Calculating Stress Intensity Factors for Countersunk Holes

AFGROW Users Workshop

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September 12, 2012
Acknowledgments

- T-38 ASIP
- A-10 ASIP
- NASGRO
- SwRI
- UTSA (Thesis advisor Dr. Yusheng Feng)
Presentation Outline

- Background
- Research Objectives
- Assumptions/Questions
- Experimental Investigation
- Solution Space Investigation and Selection
- Computational Methodology
- Final Solution Space Interpolation
- Validation and Error Analysis
- Results/Conclusions
Background

- Countersunk (CS) holes are commonly found in aerospace structures
- There is currently no standardized, validated approach for approximating CS SIFs
- CS SIF approximation rules-of-thumb produced inconsistent results
Research Objectives

1. Develop optimized solution space for the most common aerospace countersunk hole geometries
2. Establish a database of stress intensity factors (SIFs) for solution space
3. Propose a simplified, easy-to-implement, accurate means for approximating SIFs from within the solution space
4. Validate and verify results plus basic error analysis
5. Compare CS vs. Non-CS crack growth rates
Other Assumptions/Questions

- Elliptical crack shape, “a” become a virtual length after through crack transition
- Center of ellipse does not shift as crack grows
Experimental Investigation

- Back calculating $K_1$
- $K_1 = \Delta K_1 / (1 - R)$
- The experimentally derived $K$ values can be compared to $K_1$ values from FE models
Experimental Investigation - Continued

- Countersink hole experimental results
- Results show good repeatability
- Jitter due to through crack transition
Experimental Investigation - Continued

- Non-countersink experimental results
- Results show good repeatability
- Jitter due to through crack transition
Countersunk hole marker band test results showed that the ellipse approximation was valid for geometry, material and loading that was tested.

Center of ellipse did not shift as crack grew (short cracks)
Non-countersunk hole marker band test results showed that the ellipse approximation was valid for geometry and loading that was tested.

Center of ellipse did not shift as crack grew (short cracks)
Solution Space Investigation

- 12 loading and geometry variables
- Inputs can be divided into 3 categories
- A solution space for 12 inputs is extremely large!
- Number of inputs must be reduced to a manageable number

<table>
<thead>
<tr>
<th>Category</th>
<th>Input Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading</td>
<td>Remote Tension ($\sigma$)</td>
</tr>
<tr>
<td></td>
<td>Bending</td>
</tr>
<tr>
<td></td>
<td>Bearing</td>
</tr>
<tr>
<td>Part Geometry</td>
<td>Hole diameter (d)</td>
</tr>
<tr>
<td></td>
<td>Countersink depth (cd)</td>
</tr>
<tr>
<td></td>
<td>Countersink angle ($\alpha$)</td>
</tr>
<tr>
<td></td>
<td>Plate thickness (th)</td>
</tr>
<tr>
<td></td>
<td>Plate width (w)</td>
</tr>
<tr>
<td></td>
<td>Edge distance (ed)</td>
</tr>
<tr>
<td>Crack Geometry</td>
<td>Initiation location(s)</td>
</tr>
<tr>
<td></td>
<td>c Crack length</td>
</tr>
<tr>
<td></td>
<td>a Crack length</td>
</tr>
</tbody>
</table>

$4^{12} = 16,777,216$ solutions. 320 years to solve @ 10min/sol
Solution Space Investigation - Continued

- Four possible crack initiation locations
- Knee or base of CS has the highest stresses for remote tension loading
- Only base or knee of CS is evaluated

![Diagram showing countersink knee or base, faying surface, bore, and countersink surface intersection]
Solution Space Investigation - Continued

- Aircraft survey used to investigate hole diameter \((d)\), plate thickness \((th)\), and countersink depth \((cd)\). Real-world aircraft data
- Four aircraft were surveyed
- Results show a very organized distribution that has an engineering explanation
Solution Space Investigation - Continued

- 25 d/th vs. cd/th points
- cd/th ranges from 0.001 to 0.95

![Graph showing the relationship between d/th and cd/th with different aircraft data points.](image)
Solution Space Investigation - Continued

- Five inputs reduced to four inputs for final solution space
- Total number of FE solutions optimized 1750

<table>
<thead>
<tr>
<th>Category</th>
<th>Input Description</th>
<th>Solution Space Input</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part Geometry</td>
<td>Hole diameter (d)</td>
<td>d/th</td>
<td>0.3</td>
<td>2.6</td>
<td>5</td>
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<tr>
<td></td>
<td>Countersink depth (cd)</td>
<td>cd/th</td>
<td>0.001</td>
<td>0.15 to 0.99</td>
<td>5</td>
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<tr>
<td></td>
<td>Plate thickness (th)</td>
<td>th/th</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Crack Geometry</td>
<td>c Crack length</td>
<td>c/th</td>
<td>0.002</td>
<td>2.5(d/th)</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>a/c</td>
<td>a/c/th</td>
<td>0.5</td>
<td>4</td>
<td>5</td>
</tr>
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</table>
Computational Methodology

- The hp-version of the FEM was implemented
- Half symmetry used to reduce model DOF
Computational Methodology - Continued

- CIM K1 extraction study revealed K1(r) (3D only)
- For points near a free surface, extraction points were manually selected to be slightly inside to mitigate free surface effects
Computational Methodology - Continued

- **Improving K1 accuracy**

- **Note that extraction angle changes K1 results**

- **a/c vs. % difference**

```
<table>
<thead>
<tr>
<th>a/c</th>
<th>Mean %</th>
<th>Dif.</th>
<th>STDV</th>
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<tr>
<td>0.5</td>
<td>4.02</td>
<td>1.90</td>
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<td>1.0</td>
<td>2.12</td>
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<tr>
<td>1.5</td>
<td>1.32</td>
<td>1.00</td>
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<tr>
<td>2.0</td>
<td>0.88</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>0.56</td>
<td>0.56</td>
<td></td>
</tr>
</tbody>
</table>
```

"c" crack tip data
Shift for K1 extraction location for through crack transition
Solution Space Interpolation

- All calculations are done in real-time from an excel worksheet
- Real-time feedback given to mitigate errors
- Countersunk beta compounding factor ($\beta_{csx}$) used to quantify the effect of adding a SC to a straight hole (SH), c tip only.

\[
\beta_{csx} = \frac{K_{1cs}}{K_{1SH}}
\]
Solution Space Interpolation - Continued

- $d/\theta$ vs. $c_d/\theta$ is not rectangular
- Partial coordinate transformation used to "square" this part of the solution space
- Horizontal lines $H_1$ through $H_5$ fit for each $d/\theta$ ratio in solution space

**$a/c = 0.5$**

![Graph showing $Y = 2X$ relationship](image)

- $H_5$
- $H_5$ Interp K1
- $H_4$
- $H_4$ Interp K1
- $H_3$
- $H_3$ Interp K1
- $H_2$
- $H_2$ Interp K1
- $H_1$
Solution Space Interpolation - Continued

- Interpolation steps
  - Interpolate $K_1(c)$ for each point on each H line, do again for all $a/c$ ratios
  - Interpolate $K_1(o)$ on each of the above lines, for all $a/c$ ratios.
  - Interpolate $K_1(a/c)$ from above step
  - Multiply $K_1$ by $\sqrt{th}$ to convert $K_1$ to real world space
Solution Space Interpolation - Continued

- Countersunk beta compounding factor ($\beta_{csx}$) used to quantify the effect of the CS
- The $\beta_{csx}$ factors can be used directly in AFGROW or NASGRO to approximate the effects of the CS
- Double parabolic interpolation used for all calculations

\[ \beta_{csx} = \frac{K_{1cs}}{K_{1sh}} \]
Experimental validation results showed good agreement between experimentally calculated K1 value and FE derived K1 values.

Material properties may be reason for slight offset observed for longer crack lengths.
Validation & Error Analysis - Continued

- Results compared to other published and industry standard solutions
- Results show good correlation
- Convergence study used to minimize discretization and modeling error
- Global energy norm error was low (<1%) for all solutions
Interpolation error less than 3.3% for 95% of approximated solutions

Some of this error may be due to variations in K1 extraction angle
Total or maximum real world error approximated at 7.3% for 95% of solutions.

Apparent error should be less than this as some errors may counteract each other.

\[ \|E_{SIFtot}\| \leq \|E_{interp}\| + \|E_{K1EXT}\| + \|E_{modeling}\| + \|E_{discretization}\| \]
Results, Discussion, Conclusion

- How does adding a CS to a hole effect crack growth rates?
- For a CS hole in a wide plate, fatigue life can be reduced by 23.2% if CS is deep.
- These results will vary based on CS depth and other factors.
Countersunk hole K1 SIFs and beta estimates can now be quickly and approximated for the most common aerospace geometries.

A first for this type of research.
Elliptical crack shape approximation was shown to be good within the limits of the geometry tested (short cracks).

Future work should include:

- Evaluating pin/bearing and bending loads
- Careful look at the solution space developed for this research to see how it may be further optimized
- Possibly do some additional 'cleaned up' of SIF solutions
Questions?
Appendix Slides
Solution Space Dimensions

- Five input dimensions
  - Plate thickness (th)
  - Hole diameter (d)
  - Countersink depth (cd)
  - c-tip Crack length (c)
  - a-tip Crack length (a) or faying surface crack length (ct)

- Normalized by th to reduce to four input dimensions
  - d/th
  - cd/th
  - c/th
  - a/th
LEFM Overview

- Linear elastic fracture mechanics (LEFM) theory:
  - Used to make fatigue life predictions
  - Assumes an initial flaw size and shape
  - Calculates the incremental crack growth, $\Delta a$, as a result of applying load cycles

\[ a_{i+1} = a_i + \Delta a \]
LEFM theory suggests that the stress intensity (SIF) at the crack tip, $K$, can be calculated as a function of the remote stress, $\sigma$, $\sqrt{\pi a}$ and the geometry factor, $\beta$.

\[ \Delta K = \Delta \sigma \sqrt{\pi a \beta} \]

\[ \Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}} \]

\[ R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \]
1965: Richard E. Whaley, birefringent plastic coatings around countersunk holes to develop stress-concentration factors

- Surface only
- Fatigue tests showed higher stress in countersink

1978: Y. F. Cheng, stress freezing technique to measure the stress through the thickness

- Highest stress for tension is at base of countersink
1983: J.C. Newman and I.S. Raju used FEA to evaluate cracks growing in 3-D bodies

- Four geometries (no countersink)
- Empirical SIF equations
- First widely accepted research using FEA to calculate SIFs

1987: FRANC3D, program for automated crack growth research begins

1992: K. N. Shivakumar and J.C. Newman applied FEA to get stress-concentrations for straight-shank and countersunk holes in a large plate subjected to various loading conditions

- No cracks
- FORTRAN program still used today
1994: J.B. Young and K.K. Lee published stress concentration design equations for countersunk hole
- Only tension loading, no crack
- Coarse mesh used and equation was overly conservative

1994: P. W. Tan et al published set of SIF solutions for cracks growing from CS hole
- Remote tension only
- FEA
- Three crack locations
1998: M. Gosz and B. Moran use FEM to extend set of SIF solutions for countersunk holes
  - Only tensile loading

1998: S.A. Fawaz used Three-dimensional virtual crack closure (3D VCCT ) to evaluate large cracks
  - No countersink
  - Three types of loading
  - Used in AFGROW SIF database
1999: A. Rahman et al

- Three types of loading
- Faying surface only
- Global, Intermediate, Local (GIL) meshing
- J-integral using equivalent domain integral method (EDIM)

2002: C. Brooks et al use p-version FEM

- Countersunk hole with three types of loading
- Faying surface only
2002: G. P. Nikishkov and S. N. Atluri combine SGBEM and FEM to model 3-D crack growth

2003: C. Y. Park combine FEM and lab testing to evaluate countersunk holes
- Poly(methyl methacrylate) (PMMA)
- V&V
- h-version FEM
- Three types of loading
- Limited geometry

2004: J. Rungamornrat, Mark E. Mear use SGBEM to calculate SIFs for crack in 3-D bodies
2005: J. Suh expands on Park’s 2003 work adding
  - P-version FEM
  - Additional testing
  - FRANC3D BEM
  - Limited geometries

2007: K. N. Shivakumar develop new stress concentration equation for countersunk holes
  - No crack
State-of-the-Art

Computer Analysis Tools

- **FEM**
  - p-version (StressCheck and others)
  - h-version (ANSYS, ABAQUS, …)

- **BEM**
  - FADD3D (UT Austin)
  - FRANC3D (Cornell)
  - BEASY (Commercial)
Proposed Solution Approach

1. Calculate comprehensive set of SIF solutions for cracks emanating from countersunk holes
   - Primary computation with p-version FEM
   - Convergence study for error
   - Lab testing for validation
   - Literature and BEM comparison for verification

2. Develop easy to implement method for applying SIFs
   - Empirical equations?
   - Database?
   - Software?
This study compares three analysis methods:

- Hole with no countersink
- Hole with countersink, T-38 stress compounding applied to account for countersink
- Hole with countersink, FEM used to account for countersink
Linear Elastic Fracture Mechanics (LEFM) and the FEM

- Must use FEM to calculate $\beta(a)$ for different crack lengths.

For mode I crack growth:

$$\beta = \frac{K_I}{\sigma \sqrt{\pi a}}$$

- $K_I$ SIF is obtained from the finite element analysis (FEA)
LEFM and the FEM (Continued)

- LEFM is used to calculate crack growth rates
- Crack growth rate:
  \[ \frac{da}{dN} = f(\Delta K) \]
- \( a \) = crack length
- \( N \) = cycle number
- \( \Delta K = \Delta \sigma \sqrt{\pi a \beta(a)} \)
- \( \sigma \) = remote stress
- \( K \) = stress intensity factor (SIF)
- \( \beta(a) \) = geometry correction factor
Mathematical Model and Solution Approach

- Used StressCheck (SC) to model the crack
- SC is a “p” element FEA program.
- p elements are well suited for modeling cracks
- SC utilizes the contour integral method (CIM) to compute SIFs

CIM Overview

\[ K_I = \sqrt{2\pi} A_1^{(1)} \]
\[ K_{II} = \sqrt{2\pi} A_1^{(2)} \]

\[ A_i^{(m)} \approx \int_{\Gamma} (w_m T_{FE} - u_{FE} T^{(Wm)}) ds \]

Where:
- \( w_m \) = extraction function
- \( T_{FE} \) = traction vector computed from the FE solution \( u_{FE} \) along the circle \( \Gamma \)
- \( T^{(Wm)} \) = traction vector computed from \( w_m \) along \( \Gamma \)

For more information about how SC applies the CIM, see the SC users manual.
Finite Element Model

- Model symmetries
- Geometry came from T-38 fatigue critical location (FCL)
- Parametric and hand meshed
  - Pentahedron elements
  - Hexahedron elements
Finite Element Model (Continued)

- Tension loading
- Boundary conditions
- Material properties
  - Al 7075-T6
Finite Element Model (Continued)

K1 convergence study used for error estimation

![Graph showing K1 convergence to P = 6](image-url)
Finite Element Model (Continued)

- SC models showed good convergence at \( p = 6 \)
Results and Discussion

- SC was used to calculate beta values for 17 different crack lengths.
- SC beta values were compared to T-38 stress compounded betas and non-countersunk hole betas.
Results and Discussion (Continued)

- Crack growth results

Non-Countersunk hole results:
Life = 2,094

Using the traditional T-38 approach we get:
Life = 1,455

Using the SC betas we get:
Life = 1395
Example Problem Conclusions

- Adding a countersink to the hole evaluated in this investigation significantly shortened the hole’s inspection interval.
- The legacy T-38 stress compounding approach for beta corrections may not be conservative.
- StressCheck shows good $K_I$ convergence at $p = 6$.
- Min. detectible crack size.
- Auto meshing should be evaluated for use with future StressCheck FEA models.
Next Steps
Geometry and Loading

- Tensile loading
- Cracks emanating from countersink knee
Validation Via Lab Testing
– Test Setup –

- Back calculations will be applied to test data to experimentally determine SIFs for different crack lengths

Note: Validation will also be conducted via other published test results
SIF Solution Technique Verification

- Verify numerical solution technique by comparing solutions to other published material
  - Park 2003
  - Suh 2005
### Solution Matrix

- After V&V process is successfully completed, run complete solution matrix

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#### Plate Geometry

<table>
<thead>
<tr>
<th>CS Angle (α)</th>
<th>Finite Width Effects**</th>
<th>Hole radius to plate width ratio**</th>
<th>CS Depth Effect</th>
<th>Hole radius to thickness ratio</th>
<th>Crack depth ratio</th>
<th>Crack aspect ratio</th>
<th>Ellipse Placement**</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A</td>
<td>2*(ed-Q)/w</td>
<td>r/w</td>
<td>h/th</td>
<td>r/th</td>
<td>a/(t-h)</td>
<td>a/c</td>
<td>Ell. Org Hight/th</td>
</tr>
</tbody>
</table>

#### Crack Geometry

| Estimated Min. Permutations | 1 | 5 | 5 | 5 | 5 | 5 | 5 | 1 | = 3,125 | 22 |
| Estimated Max. Permutations | 1 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | = 1,000,000 | 6,944 |

** Note: Total Run time based on 10 minutes per solution estimate

** Indicate items that may be removed from scope

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** 7,000 to 12,000 elements per model
SIF Implementation Technique

Develop an easy to implement means of applying SIF solutions

- Empirical equation
- Computer program
- Other options?
Other Related Topics

- Hand meshing vs. auto meshing
- Poisson's ratio (0.3?)
- COM Enhancements
Conclusion

- The countersunk hole is a three-dimensional problem with many input variables
- Current simplification techniques for countersunk hole analysis are insufficient
- An accurate, easy to implement means to calculate SIFs for countersunk holes is needed