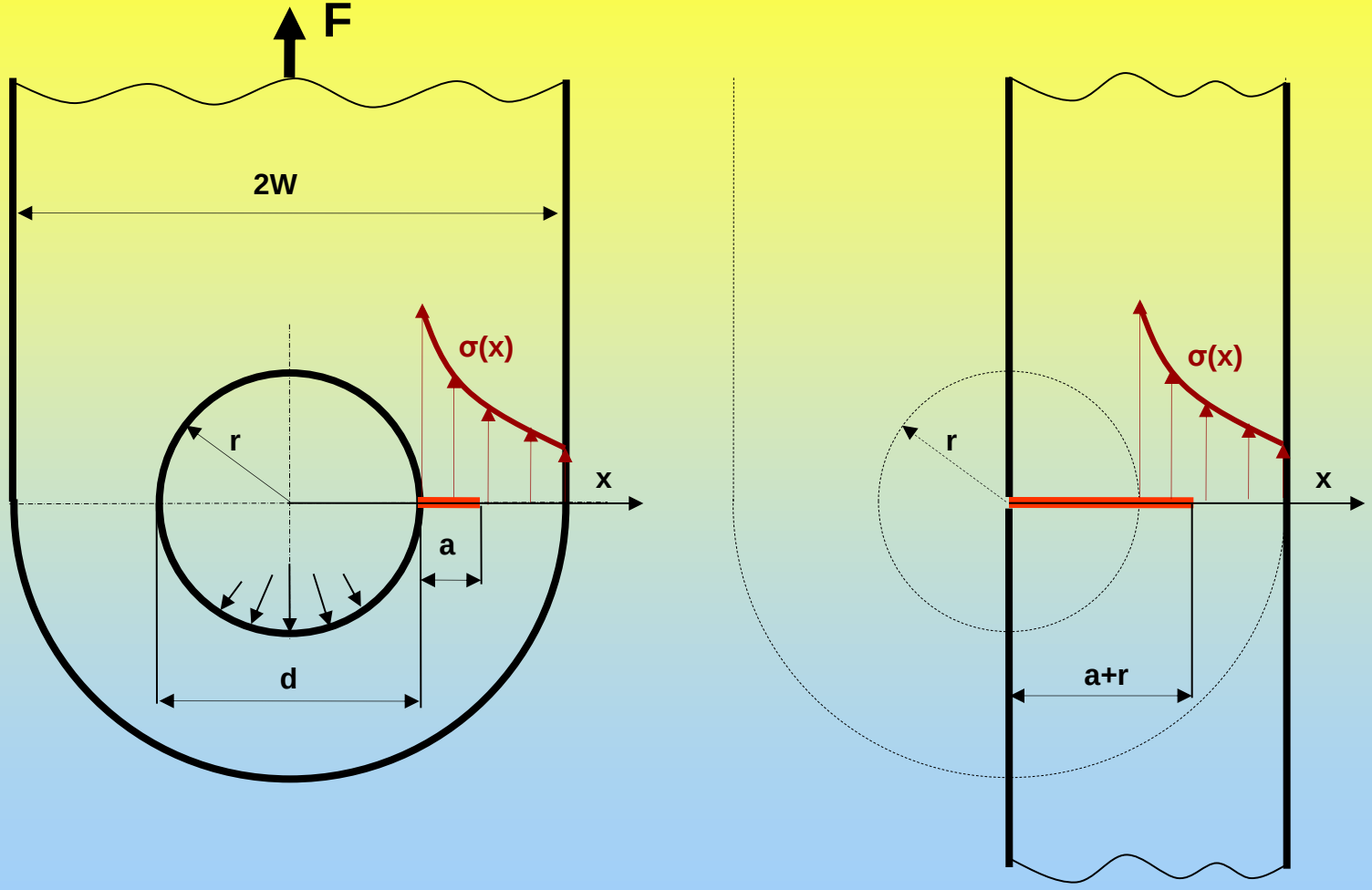
$$m_B(y,c)=\frac{2}{\sqrt{2\pi(c-y)}}\left[1+M_{1B}\left(1-\frac{y}{c}\right)^{1/2}+M_{2A}\left(1-\frac{y}{c}\right)+M_{3A}\left(1-\frac{y}{c}\right)^{3/2}\right]$$

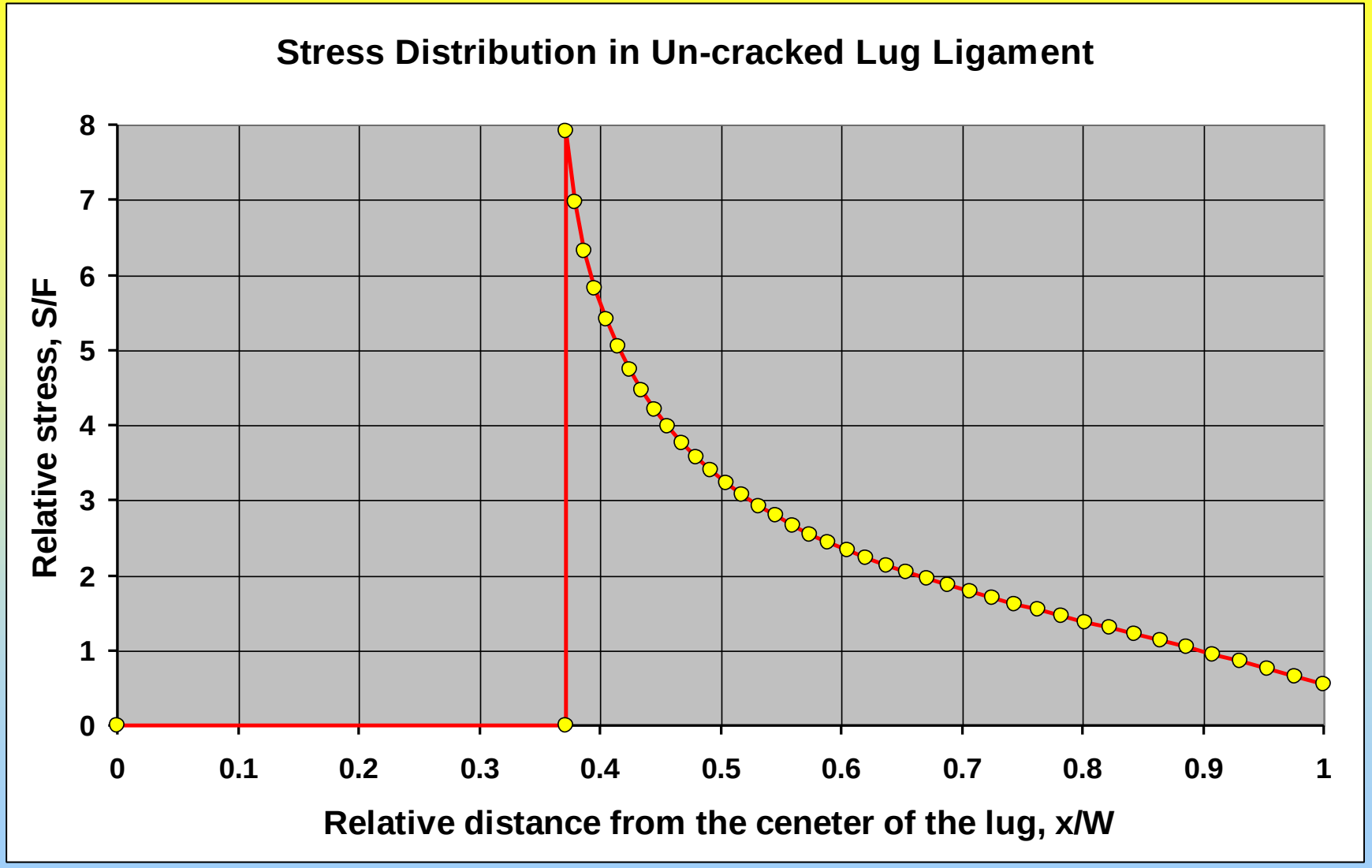
Corner Crack Under Out-of-Plane Stress Distribution



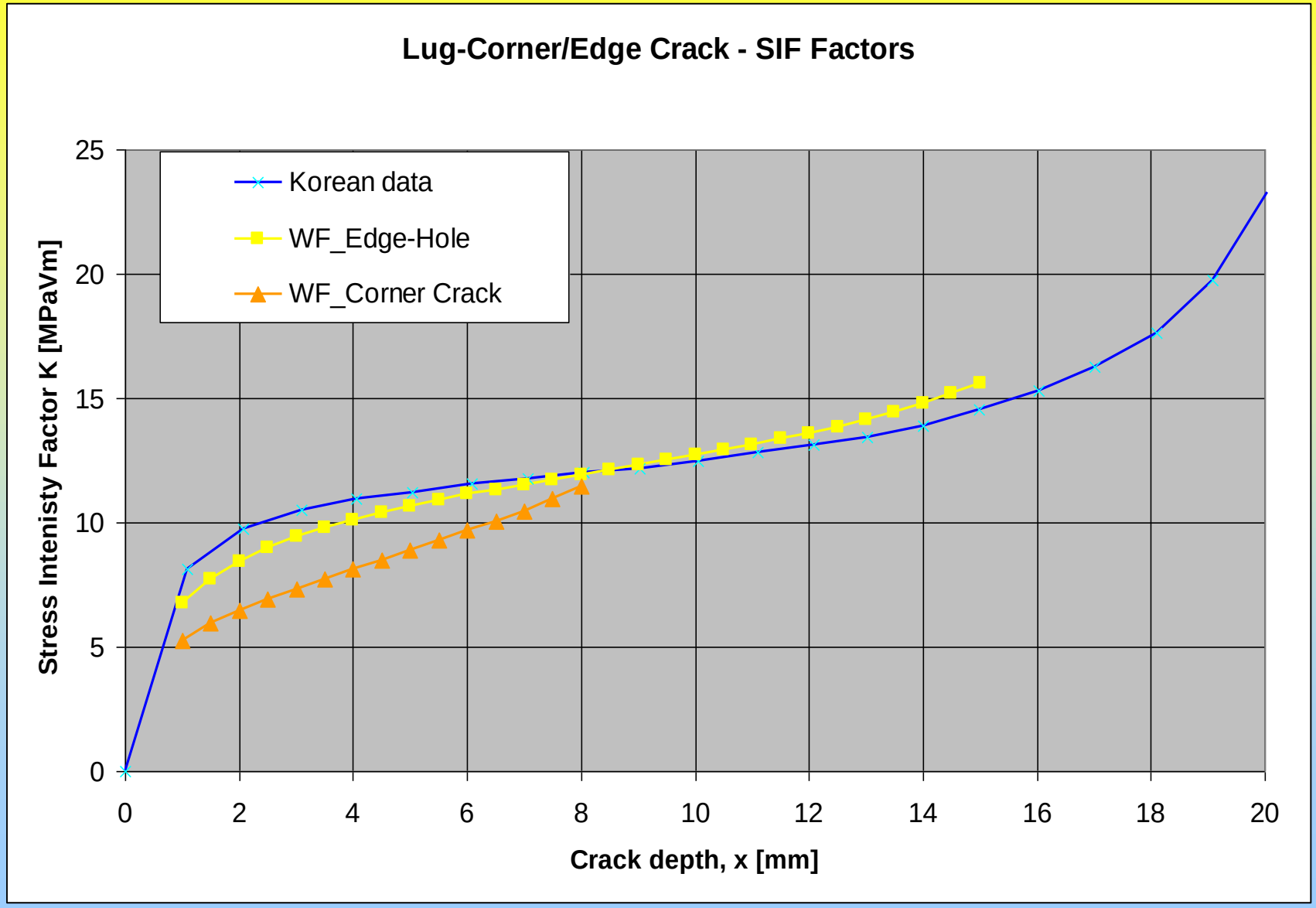


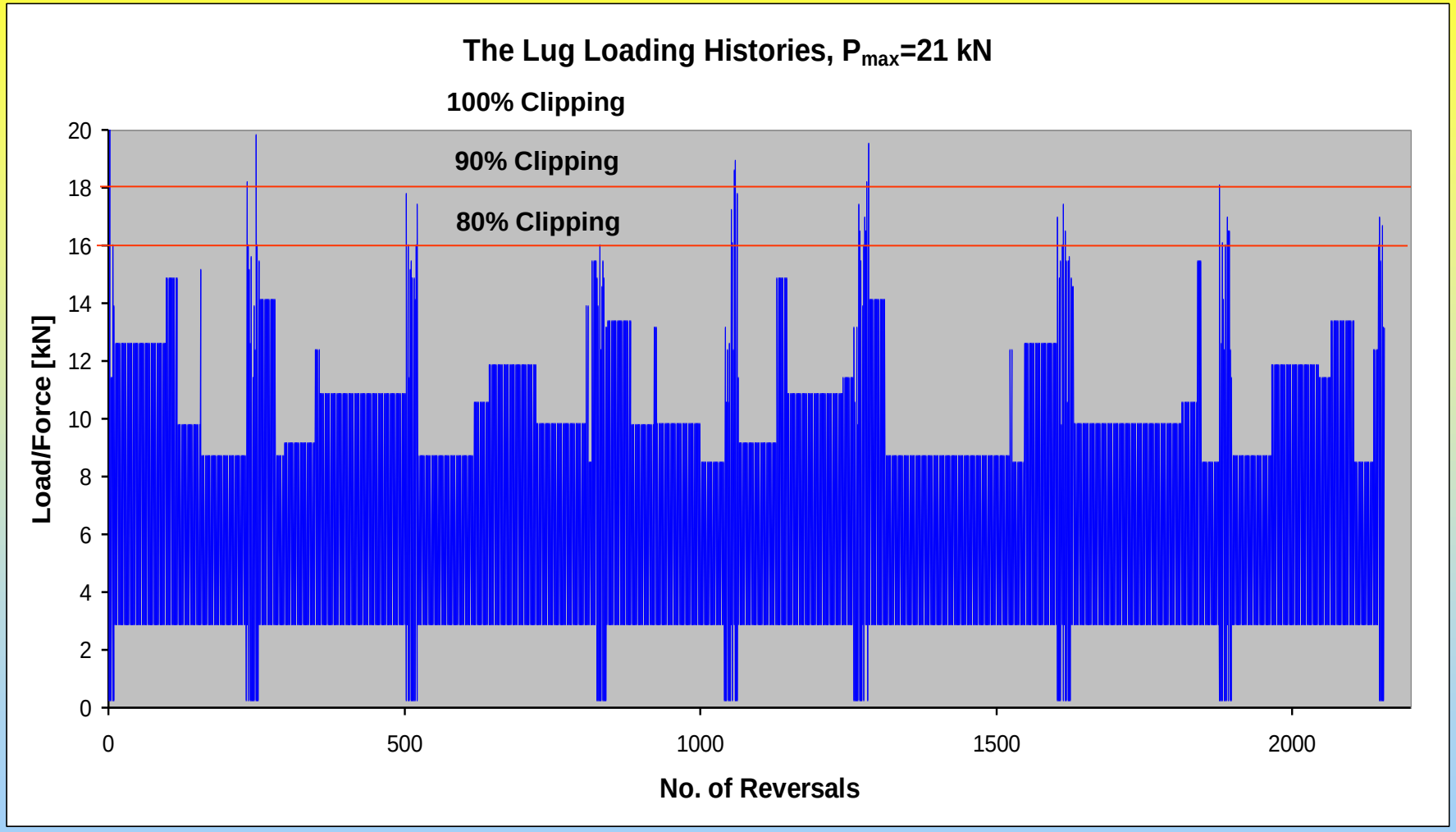
Stress Intensity Factor for an edge crack in the lug by the WF method

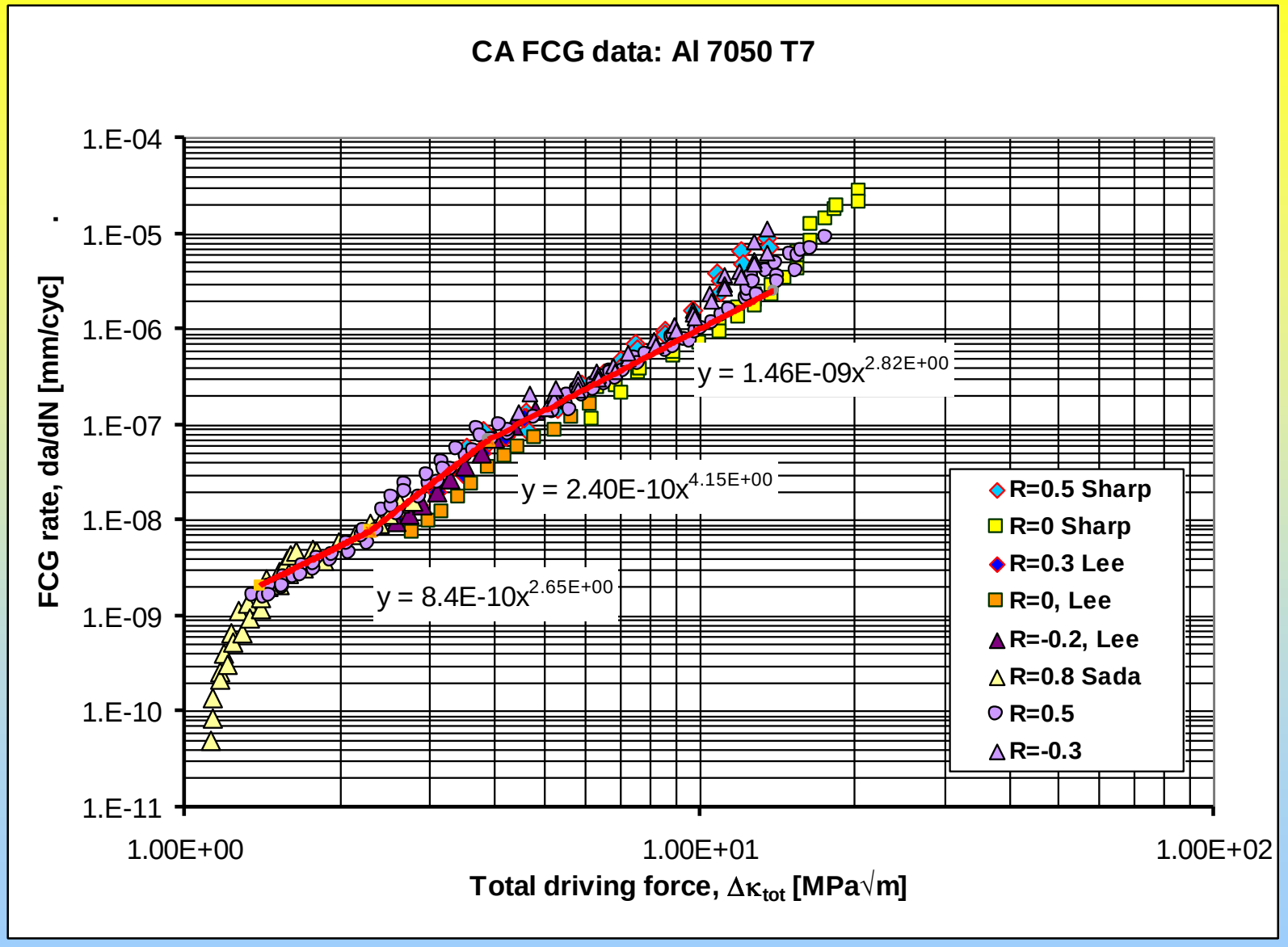


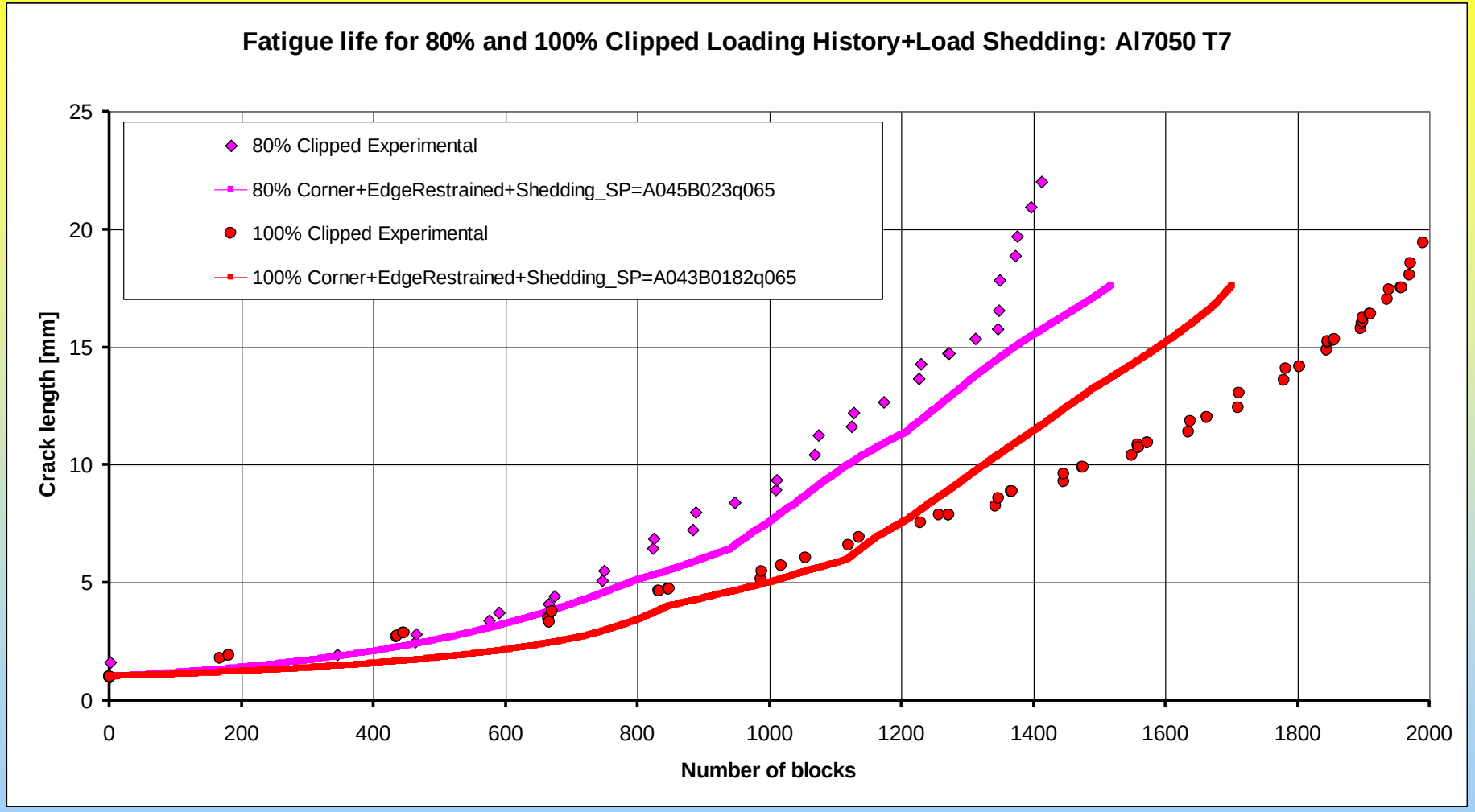

Stress Distribution in Un-cracked Lug Ligament
Relative stress, S/F
0
1
2
3
4
5
6
7
8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Relative distance from the ceneter of the lug, x/W

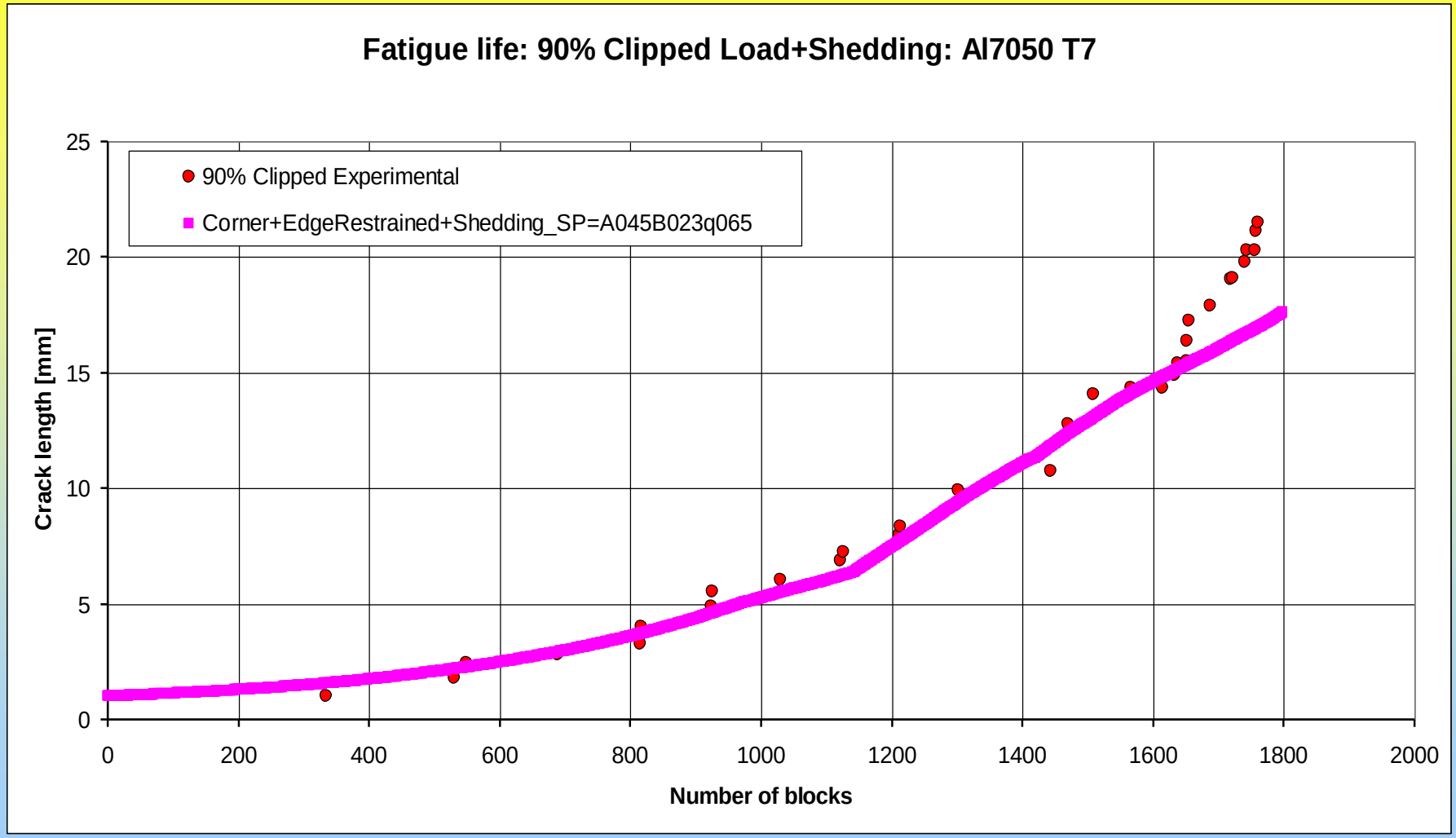
Stress Intenisty factors for Corner and Edge crack in the Korean attachment lug



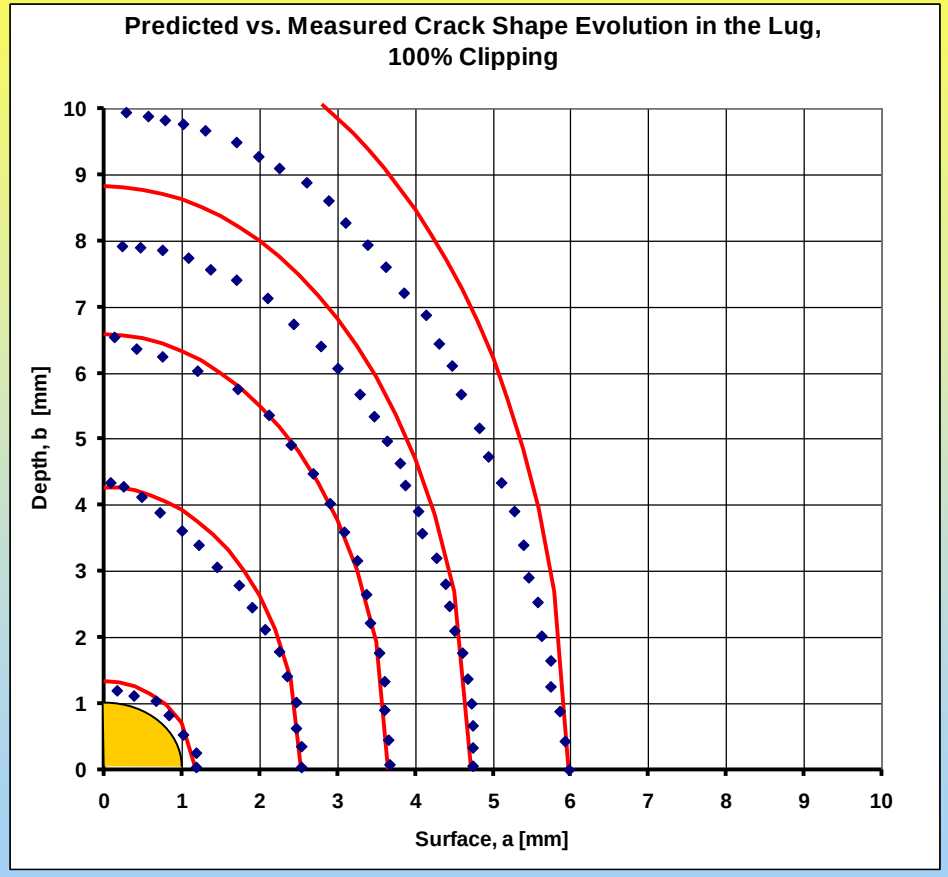

The Lug Loading Histories, P_{max}=21 kN
100% Clipping
90% Clipping
80% Clipping
Load/Force [kN]
No. of Reversals
0
2
4
6
8
10
12
14
16
18
20
0
500
1000
1500
2000

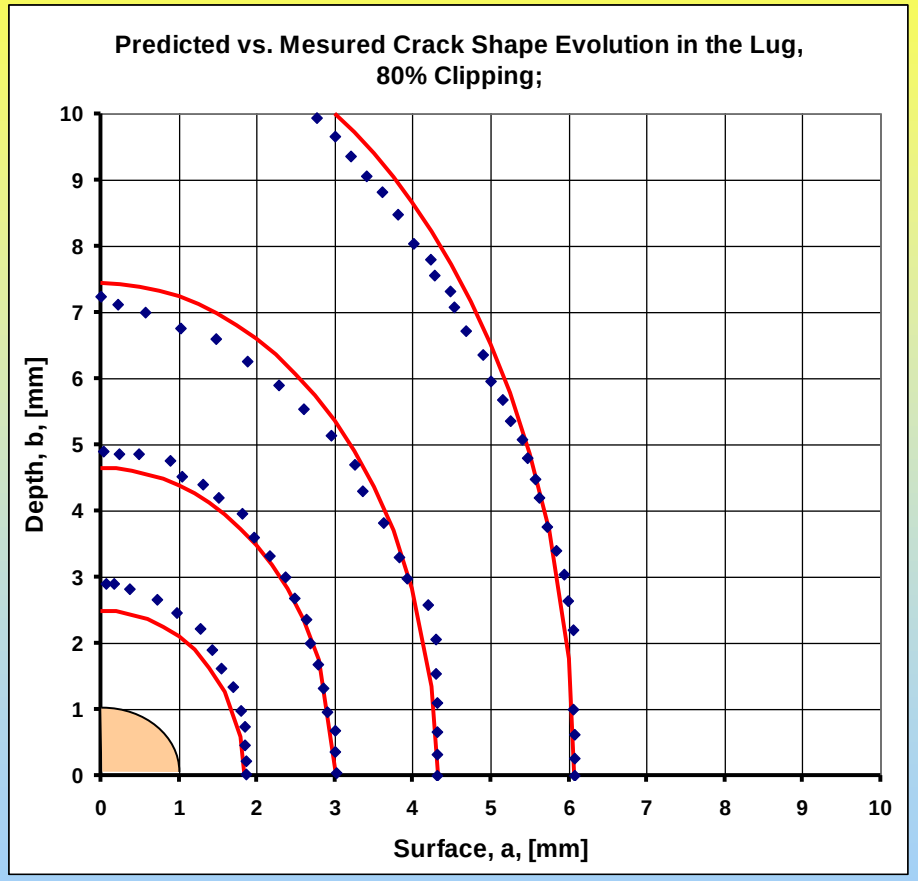

CA FCG data: Al 7050 T7
FCG rate, da/dN [mm/cyc]
1.E-04
1.E-05
1.E-06
1.E-07
1.E-08
1.E-09
1.E-10
1.E-11
1.00E+00
1.00E+01
1.00E+02
Total driving force, Δκtot [MPa√m]
y = 1.46E-09x2.82E+00
y = 2.40E-10x4.15E+00
y = 8.4E-10x2.65E+00
R=0.5 Sharp
R=0 Sharp
R=0.3 Lee
R=0, Lee
R=-0.2, Lee
R=0.8 Sada
R=0.5
R=-0.3

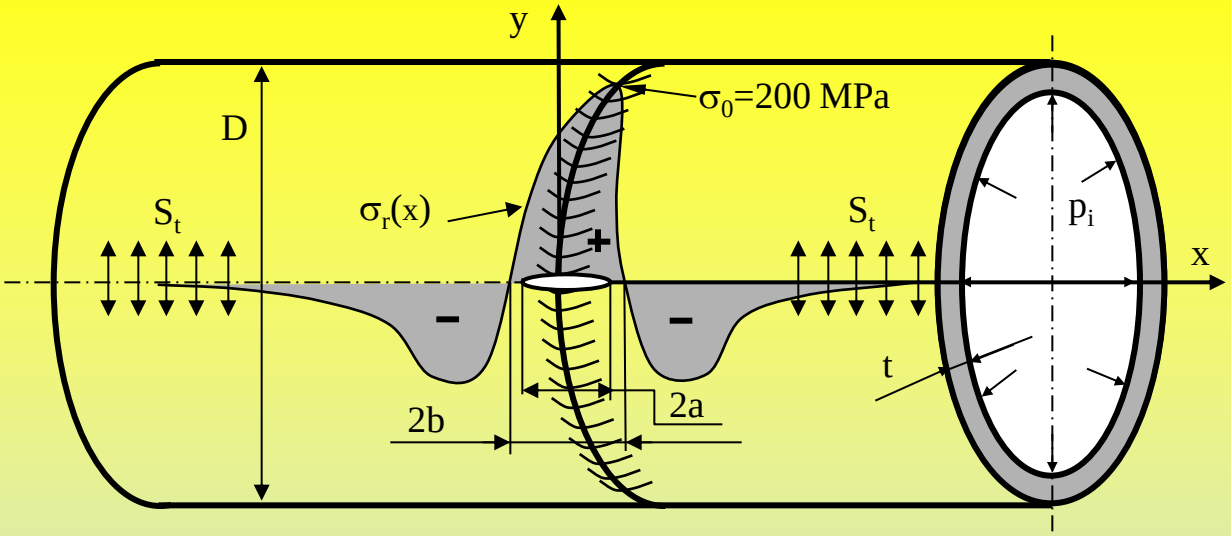

Fatigue life for 80% and 100% Clipped Loading History+Load Shedding: Al7050 T7
80% Clipped Experimental
80% Corner+EdgeRestrained+Shedding_SP=A045B023q065
100% Clipped Experimental
100% Corner+EdgeRestrained+Shedding_SP=A043B0182q065
Crack length [mm]
Number of blocks
0
5
10
15
20
25
0
200
400
600
800
1000
1200
1400
1600
1800
2000


Fatigue life: 90% Clipped Load+Shedding: Al7050 T7
90% Clipped Experimental
Corner+EdgeRestrained+Shedding_SP=A045B023q065
Crack length [mm]
Number of blocks
0
5
10
15
20
25
0
200
400
600
800
1000
1200
1400
1600
1800
2000

Crack Shape Evolution; *quarter circular initial crack*





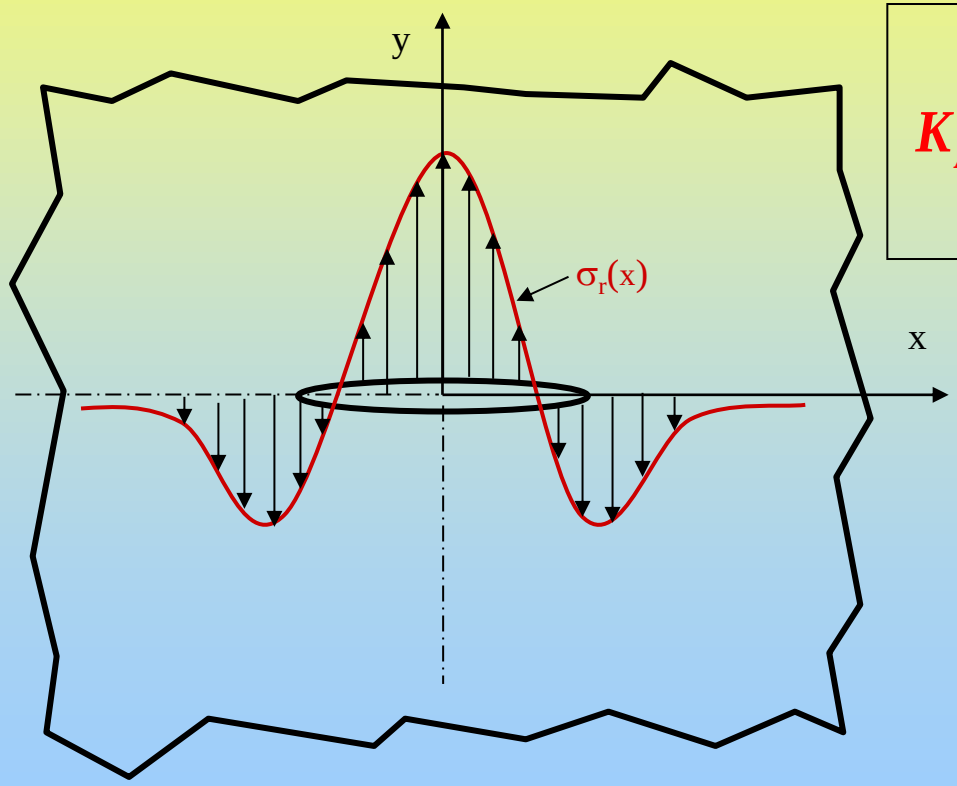


$$\sigma_r(x) = \sigma_0 \cdot e^{-\frac{1}{2}\left(\frac{x}{b}\right)^2}\left[1-\left(\frac{x}{b}\right)^2\right]$$

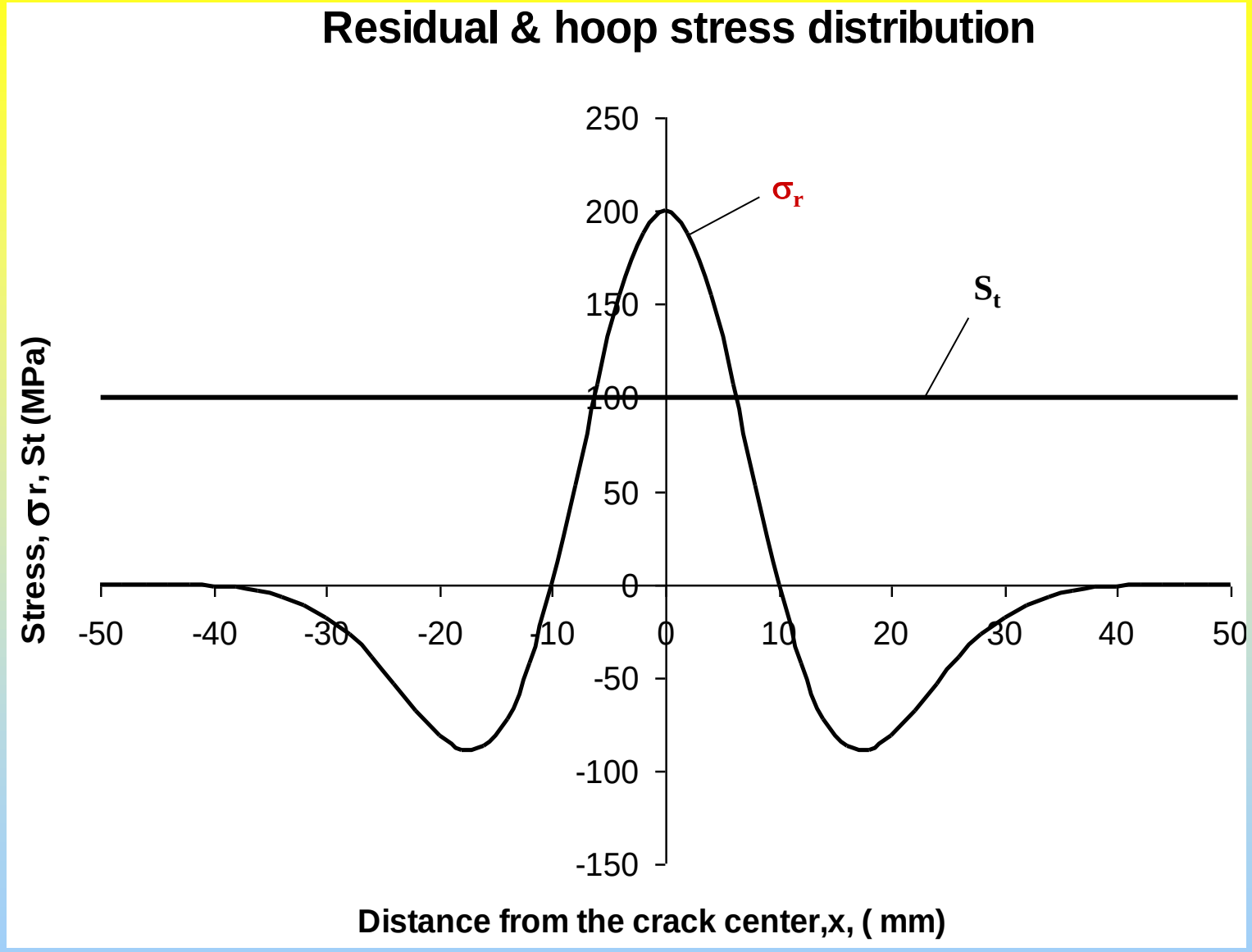
$$S_t = p_i \frac{D}{2t}$$

Residual stress, σ_r, contribution, to the stress intensity factor, $K_A^{\sigma_r}$

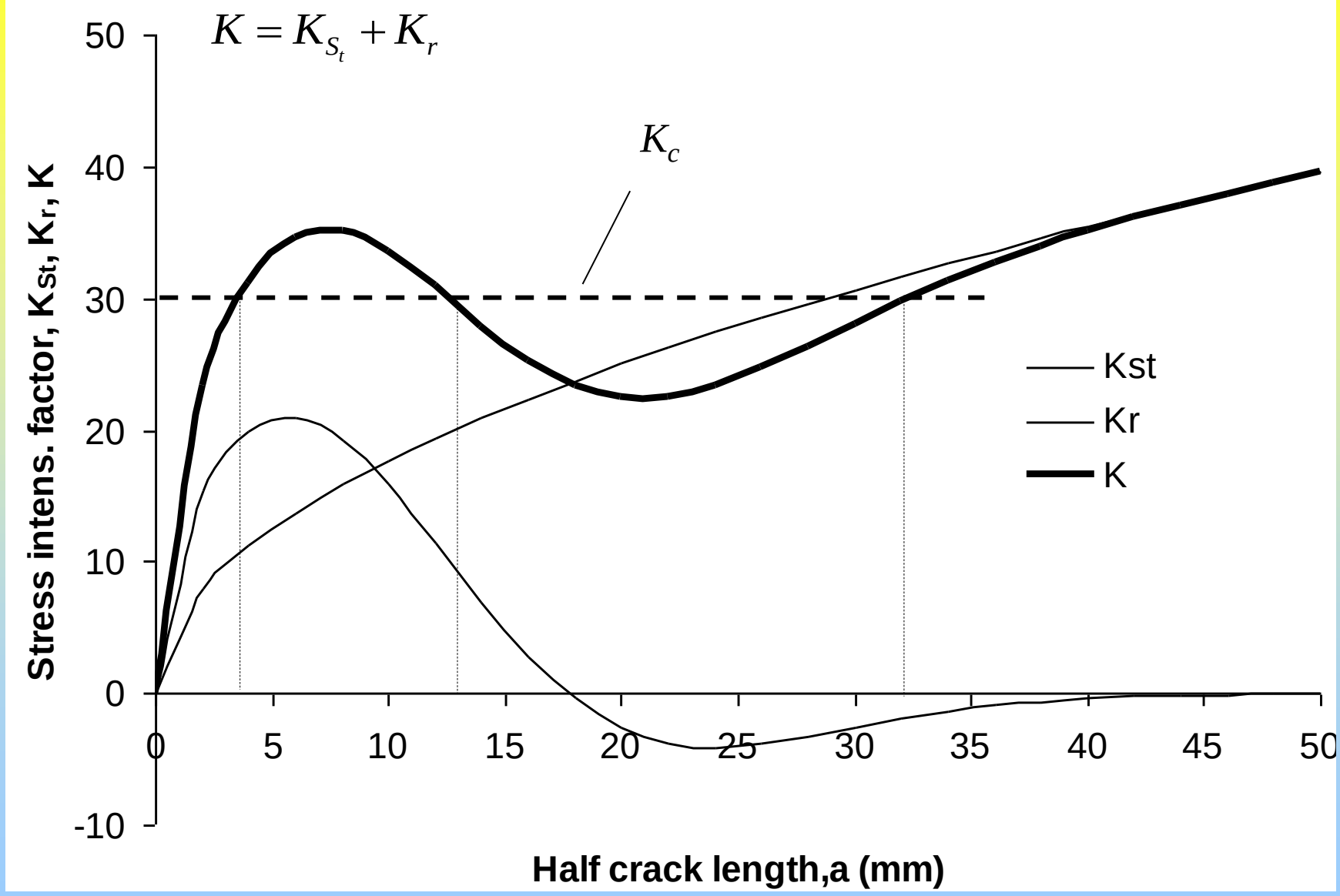
$$K_A^{\sigma_r} = \int_{-a}^{+a} \sigma_r \cdot \frac{1}{\sqrt{\pi a}} \sqrt{\frac{a+x}{a-x}} \cdot dx = \int_{-a}^{+a} \sigma_0 \cdot e^{-\frac{1}{2}\left(\frac{x}{b}\right)^2} \left[1 - \left(\frac{x}{b}\right)^2\right] \cdot \frac{1}{\sqrt{\pi a}} \sqrt{\frac{a+x}{a-x}} \cdot dx$$



$$K_A^{\sigma_r} = \sigma_0 \sqrt{\pi a} \cdot e^{-0.42\left(\frac{a}{b}\right)^2} \left[1 - \frac{1}{\pi}\left(\frac{a}{b}\right)^2\right]$$


Residual & hoop stress distribution
250
200
150
100
50
0
-50
-100
-150
-50
-40
-30
-20
-10
0
10
20
30
40
50
σr
St
Stress, σr, St (MPa)
Distance from the crack center,x, (mm)

Stress intensity factors



Summary of the WF method

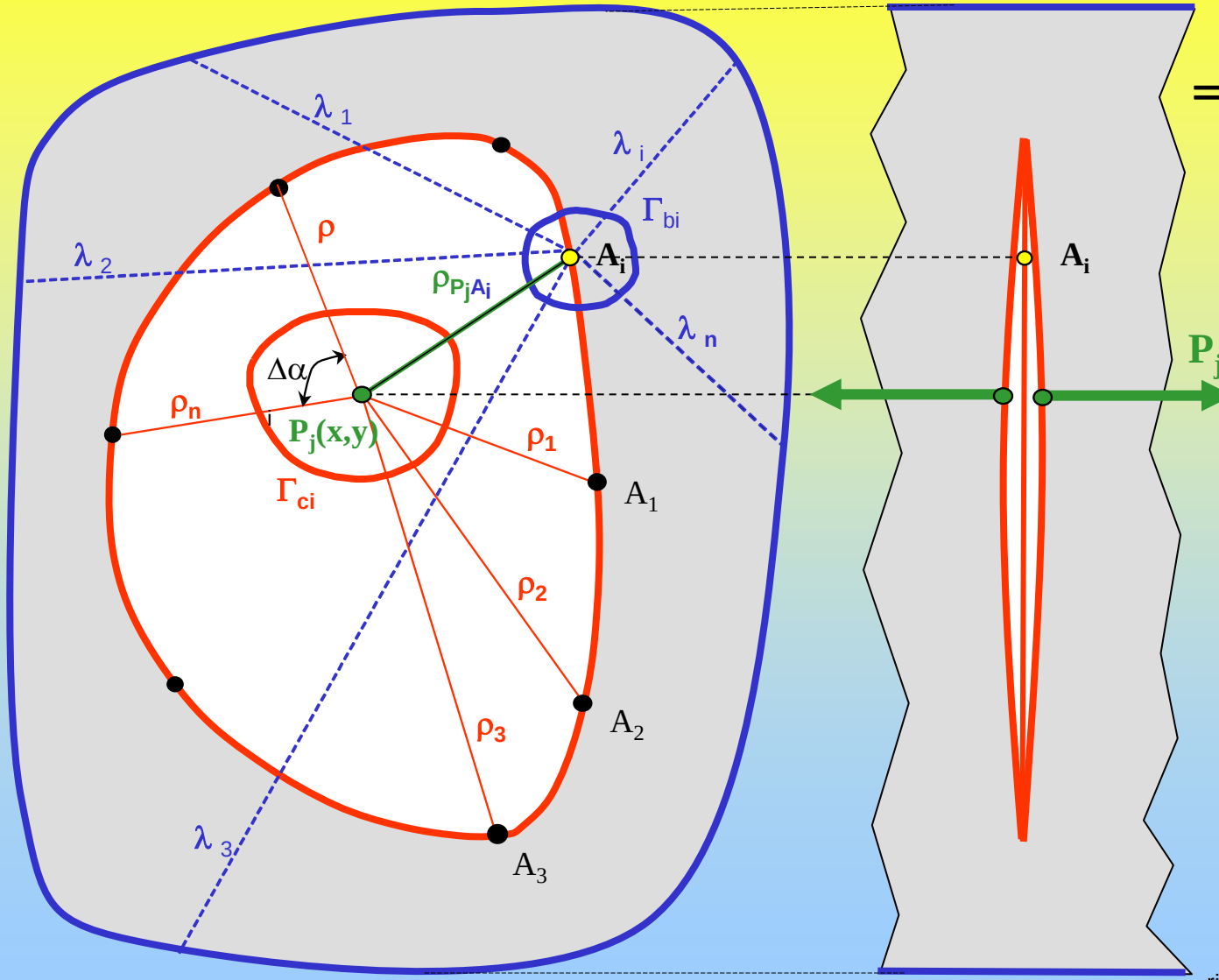
- **Very efficient computationally** ***(SIF can be calculated many million times…. after each fatigue crack increment),***
- **Can produce SIF for a wide variety of loading conditions** ***(new non-existent solutions can be produced in a small fraction of CPU time required by other methods),***
- **Can account for the residual stress effect,**
- **Can account for variable thickness effects,**
- **Can account for the load shedding effect,**
- **No necessity for stress analysis of cracked bodies** ***(i.e. analytical or FEM stress data from un-cracked bodies are only needed!),***
- **The same FEM model can be used for the stress and load shedding analyses** ***(no need for special mesh… no need for additional FE mesh modeling for the load shedding analysis!)***

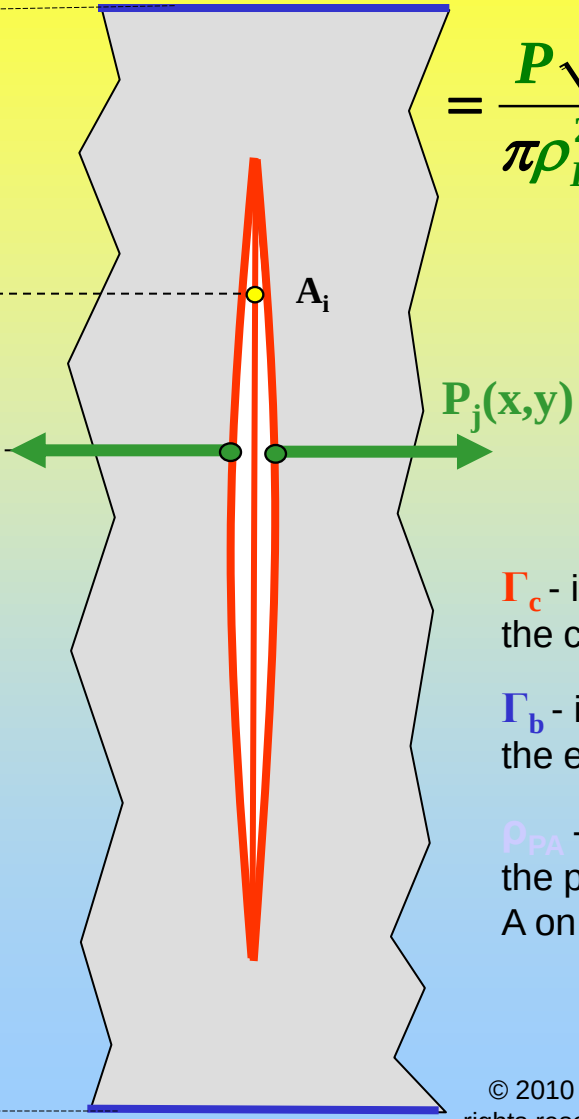
THE FUTURE

Arbitrary Planar Cracks under Arbitrary Stress Field

Weight Function for Arbitrary Planar Cracks

$$K_{A_i} = m_{A_i}(x, y; P) = \frac{P\sqrt{2}}{\pi \rho_{P_j A_i}^2} \times \frac{\sqrt{\Gamma_c + \Gamma_b}}{\Gamma_c}$$





Γ_c - inverted contour of the crack front;

Γ_b - inverted contour of the external boundary;

ρ_{PA} - distance between the point load P and point A on the crack front

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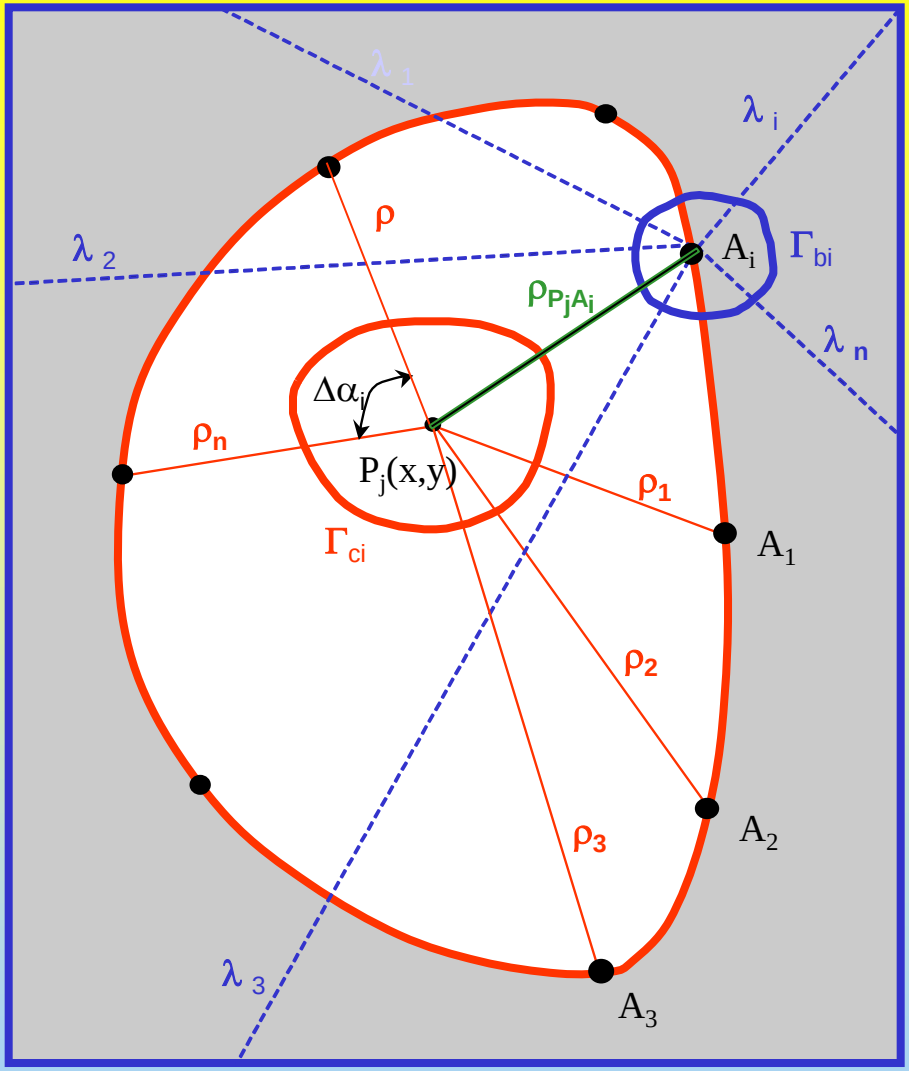
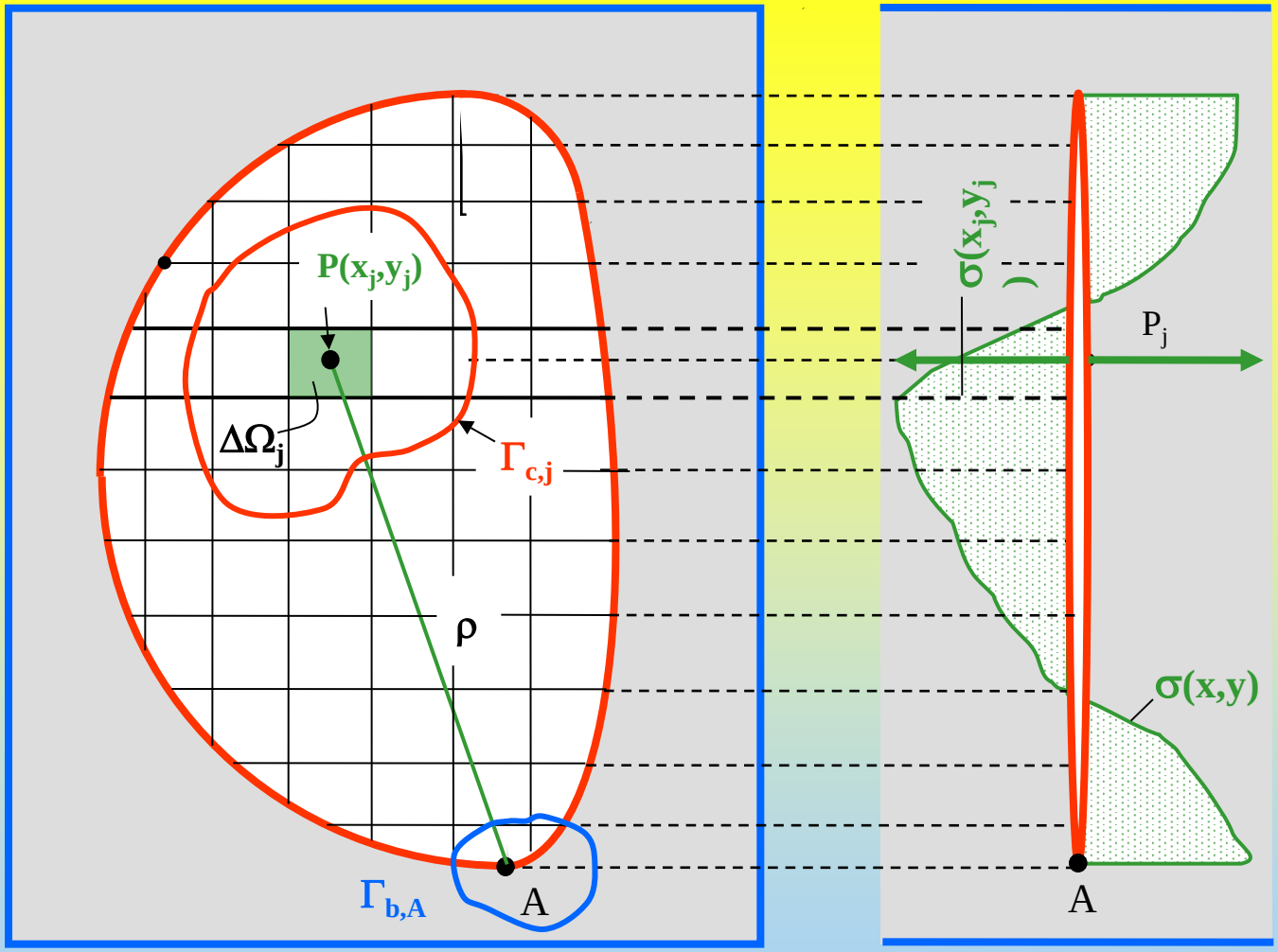
Point load Weight Function for arbitrary planar cracks under arbitrary stress distribution



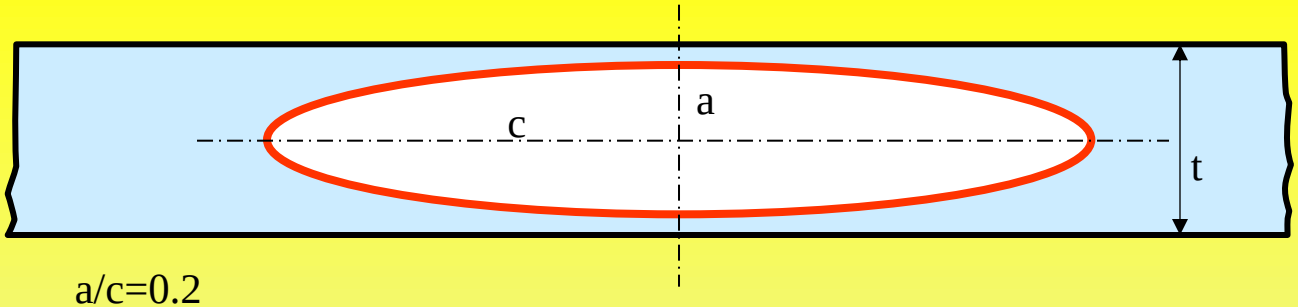
G. Glinka and W. Reinhard, "Calculation of Stress Intensity Factors for Cracks of Complex Geometry and Subjected to Arbitrary Non-linear Stress Fields", **31st National Symposium on Fatigue and Fracture Mechanics, Cleveland, OH, June 21-24, 1999. Best paper Award. ASTM STP…..**

Wu, Z., Glinka G., Jakubczak, H., and Nilsson, L., "Calculation of Stress Intensity Factors for Cracks in Structural and Mechanical Components Subjected to Complex Stress Fields" **Fatigue Testing and Analysis Under Variable Amplitude Loading Conditions, ASTM STP 1439, P. C. McKeighan and N. Ranganathan, Eds., ASTM international, West Conshohocken, PA, 2003.**

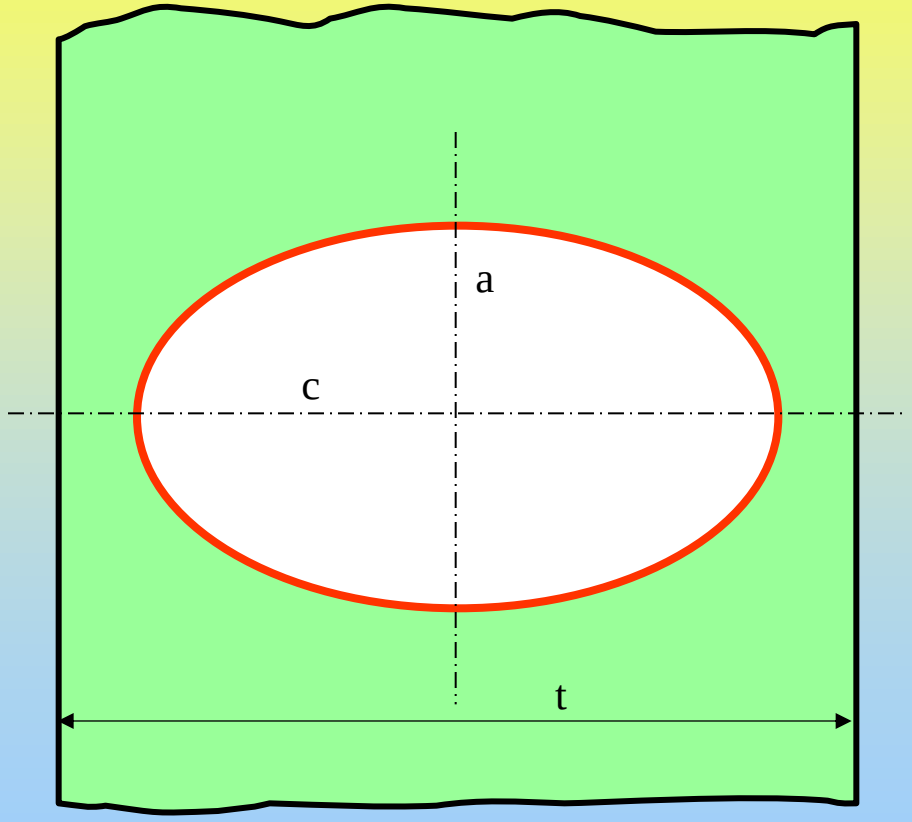
$$K_{A_i} = m_{A_i}(x, y; P) = \frac{P\sqrt{2}}{\pi \rho_{P_j A_i}^2} \times \frac{\sqrt{\Gamma_c + \Gamma_b}}{\Gamma_c}$$



$$K_A = \iint_{\Omega} \sigma(x,y) m_A(x,y;P) dxdy = \sum_{j=1}^{j=N} \left\{ \frac{\sigma\left(x_j, y_j\right) \times \Delta\Omega_j \times \sqrt{2}}{\pi \rho_{P_j A}^2} \times \frac{\sqrt{\Gamma_{c,j} + \Gamma_{b,A}}}{\Gamma_{c,j}} \right\}$$

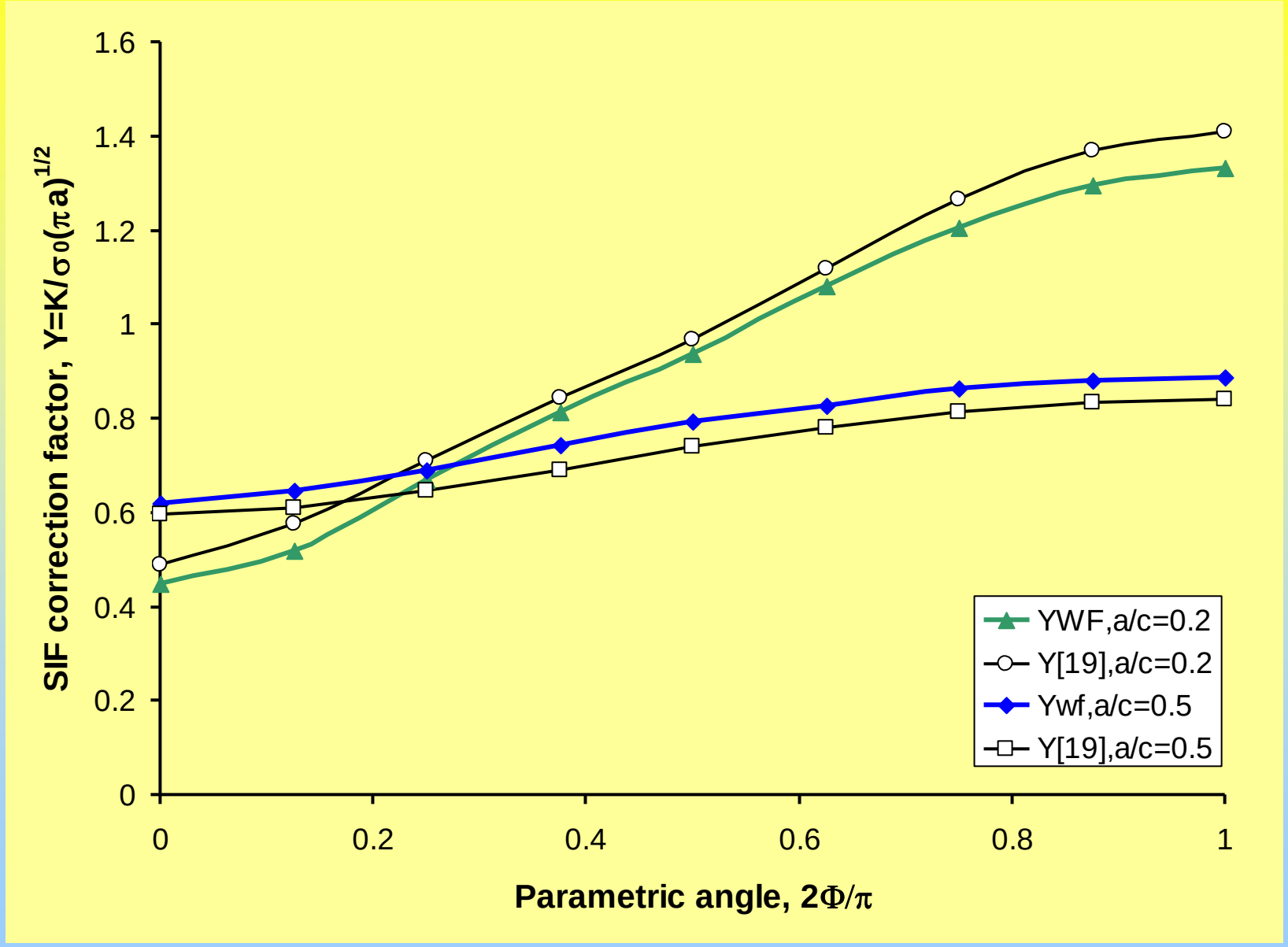


a/c=0.2
a/t=0.8

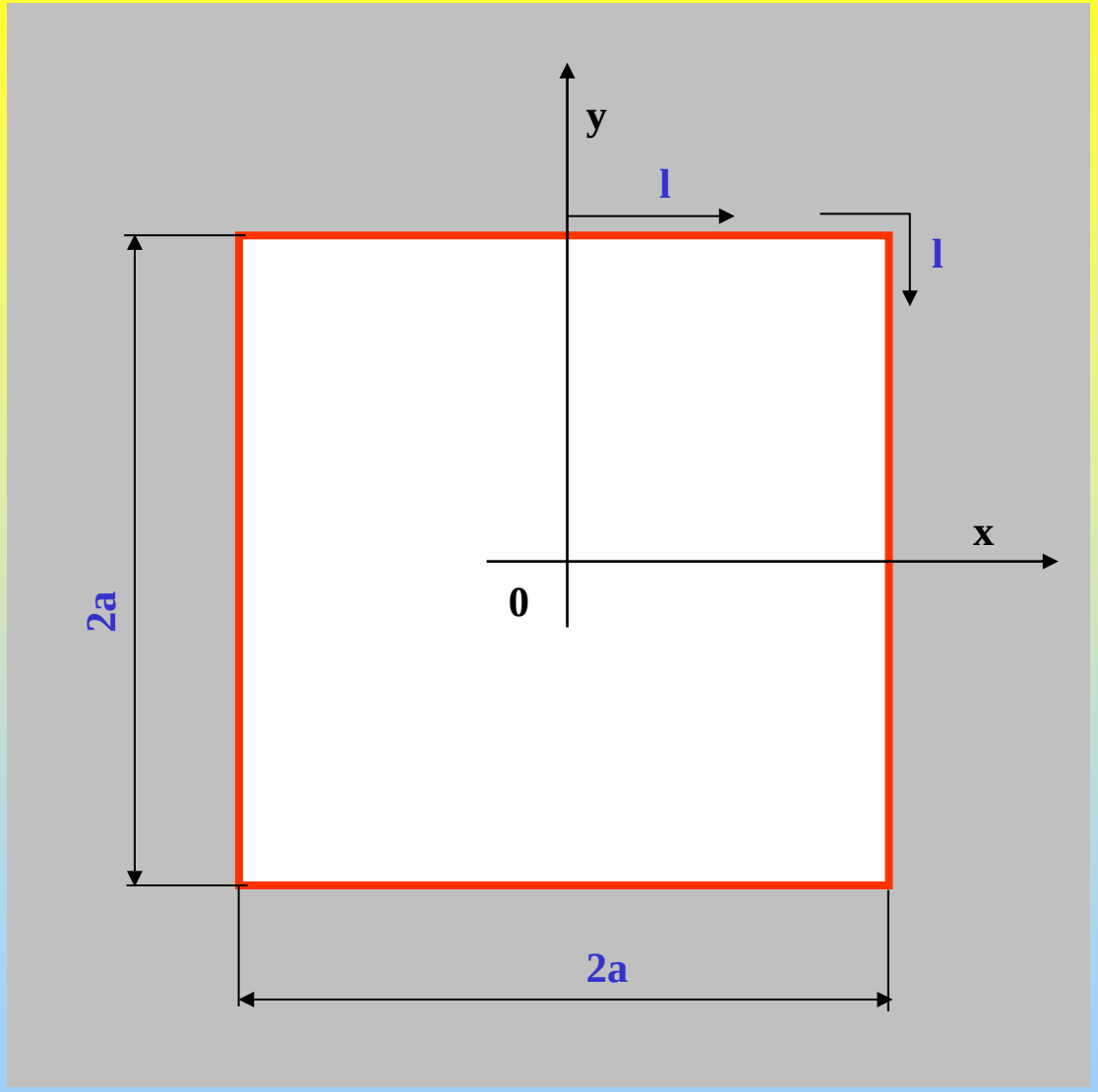


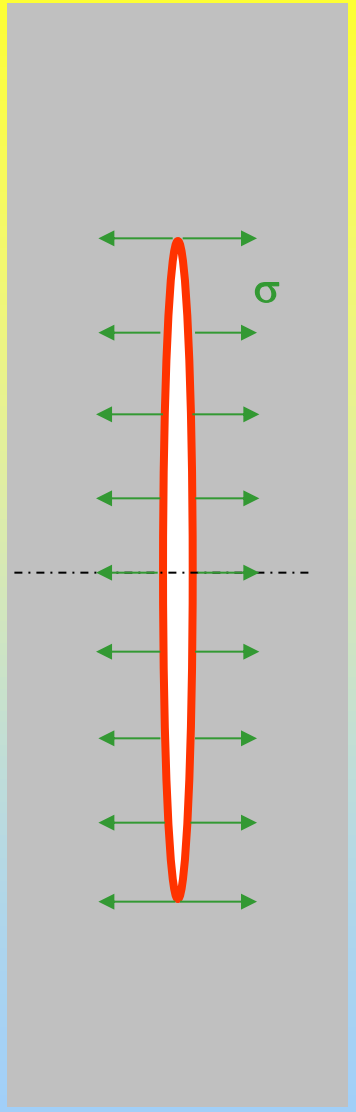
a/c=0.6
c/t=0.8

Uniform tensile stress – comparison with J.Newman FEM data



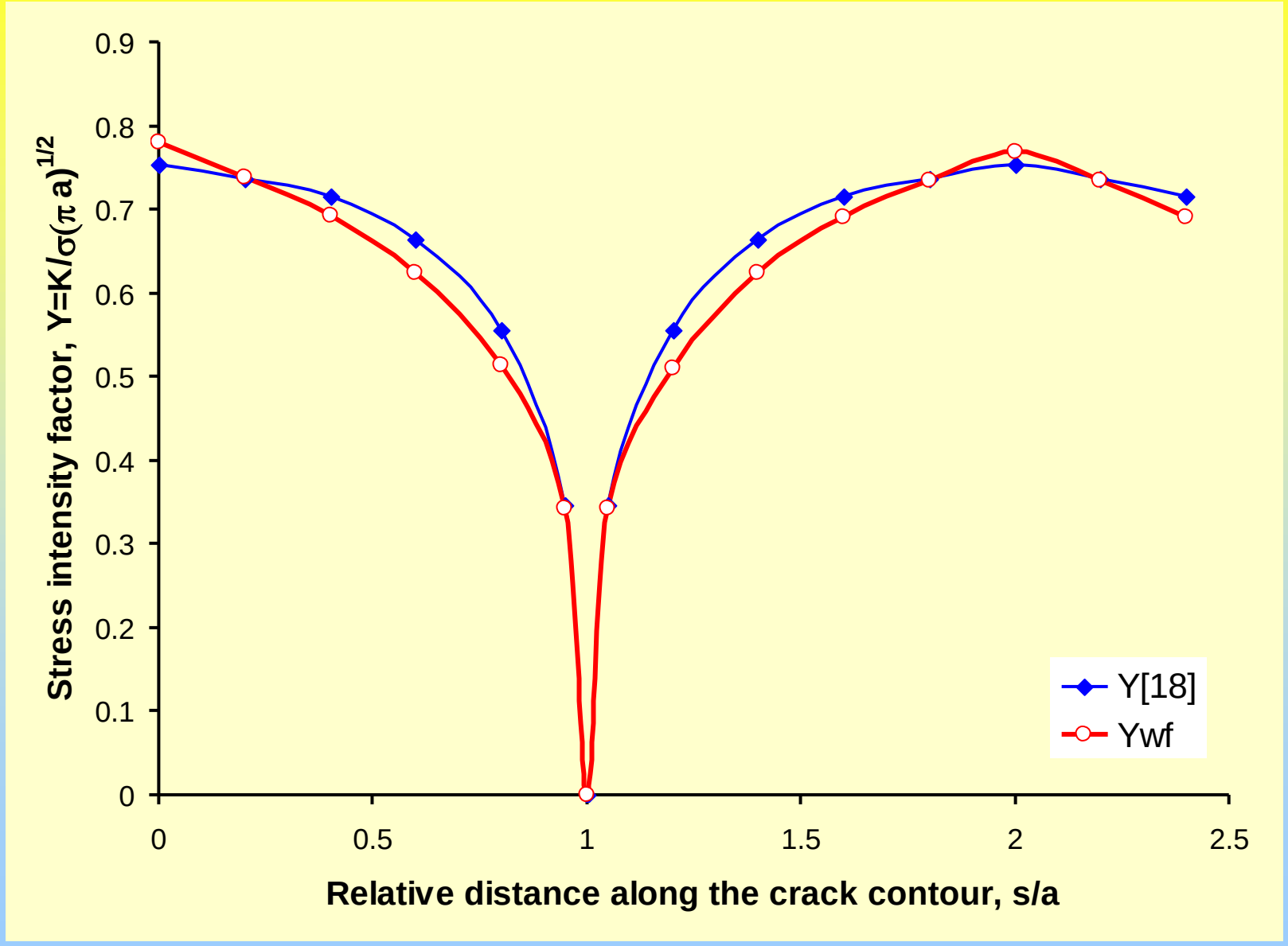
Square crack under uniform tension





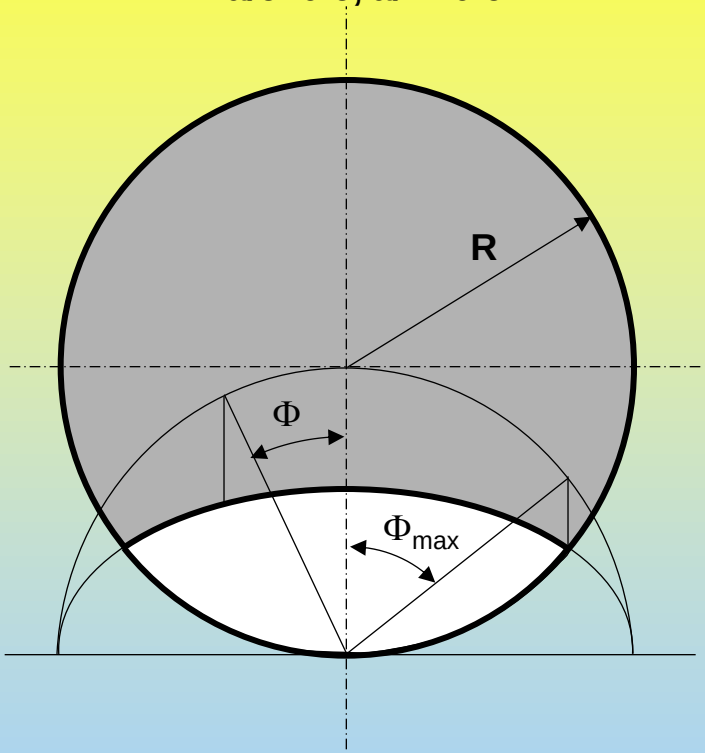
Square crack under uniform tension – comparison with FEM data

(Murakami's Handbook of SIFs)

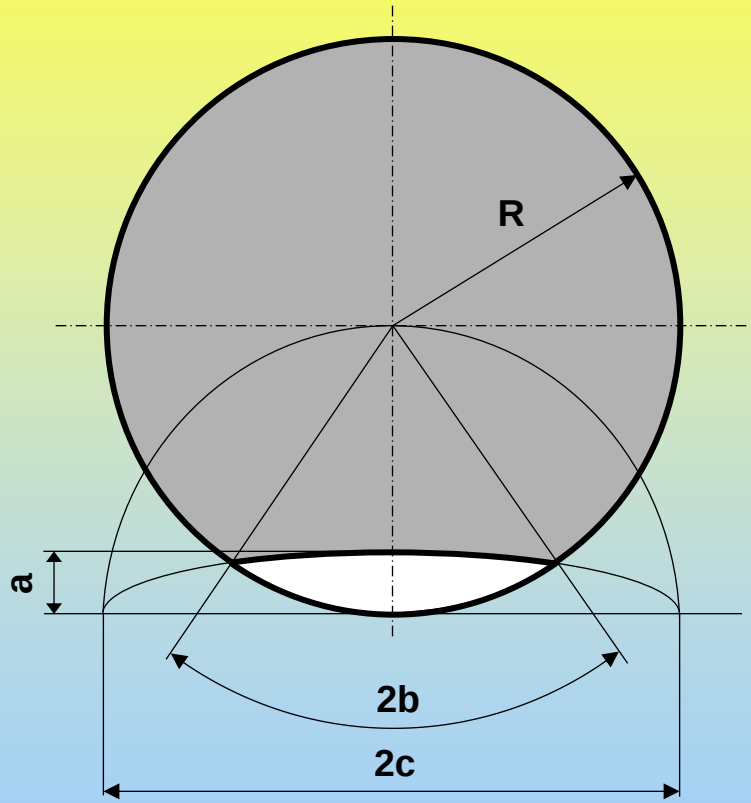


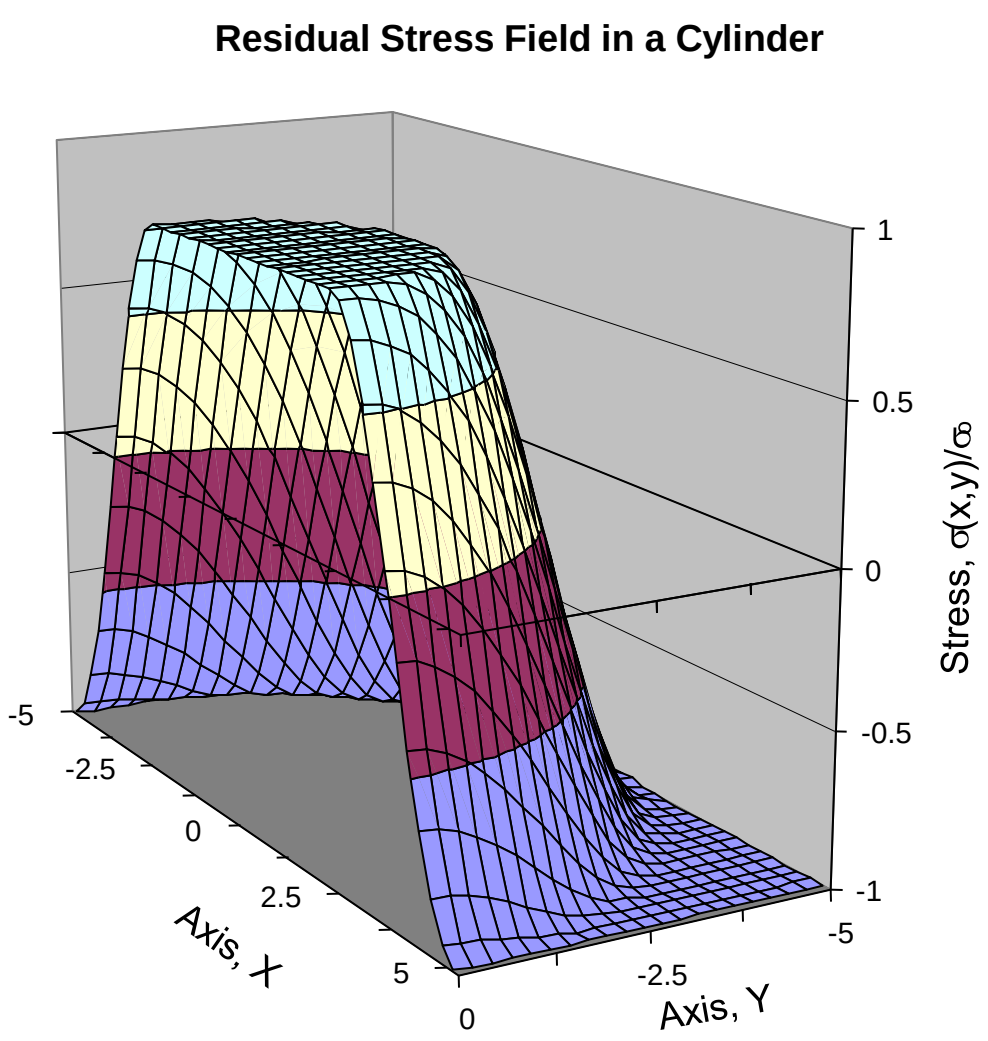
Semi-elliptical surface cracks in cylindrical bars

a/c=0.6, a/R=0.6



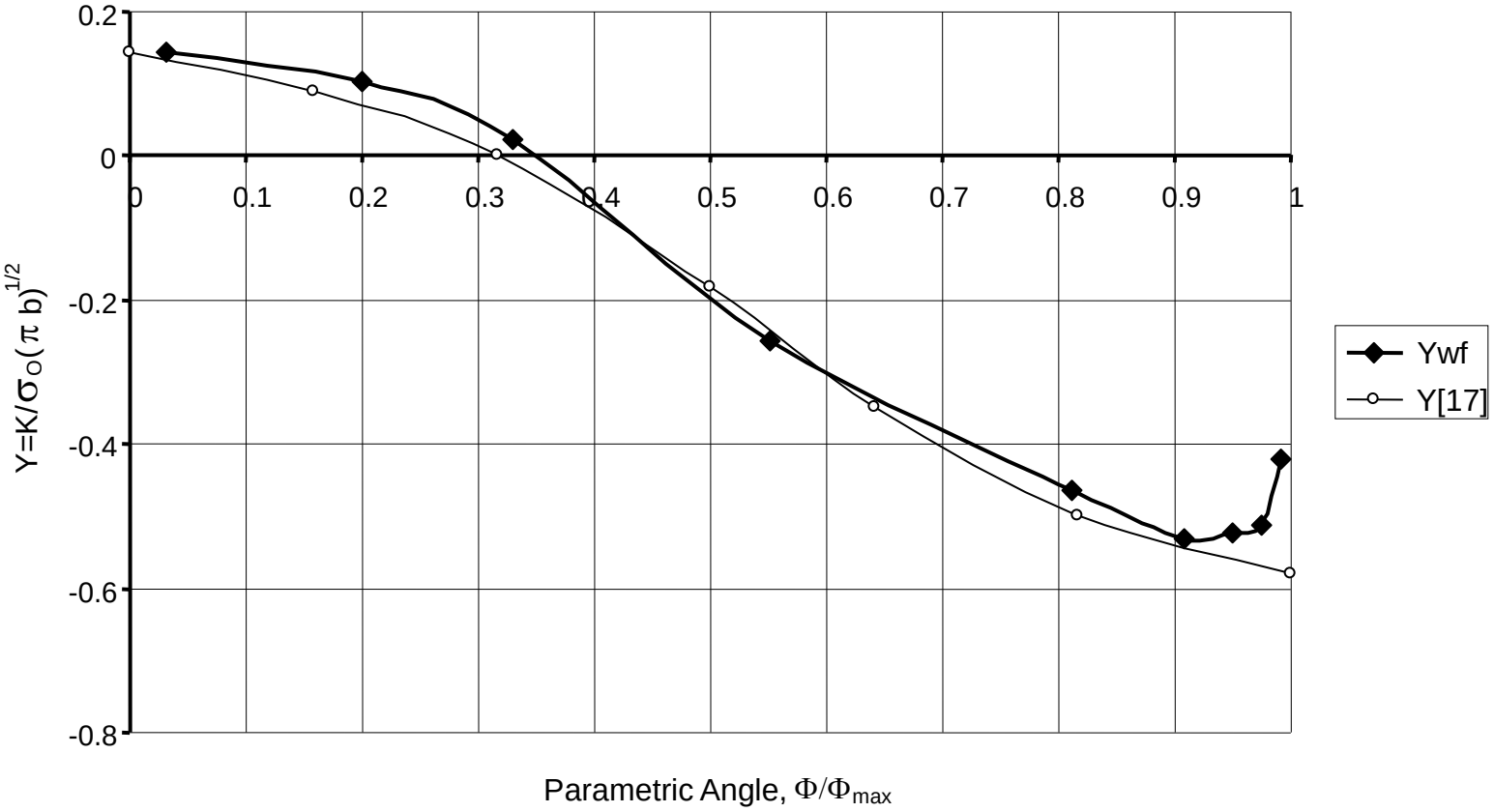
a/c=0.2, a/R=0.2





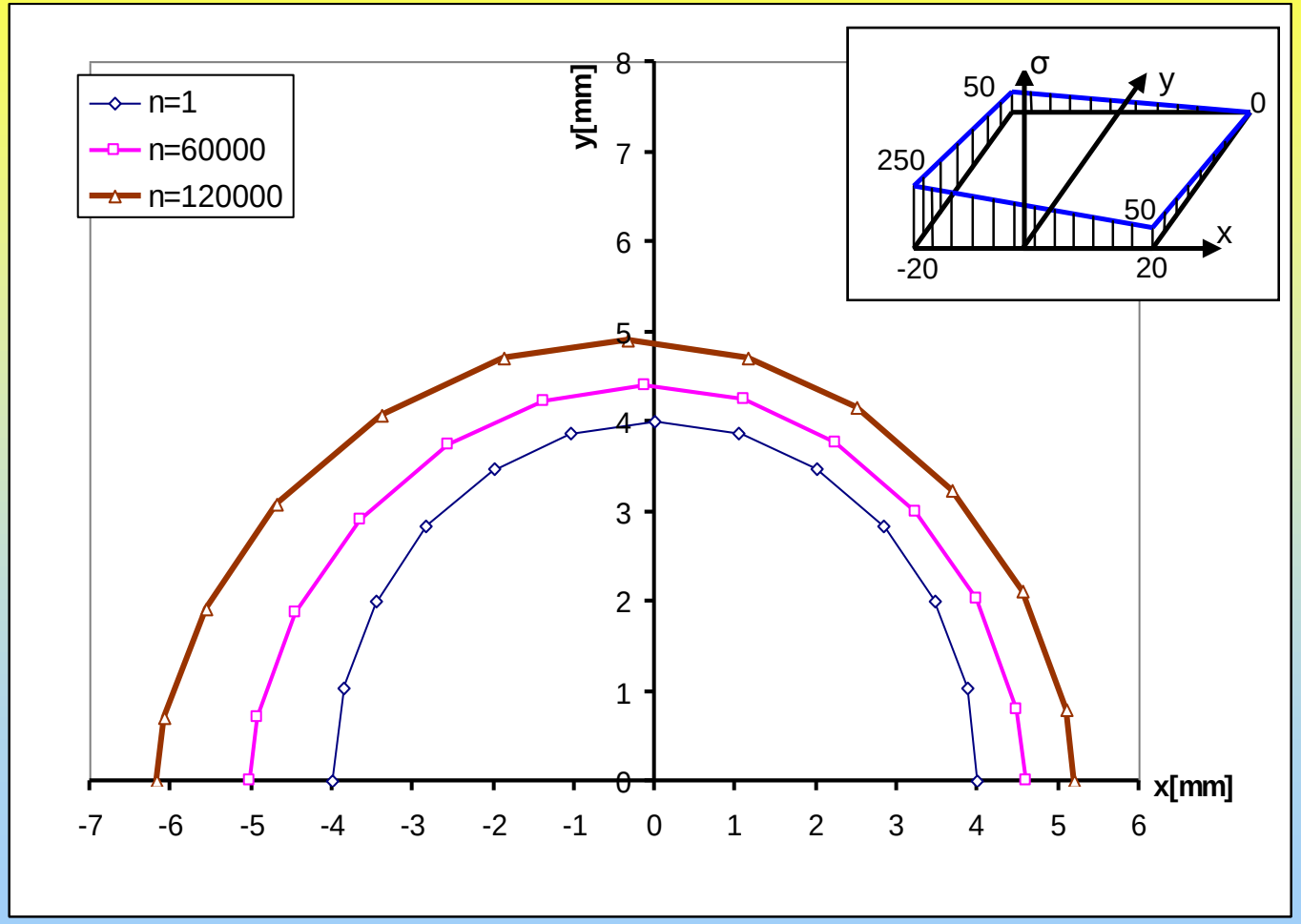
Residual stress field in a cylindrical bar [17]

SIF Comparison for Semi-elliptical Crack in a Cylinder with Residual Stresses

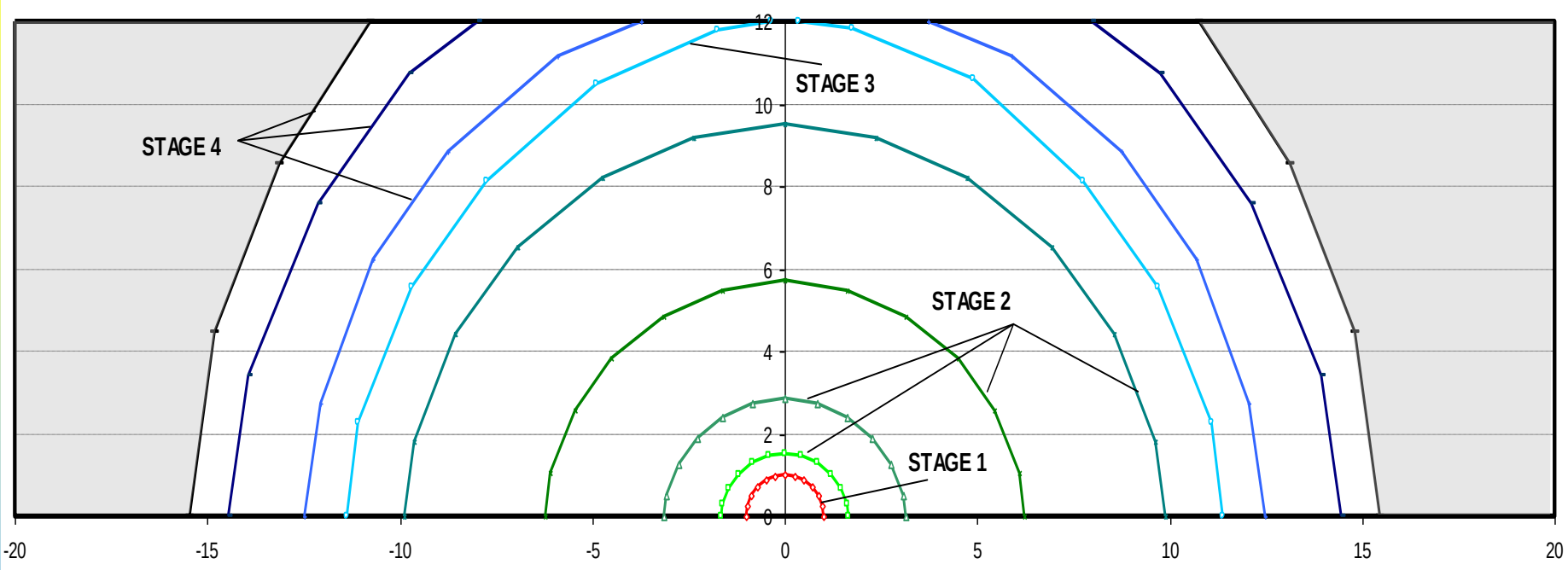


Comparison of SIF for semi-elliptical surface crack in a cylindrical bar subjected to 2-D residual stress field, ac=0.6, a/R=0.6

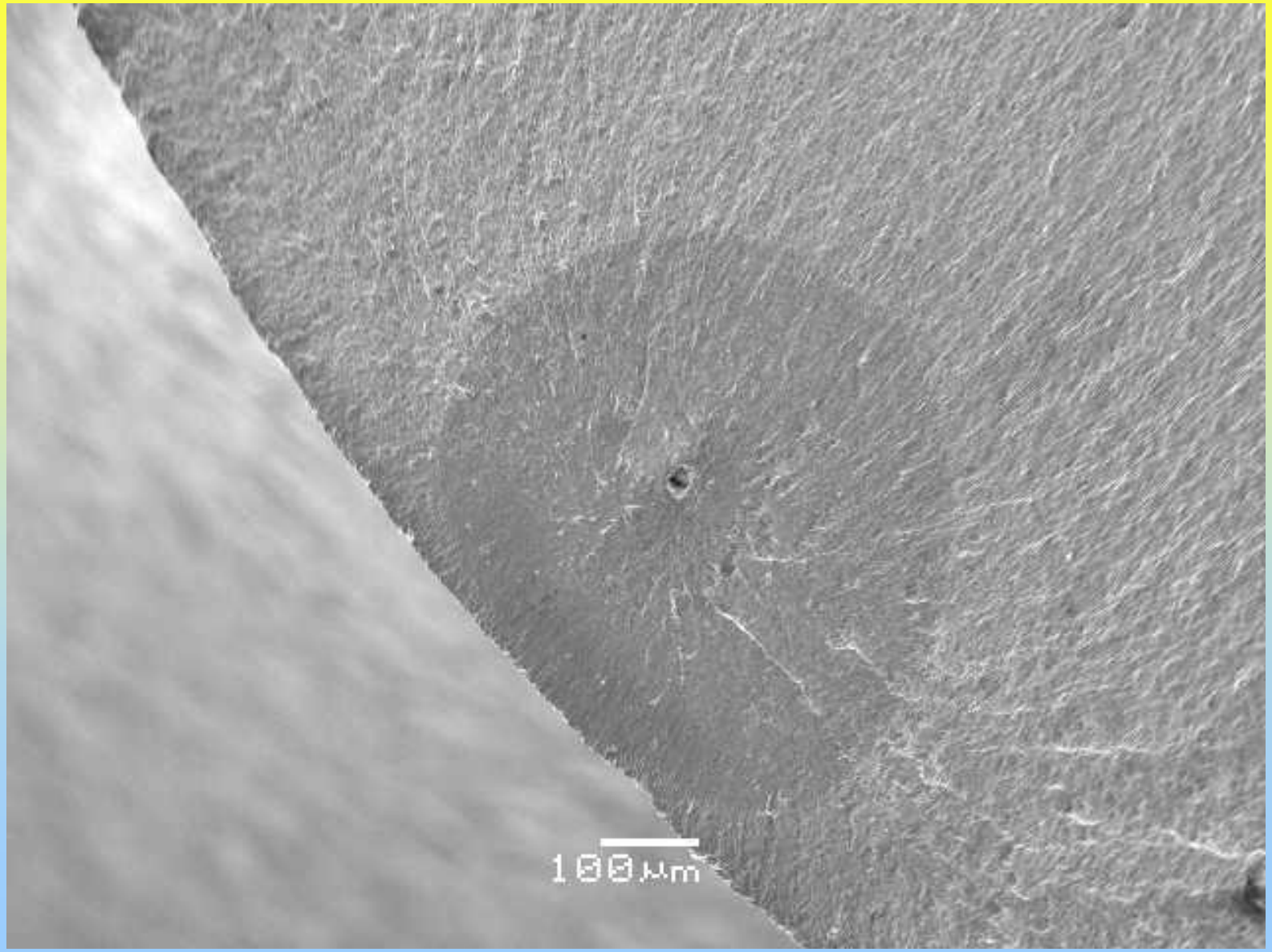
Semi-elliptical cracks in finite thickness plate

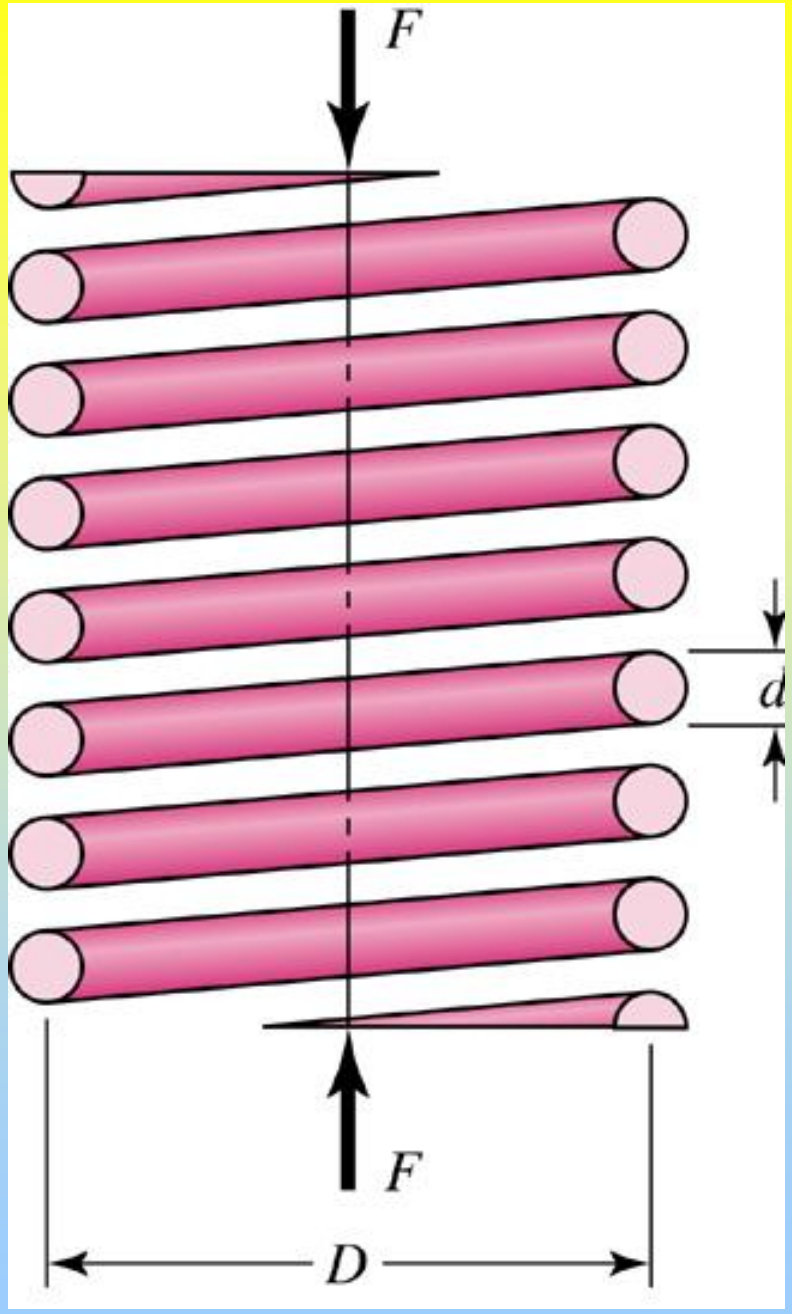


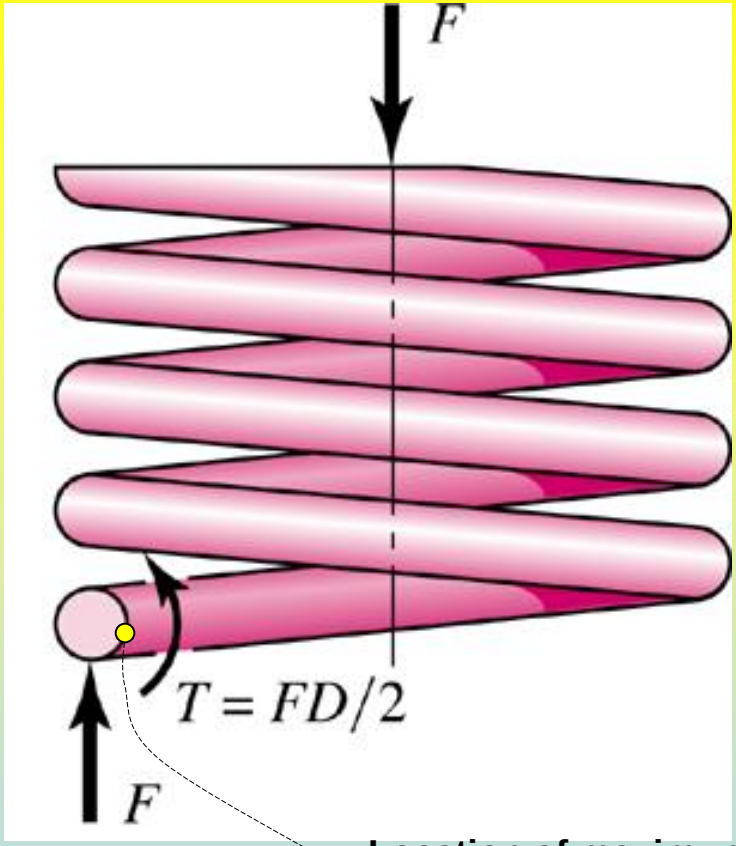
Semi- elliptical/tunnel crack under uniform stress distribution



Relative dimensions of the inclusion (d=20-30 μm) and the final crack size ($2a_f$= 700 μm)





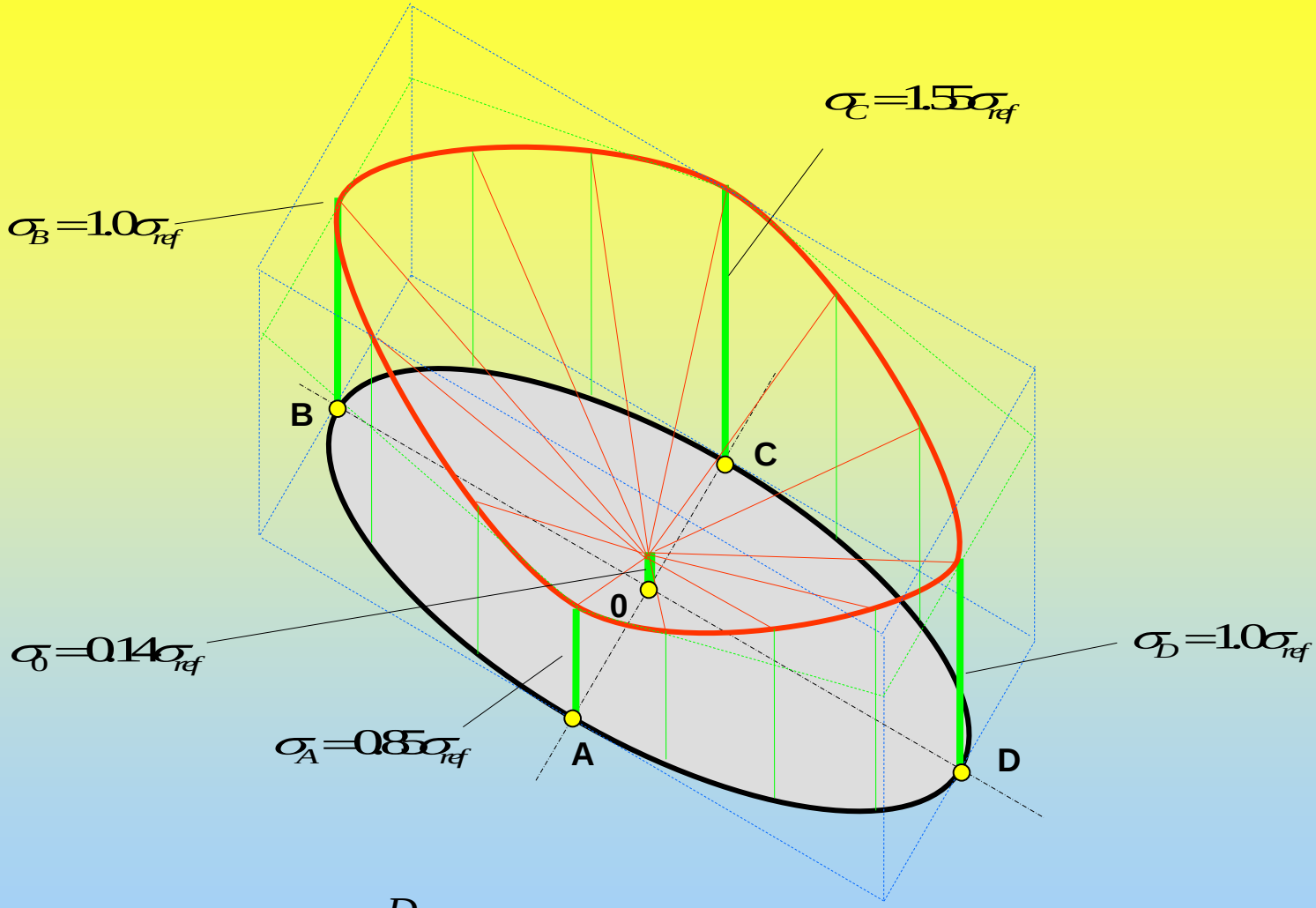


Location of maximum shear stress

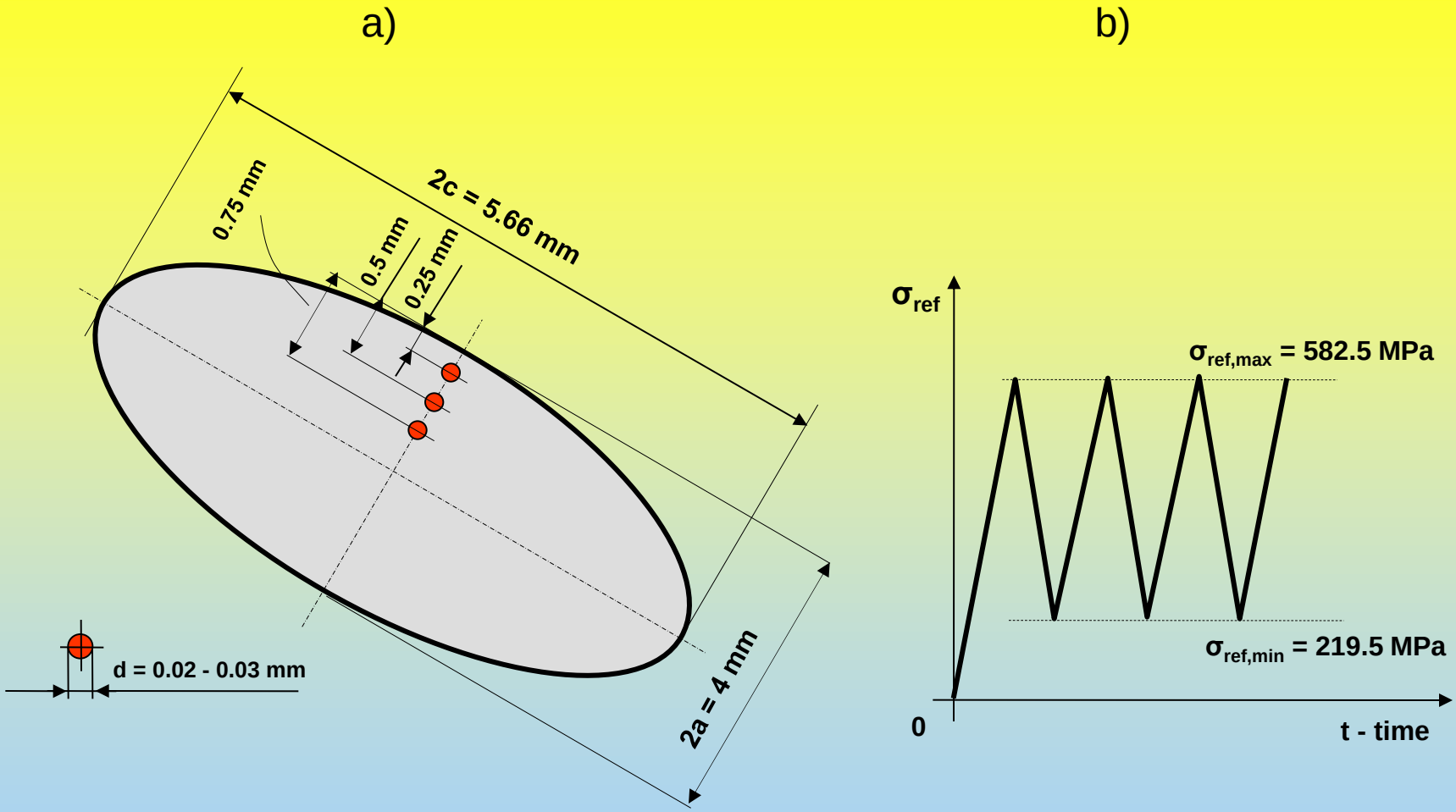
D=18.5 mm, d=4 mm

Fig. 10. Geometry and dimensions of the spring

Distribution of stresses in the critical cross section



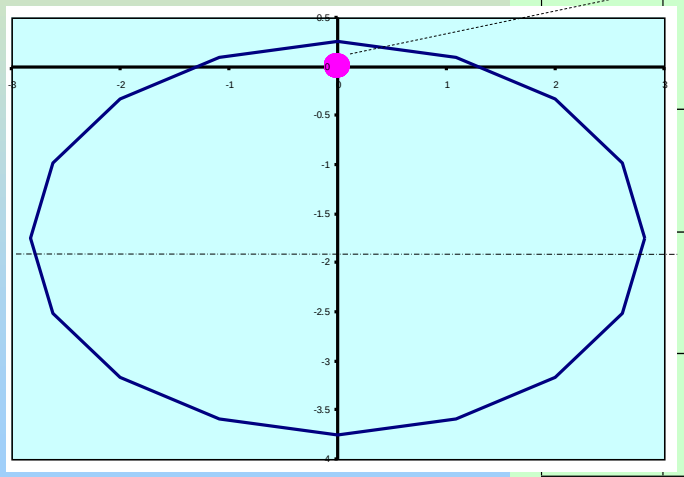
$$\sigma_{ref} = \sigma_{torque} = \frac{16T}{\pi d^3} = \frac{16F\,\frac{D}{2}}{\pi d^3} = \frac{8FD}{\pi d^3} = 4.625\,\frac{8F}{\pi d^2}$$

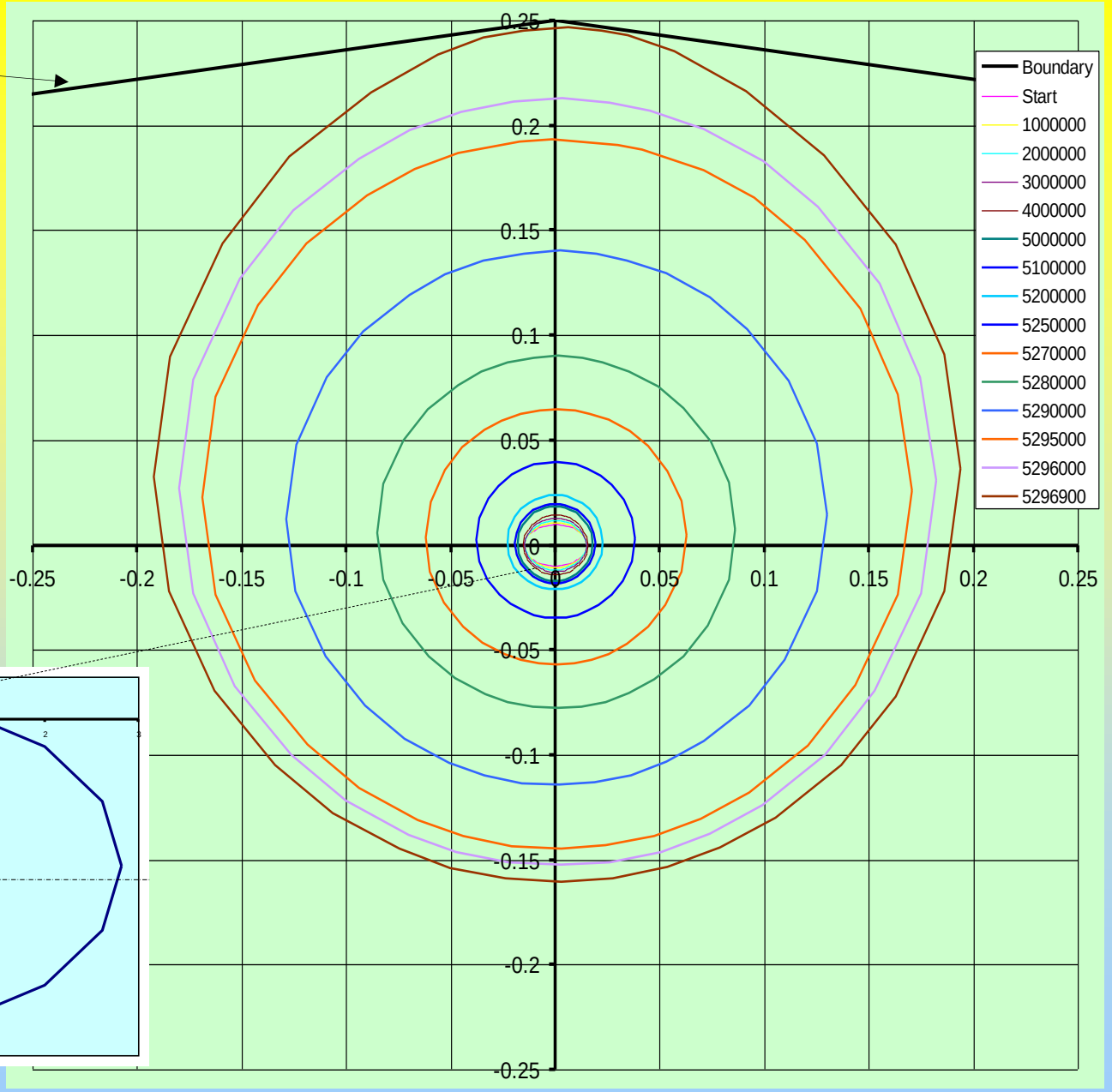


Locations of initial cracks (flaws) in the cross section; assumed penny shape crack of size $2a_i$ =0.02 mm and elliptical crack 0.02/0.03 mm, b) Cyclic history of the reference stress σ_{ref}

Edge of the cross section

N_f = 5,296,900 cycles





Fatigue crack growth; d=0.03x0.02 mm, depth 0.25 mm, $\sigma_{C,\,max}$= 1030 MPa, $\sigma_{C,\,min}$= 390 MPa