Calculating Stress Intensity Factors for Countersunk Holes

AFGROW Users Workshop

By
Jody Cronenberger
Southwest Research Institute

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Acknowledgments

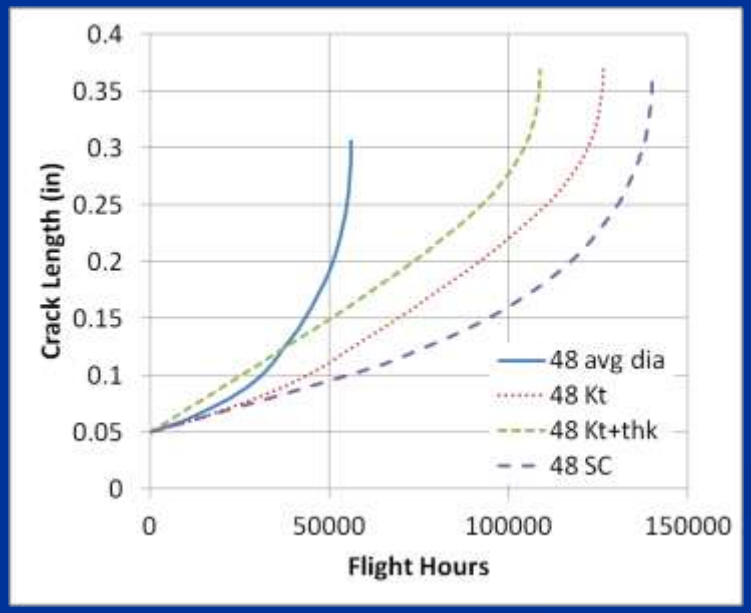
- T-38 ASIP
- A-10 ASIP
- NASGRO
- SwRI
- UTSA (Thesis advisor Dr. Yusheng Feng)

Presentation Outline

- Background
- Research Objectives
- Assumptions/Questions
- Experimental Investigation
- Solution Space Investigation and Selection
- Computational Methodology
- Final Solution Space Interpolation
- Validation and Error Analysis
- Results/Conclusions

Background

- Countersunk (CS) holes are commonly found in aerospace structures
- There is currently no standardized, validated approach for approximating CS SIFs
- CS SIF approximation rules-of-thumb produced inconsistent results

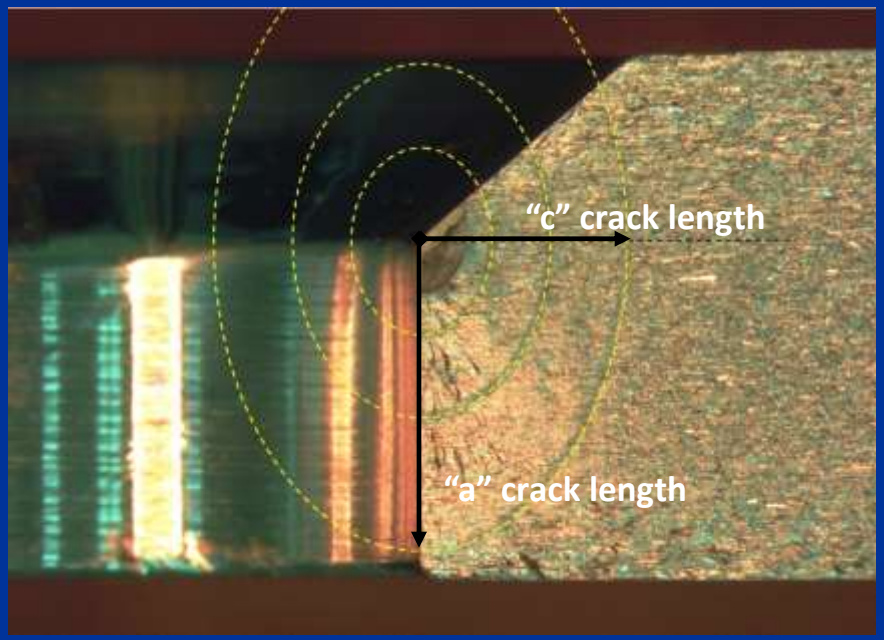


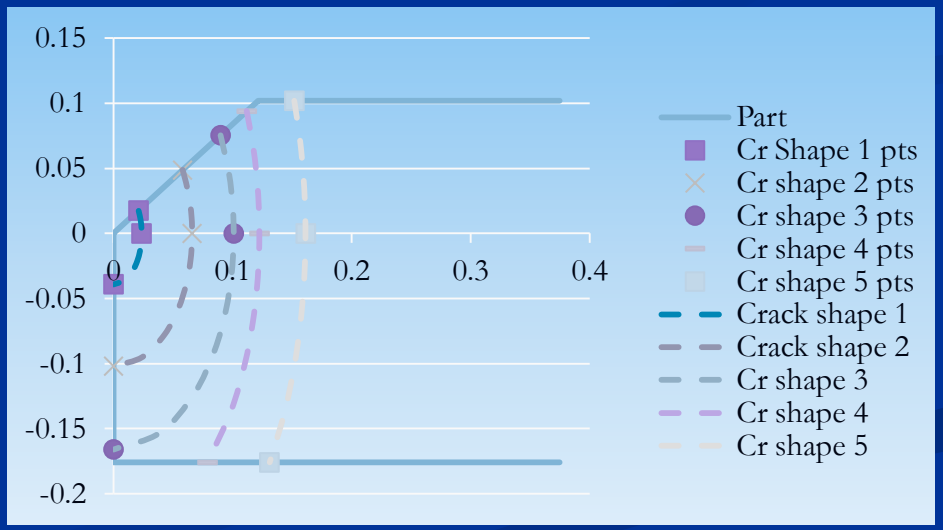
Research Objectives

1. Develop optimized solution space for the most common aerospace countersunk hole geometries
2. Establish a database of stress intensity factors (SIFs) for solution space
3. Propose a simplified, easy-to-implement, accurate means for approximating SIFs from within the solution space
4. Validate and verify results plus basic error analysis
5. Compare CS vs. Non-CS crack growth rates
6. Secondary Objectives: Elliptical shape approximation, Extraction angle…

Other Assumptions/questions

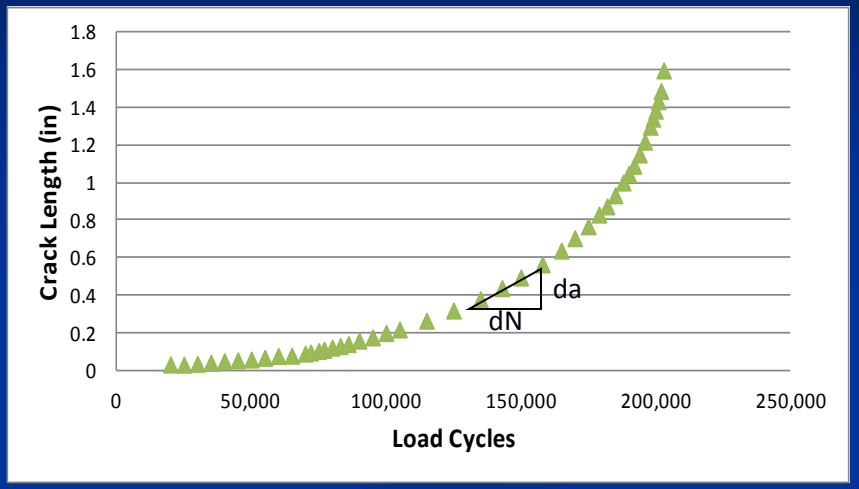
- Elliptical crack shape, "a" become a virtual length after through crack transition
- Center of ellipse does not shift as crack grows

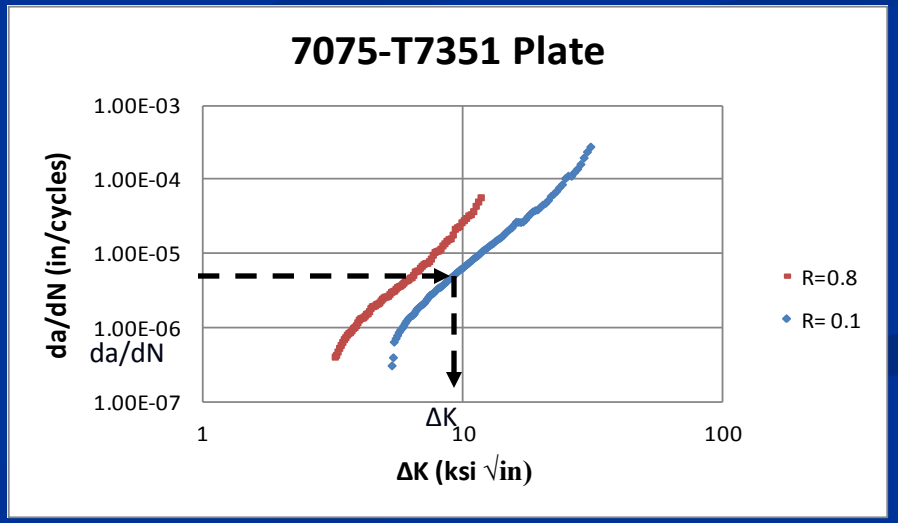




Experimental Investigation

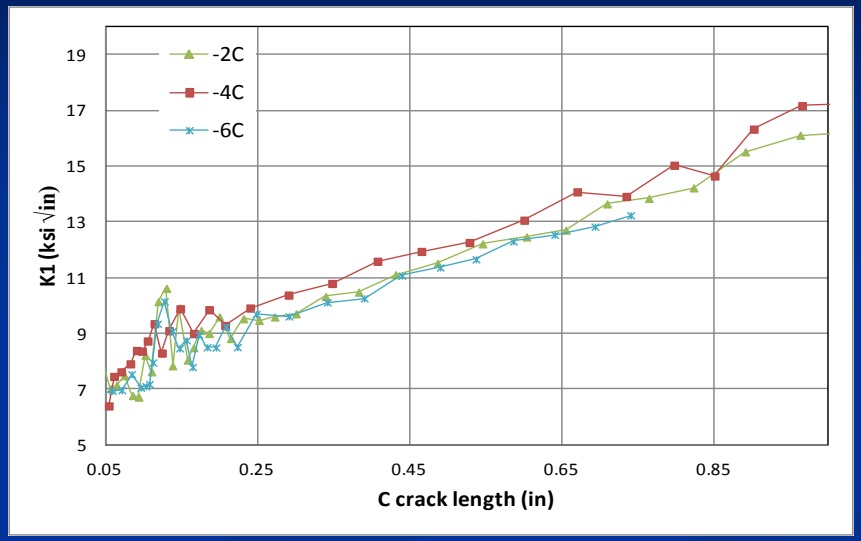
- Back calculating K1
- K1 = ΔK1/(1-R)
- The experimentally derived K values can be compared to K1 values from FE models

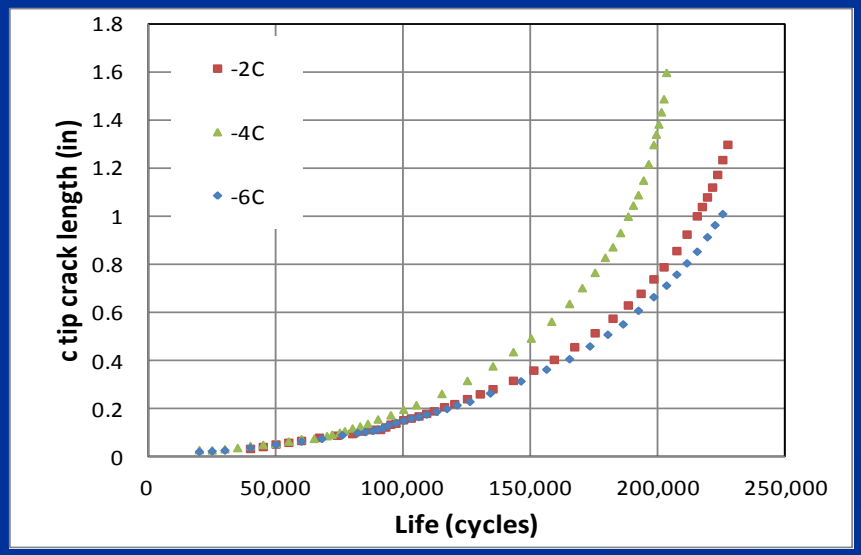


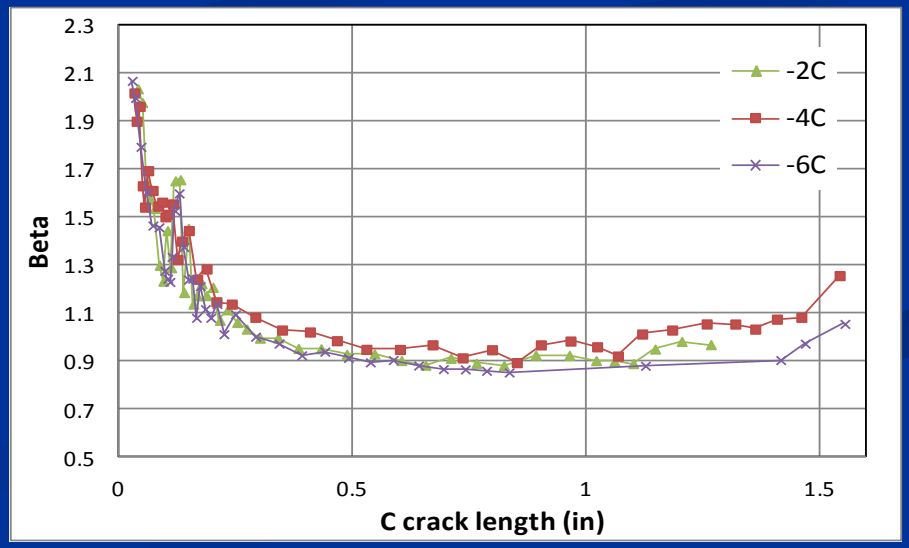


Experimental Investigation - Continued

- Countersink hole experimental results
- Results show good repeatability
- Jitter due to through crack transition

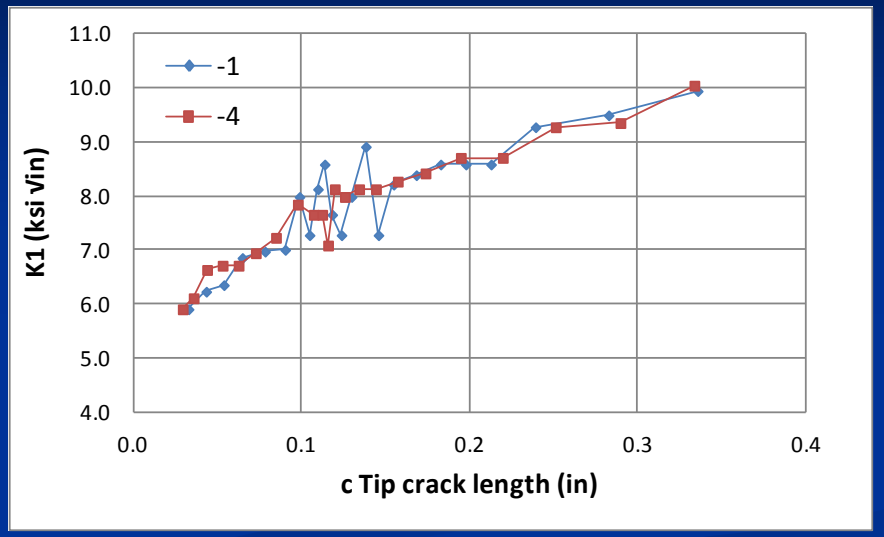


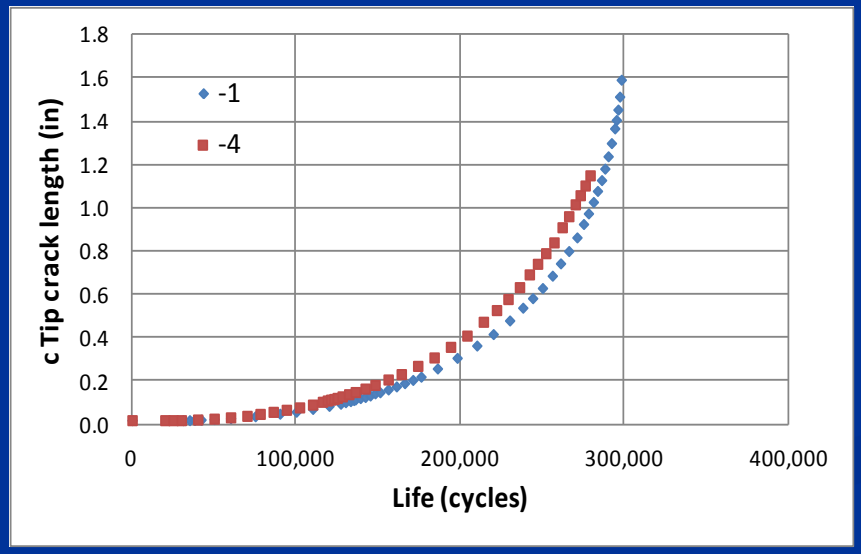


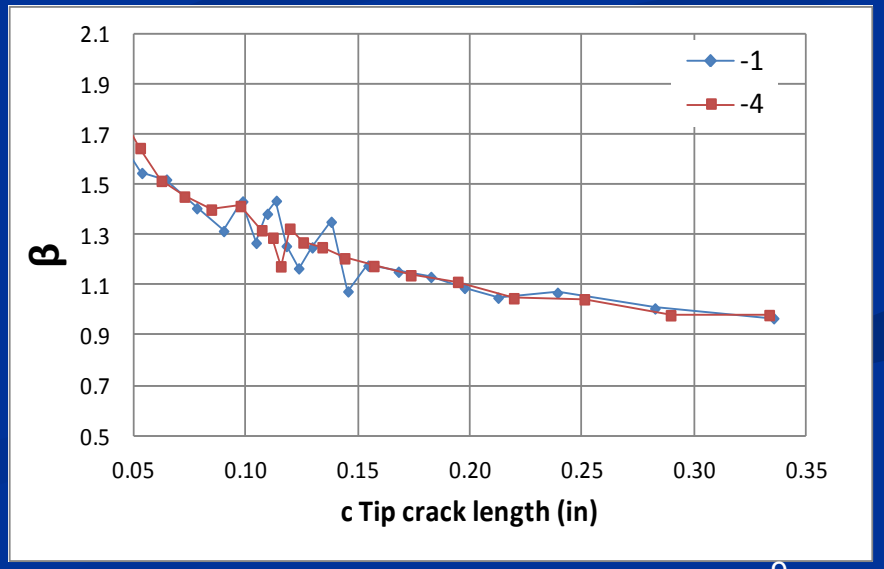


Experimental Investigation - Continued

- Non-countersink experimental results
- Results show good repeatability
- Jitter due to through crack transition

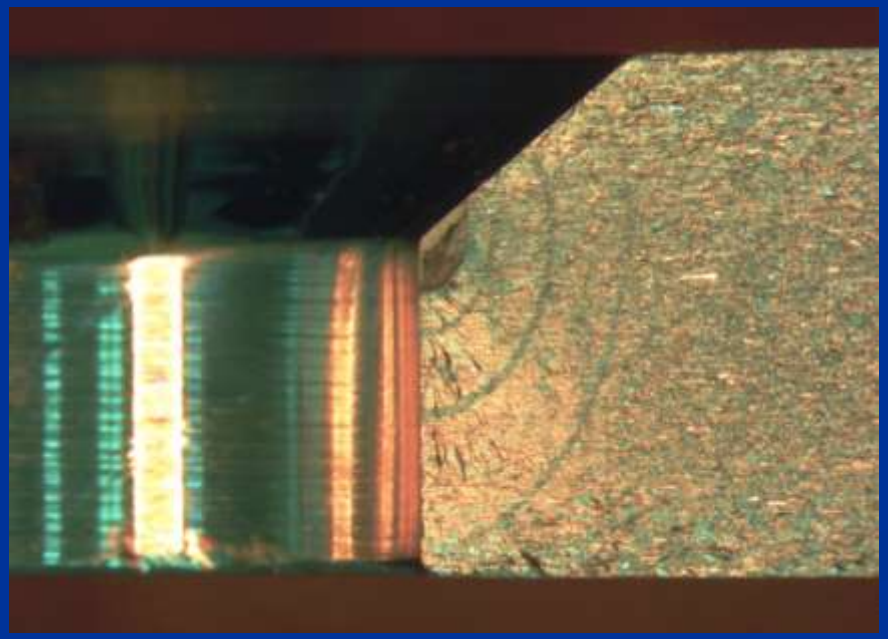


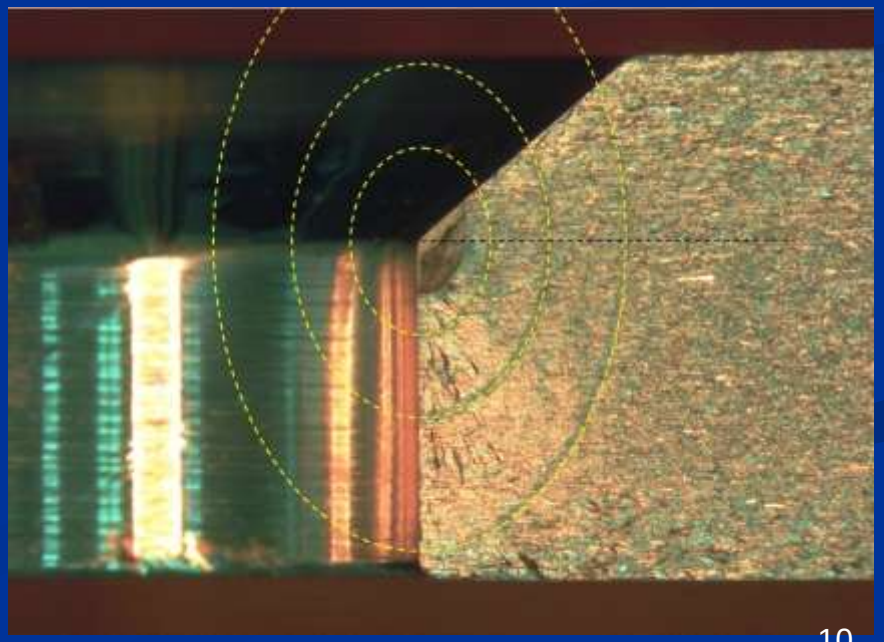




Experimental Investigation - Continued

- Countersunk hole marker band test results showed that the ellipse approximation was valid for geometry, material and loading that was tested
- Center of ellipse did not shift as crack grew (short cracks)





Experimental Investigation - Continued

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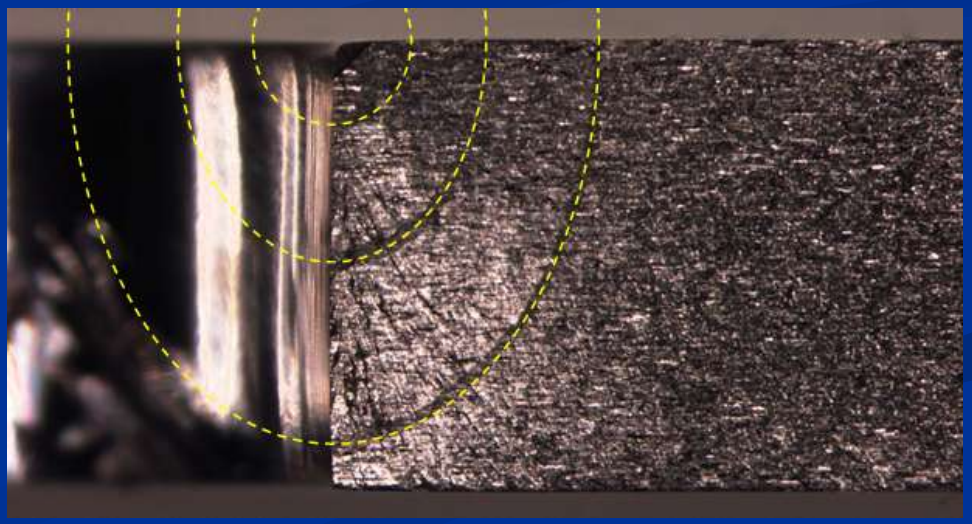
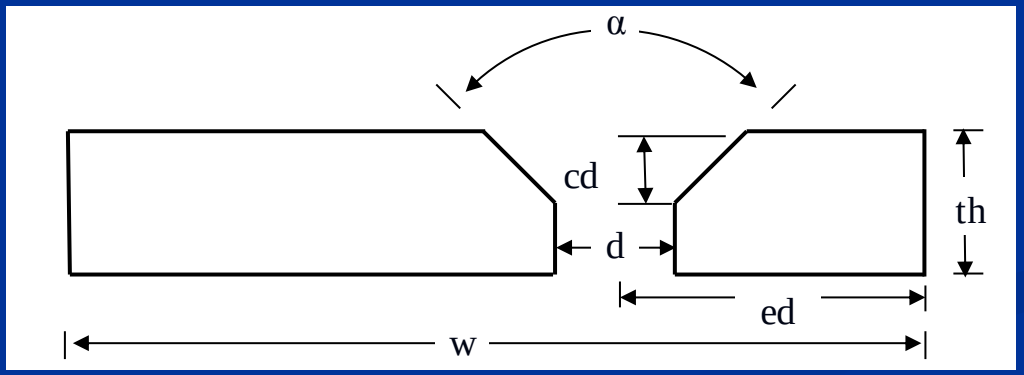


Solution Space Investigation

- 12 loading and geometry variables
- Inputs can be divided into 3 categories
- A solution space for 12 inputs is extremely large!
- Number of inputs must be reduced to a manageable number

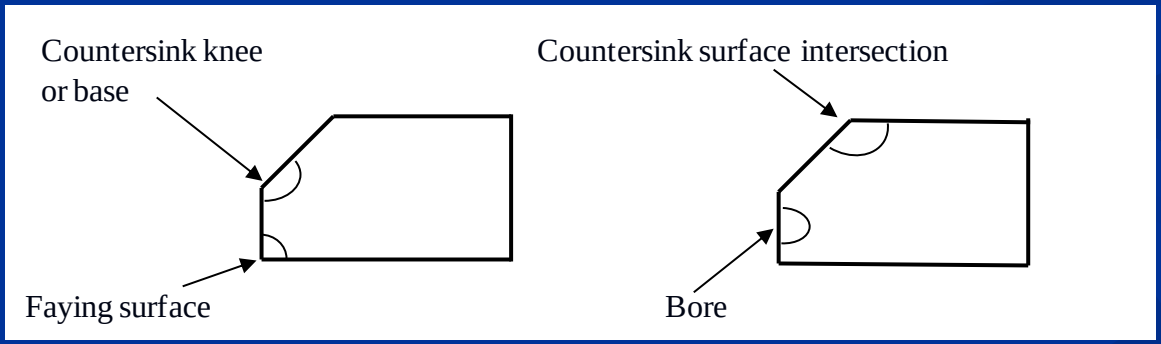
Category	Input Description
Loading	**Remote Tension (σ)**
	Bending
	Bearing
Part Geometry	**Hole diameter (d)**
	Countersink depth (cd)
	Countersink angle (α)
	Plate thickness (th)
	Plate width (w)
	Edge distance (ed)
Crack Geometry	Initiation location(s)
	c Crack length
	a Crack length



4^12 = 16,777,216 solutions. 320 years to solve @ 10min/sol

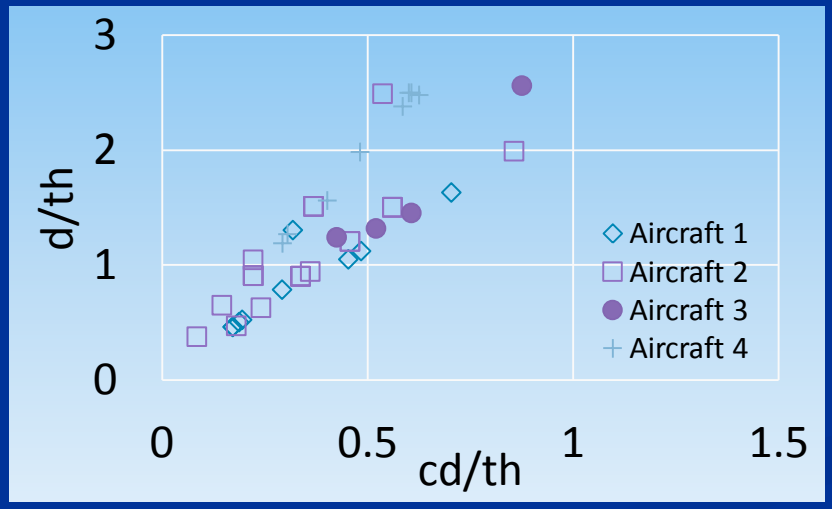
Solution Space Investigation - Continued

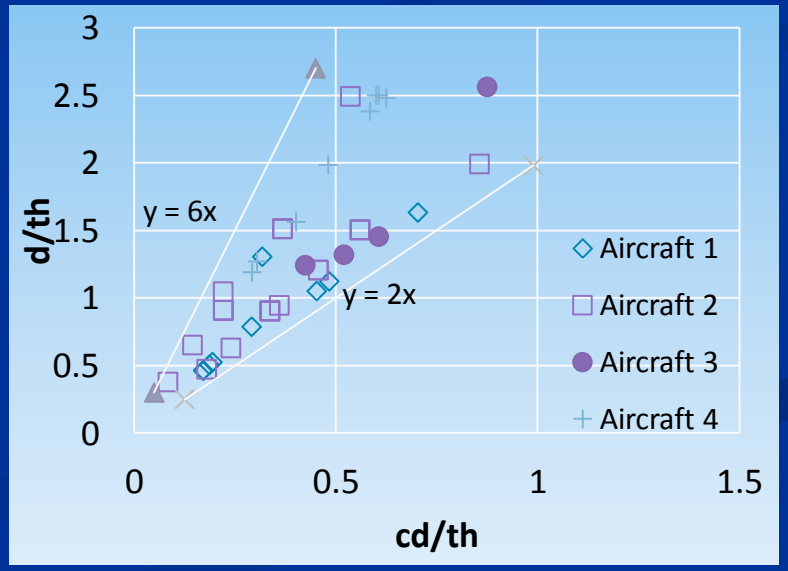
- Four possible crack initiation locations
- Knee or base of CS has the highest stresses for remote tension loading
- Only base or knee of CS is evaluated



Solution Space Investigation - Continued

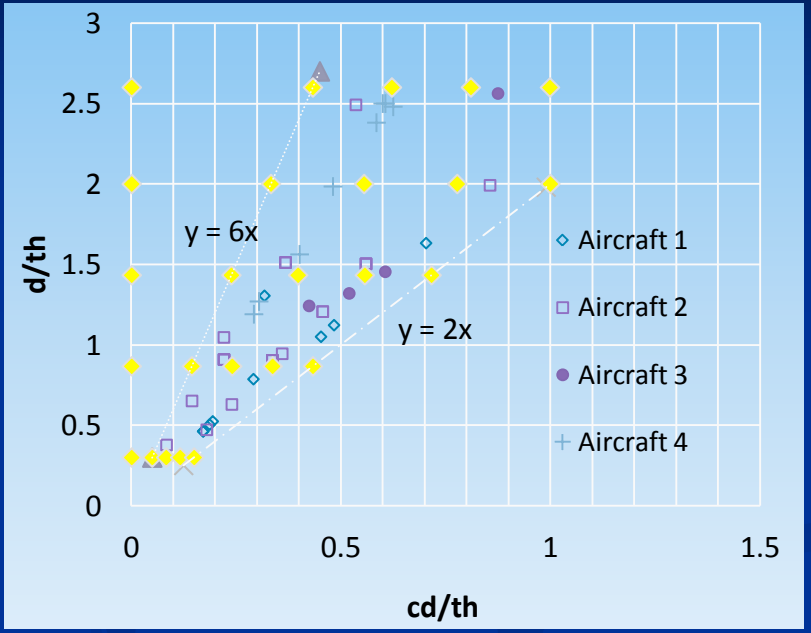
- Aircraft survey used to investigate hole diameter (d), plate thickness (th), and countersink depth (cd). Real-world aircraft data
- Four aircraft were surveyed
- Results show a very organized distribution that has an engineering explanation

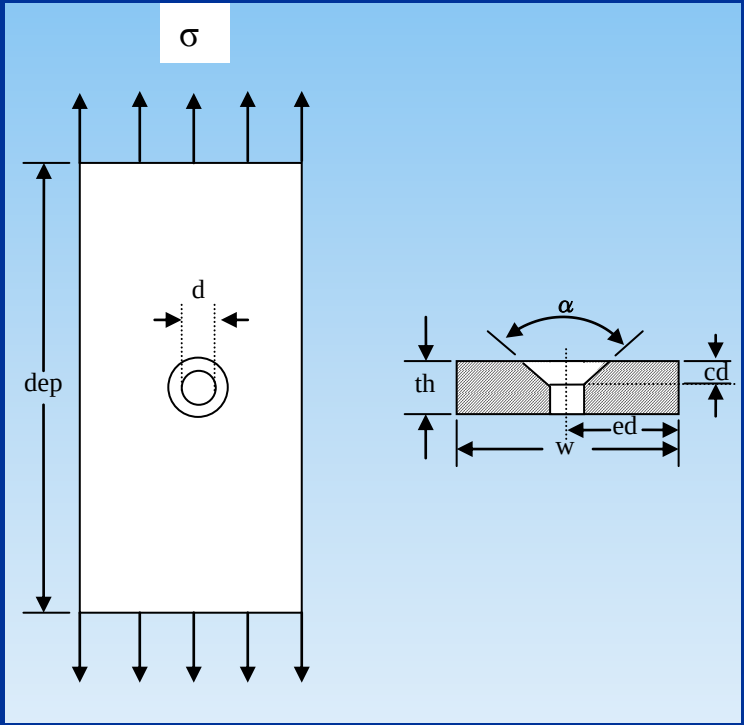




Solution Space Investigation - Continued

- 25 d/th vs. cd/th points
- cd/th ranges from 0.001 to 0.95





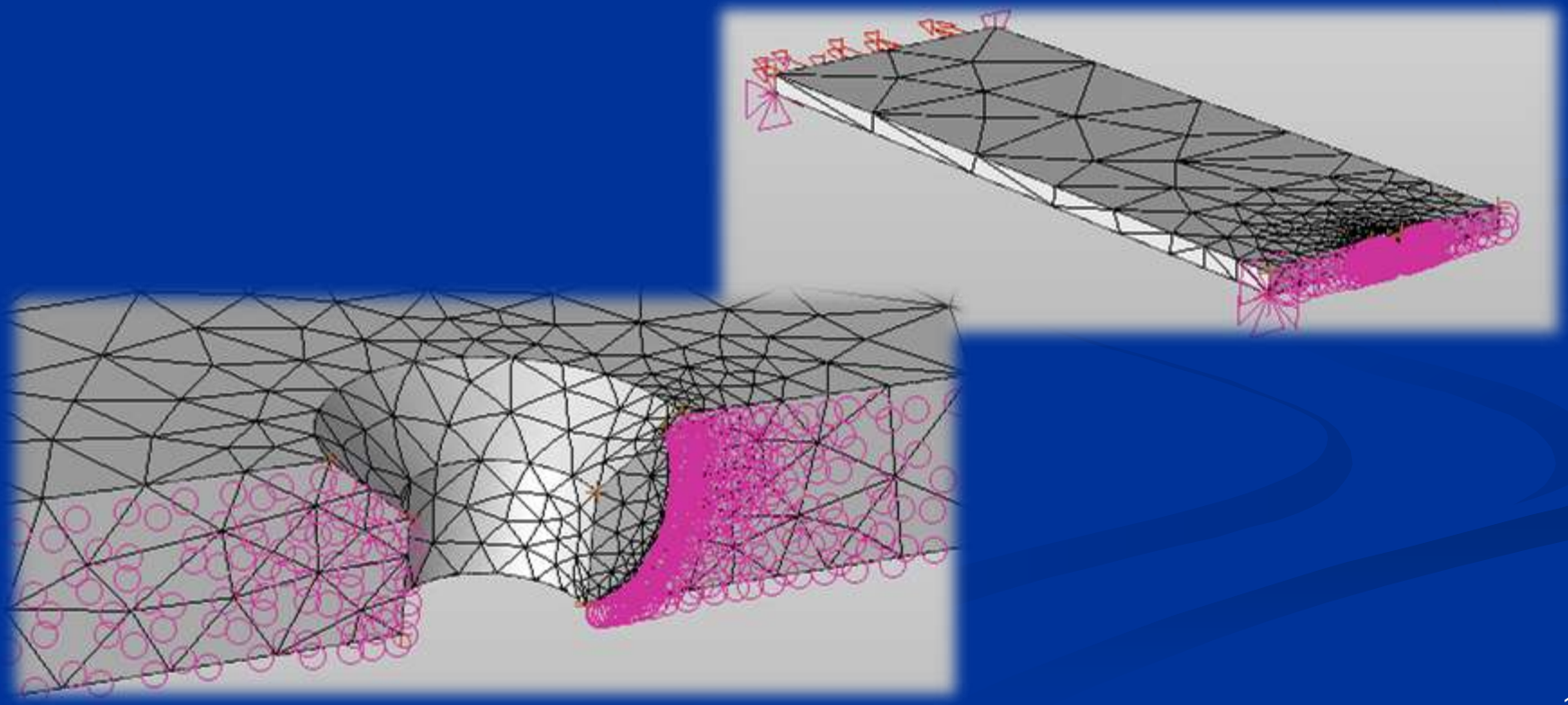
Solution Space Investigation - Continued

- Five inputs reduced to four inputs for final solution space
- Total number of FE solutions optimized 1750

Category	Input Description	Solution Space Input	Minimum	Maximum	Steps
Part Geometry	Hole diameter (d)	d/th	0.3	2.6	5
	Countersink depth (cd)	cd/th	0.001	0.15 to 0.99	5
	Plate thickness (th)	th/th	1	1	1
Crack Geometry	c Crack length	c/th	0.0125(d/th) + 0.002	2.5(d/th)	14
	a/c	a/c/th	0.5	4	5

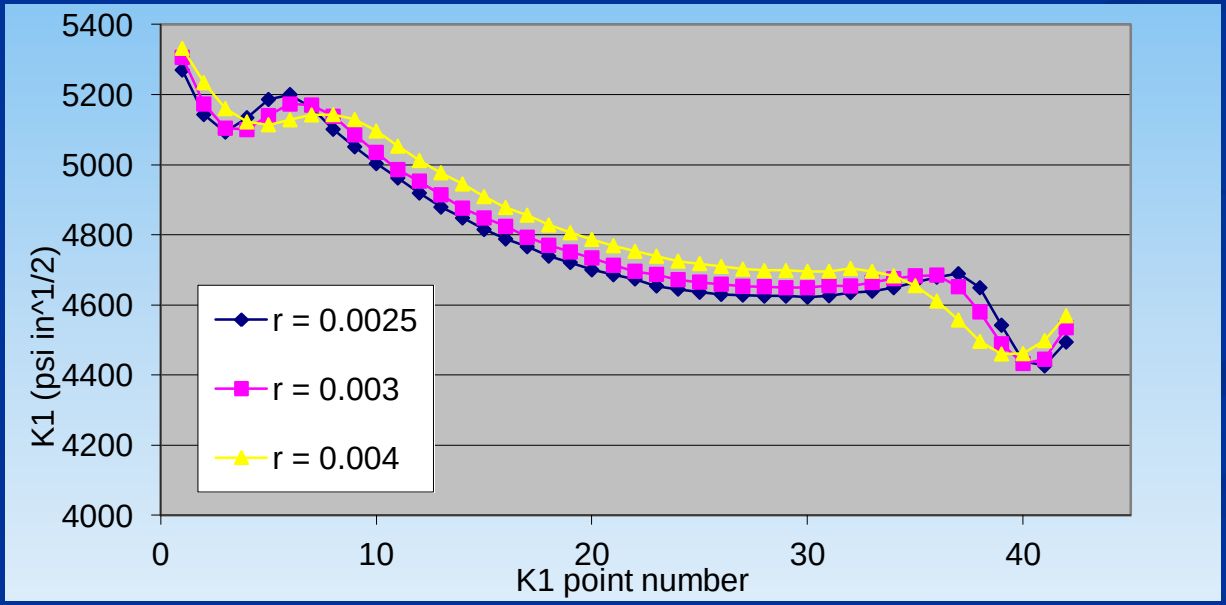
Computational Methodology

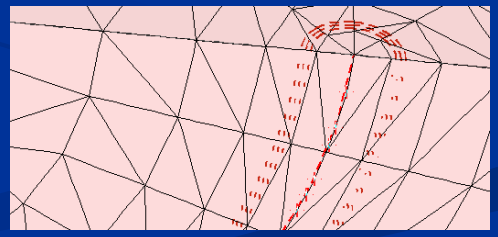
- The hp-version of the FEM was implemented
- Half symmetry used to reduce model DOF



Computational Methodology - Continued

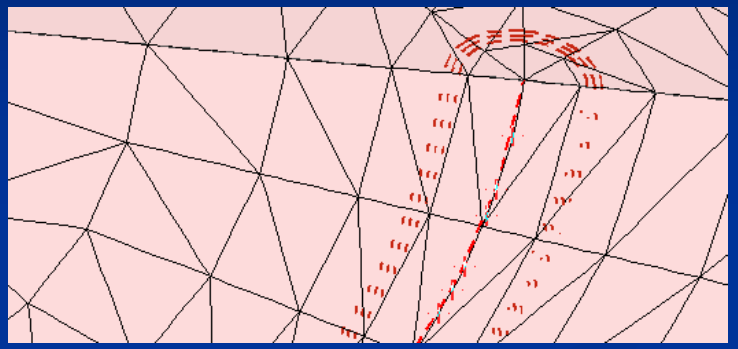
- CIM K1 extraction study reveled K1(r) (3D only)
- For points near a free surface, extraction points were manually selected to be slightly inside to mitigate free surface effects





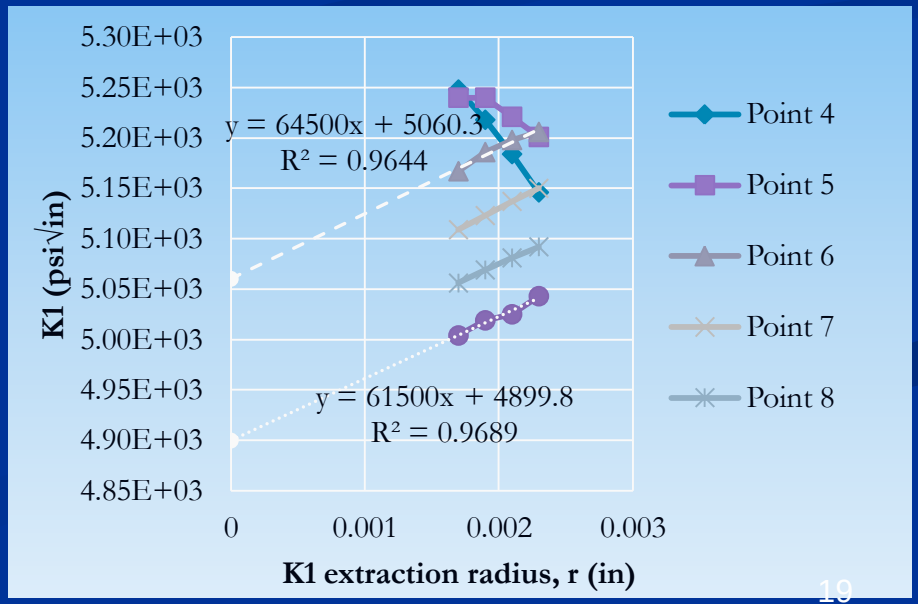
Computational Methodology - Continued

- Improving K1 accuracy
- Note that extraction angle changes K1 results
- a/c vs. % difference



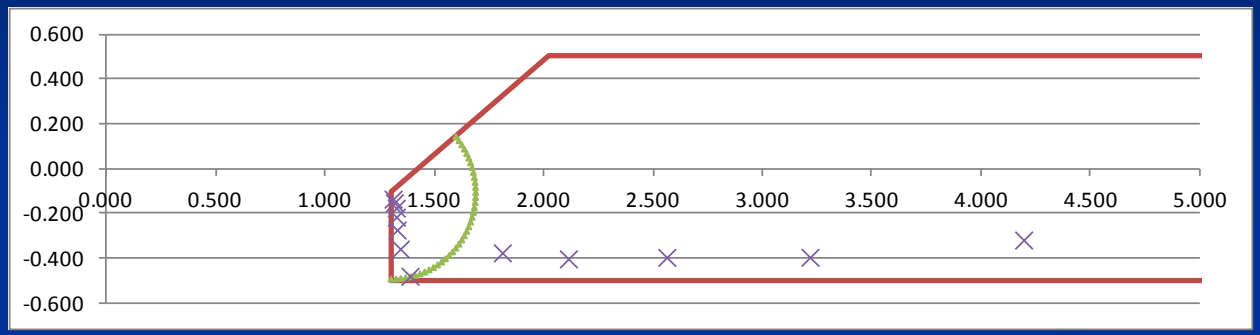
	a/c=0.5	a/c=1.0	a/c=1.5	a/c=2.0	a/c=4.0
Mean % Dif.	4.02	2.12	1.32	0.88	0.56
STDV	1.90	1.49	1.00	0.71	0.56

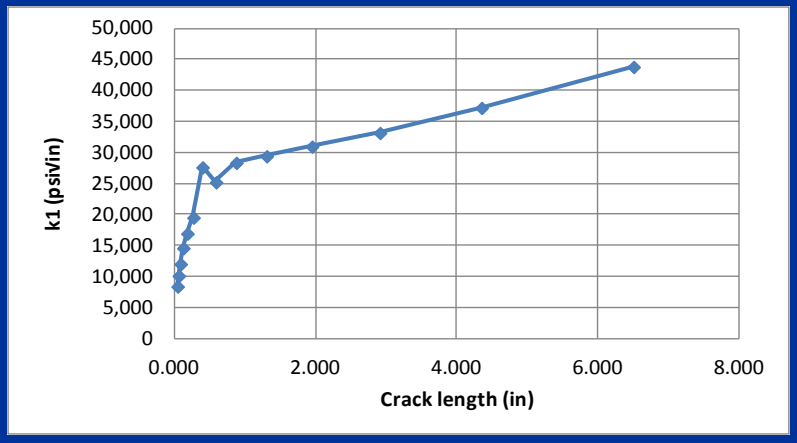
"c" crack tip data

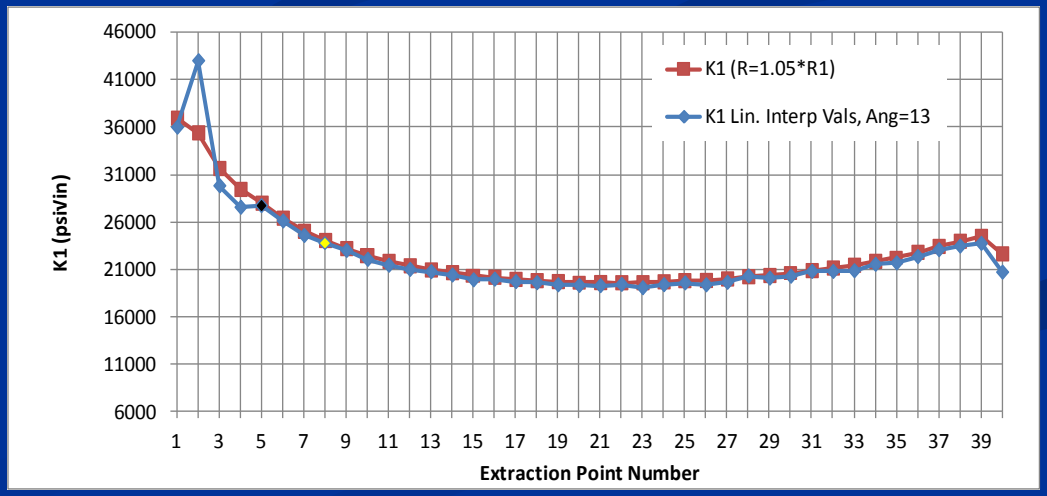


Computational Methodology - Continued

- Shift for K1 extraction location for through crack transition



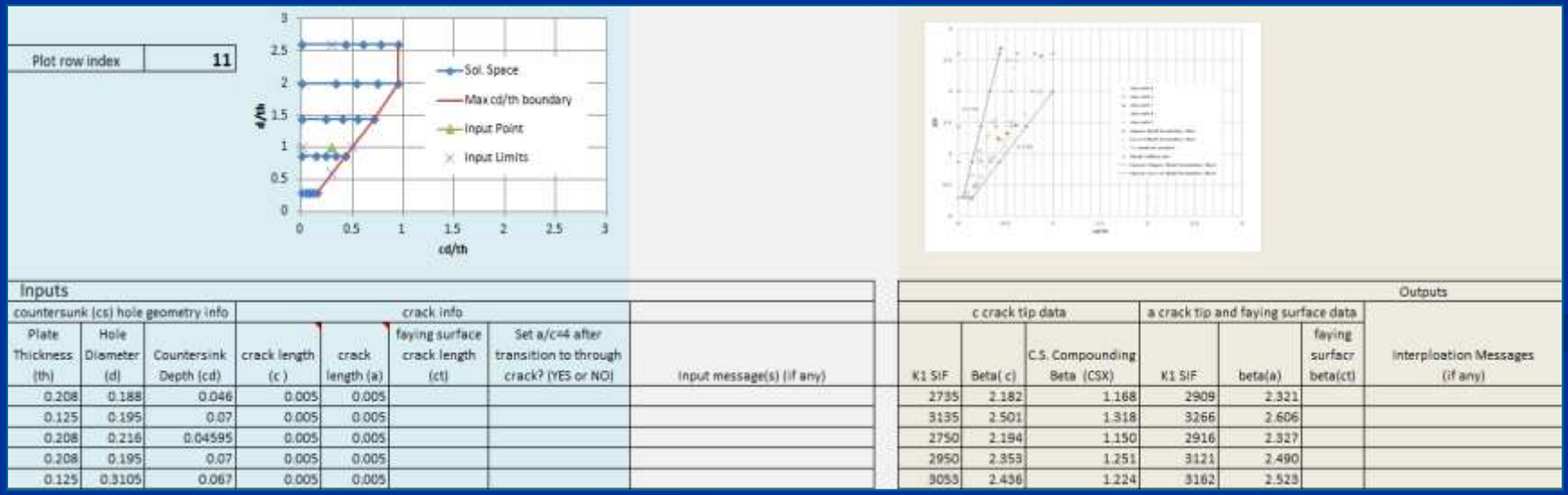




Solution Space Interpolation

- All calculations are done in real-time from an excel worksheet
- Real-time feedback given to mitigate errors
- Countersunk beta compounding factor (βcsx) used to quantify the effect of adding a SC to a straight hole (SH), c tip only.

$$\beta_{csx} = K1_{cs}/K1_{SH}$$

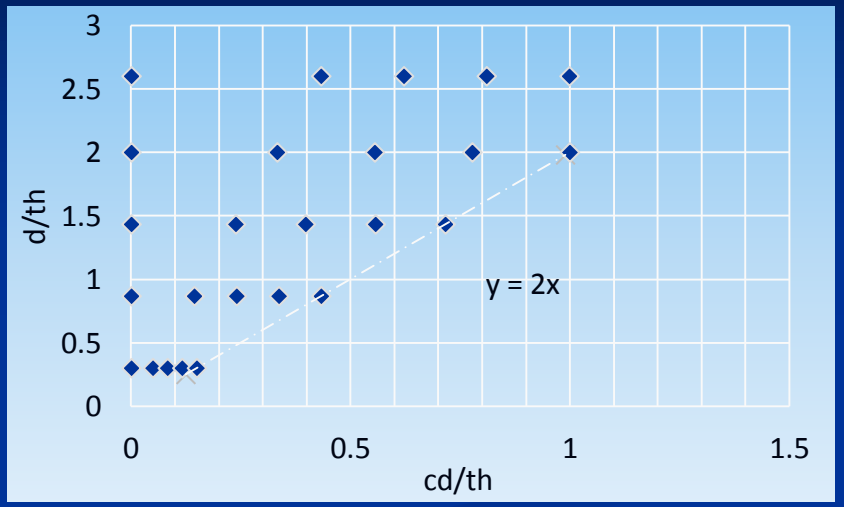


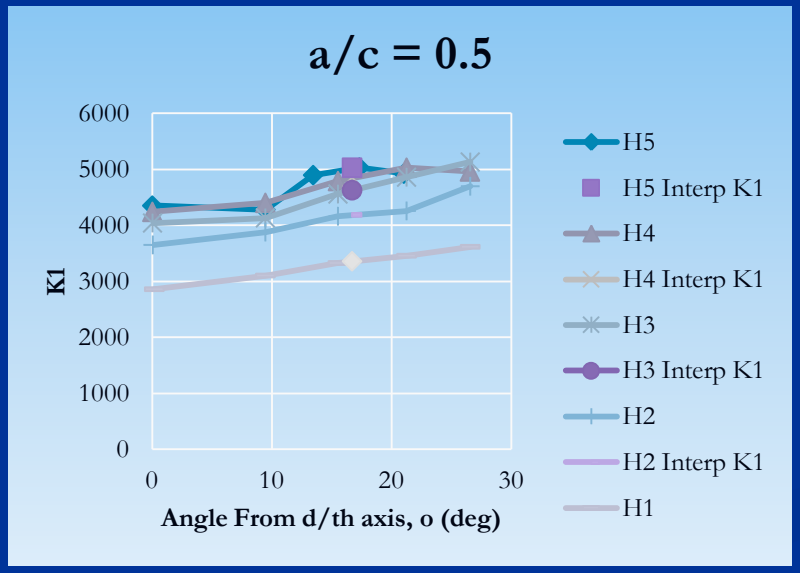
Plot row index: 11

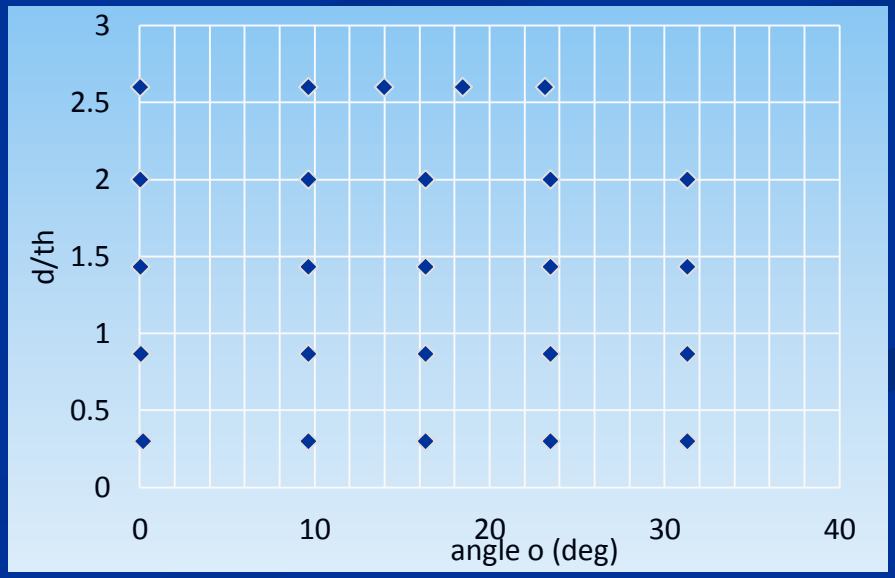
Inputs								Outputs						
countersunk (cs) hole geometry info			crack info					c crack tip data			a crack tip and faying surface data			
Plate Thickness (th)	Hole Diameter (d)	Countersink Depth (cd)	crack length (c)	crack length (a)	faying surface crack length (ct)	Set a/c=4 after transition to through crack? (YES or NO)	Input message(s) (if any)	K1 SIF	Beta(c)	C.S. Compounding Beta (CSX)	K1 SIF	beta(a)	faying surfacr beta(ct)	Interploation Messages (if any)
0.208	0.188	0.046	0.005	0.005				2735	2.182	1.168	2909	2.321		
0.125	0.195	0.07	0.005	0.005				3135	2.501	1.318	3266	2.606		
0.208	0.216	0.04595	0.005	0.005				2750	2.194	1.150	2916	2.327		
0.208	0.195	0.07	0.005	0.005				2950	2.353	1.251	3121	2.490		
0.125	0.3105	0.067	0.005	0.005				3053	2.436	1.224	3162	2.523		

Solution Space Interpolation - Continued

- d/th vs. cd/th is not rectangular
- Partial coordinate transformation used to "square" this part of the solution space
- Horizontal lines H1 through H5 fit for each d/th ratio in solution space

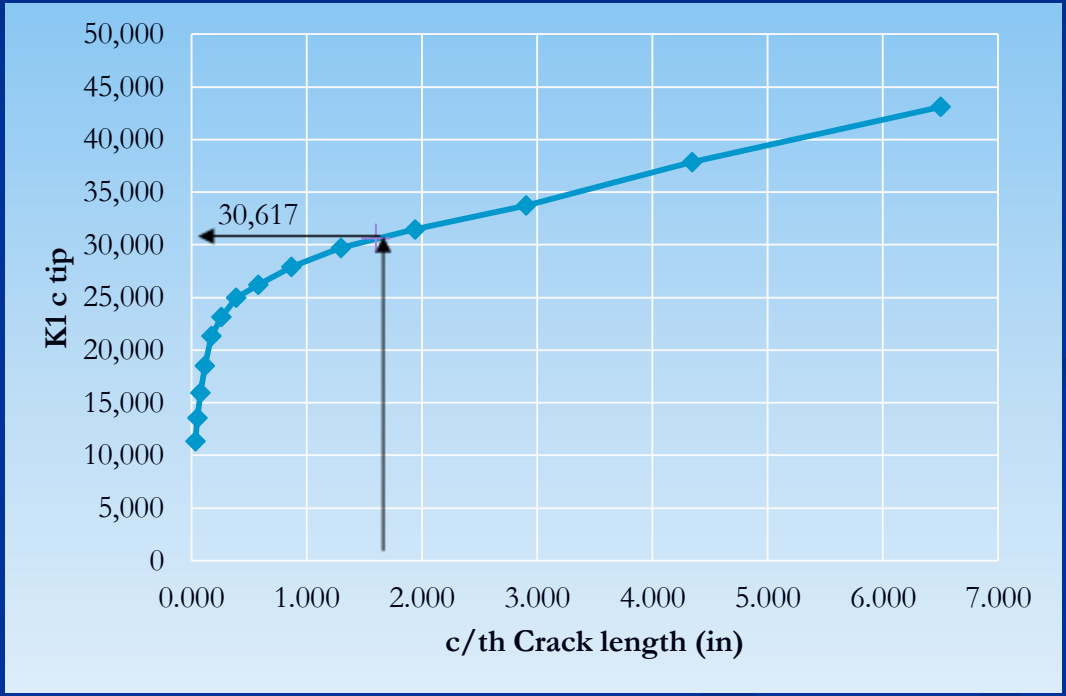






Solution Space Interpolation - Continued

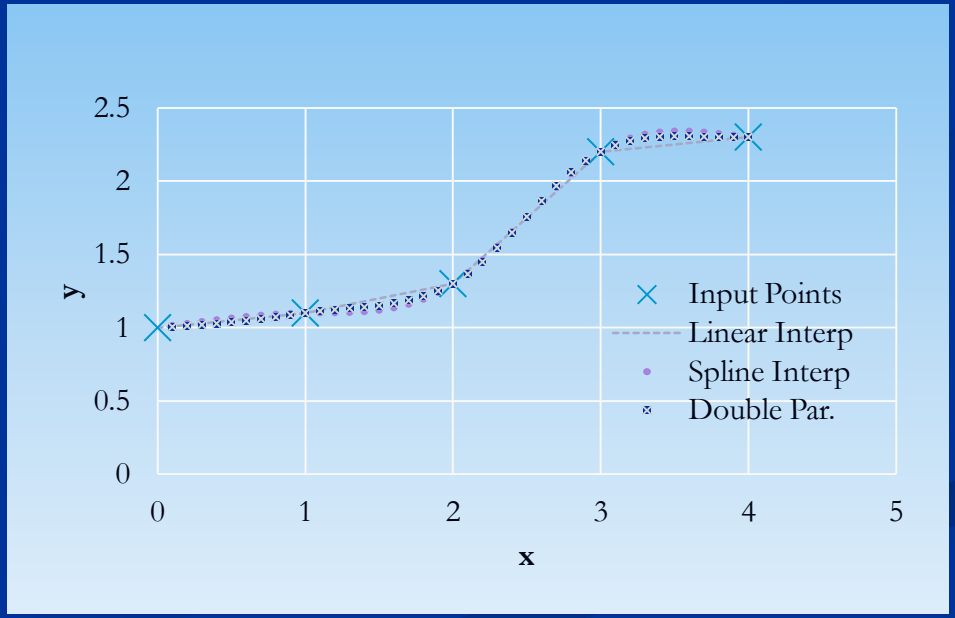
- Interpolation steps
  - Interpolate K1(c) for each point on each H line, do again for all a/c ratios
  - Interpolate K1(o) on each of the above lines, for all a/c ratios.
  - Interpolate K1(a/c) from above step
  - Multiply K1 by √th to convert K1 to real world space



Solution Space Interpolation - Continued

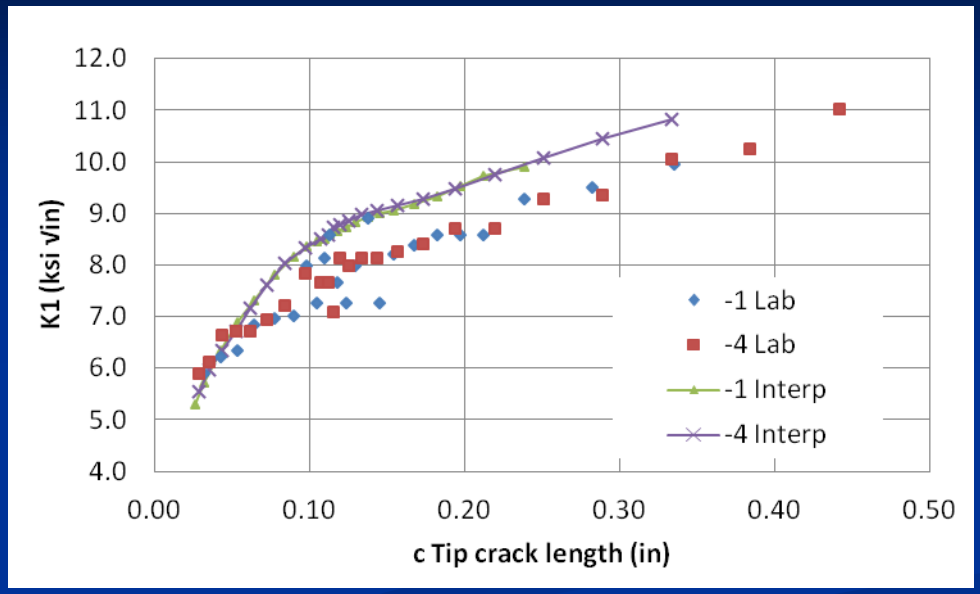
- Countersunk beta compounding factor (βcsx) used to quantify the effect of the CS
- The βcsx factors can be used directly in AFGROW or NASGRO to approximate the effects of the CS
- Double parabolic interpolation used for all calculations

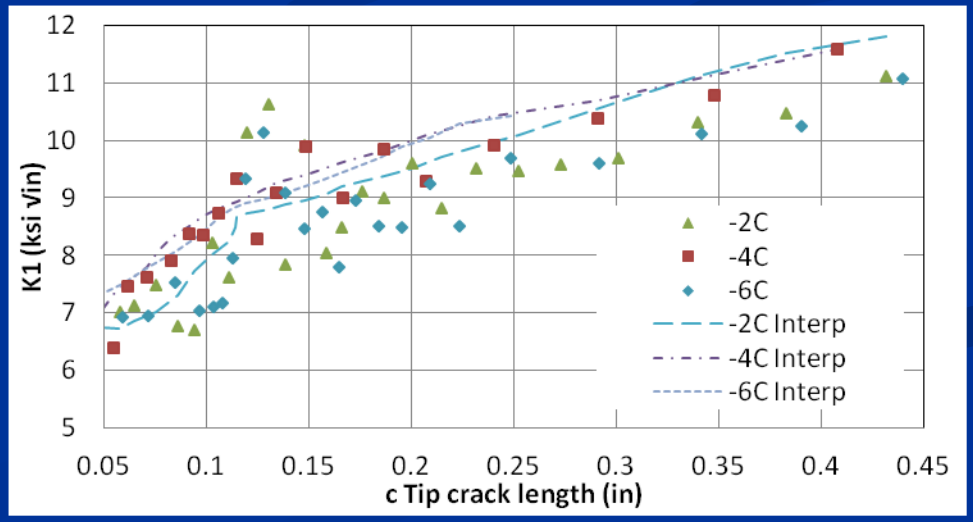
$$\beta_{csx} = K1_{cs}/K1_{sh}$$



Validation & Error Analysis

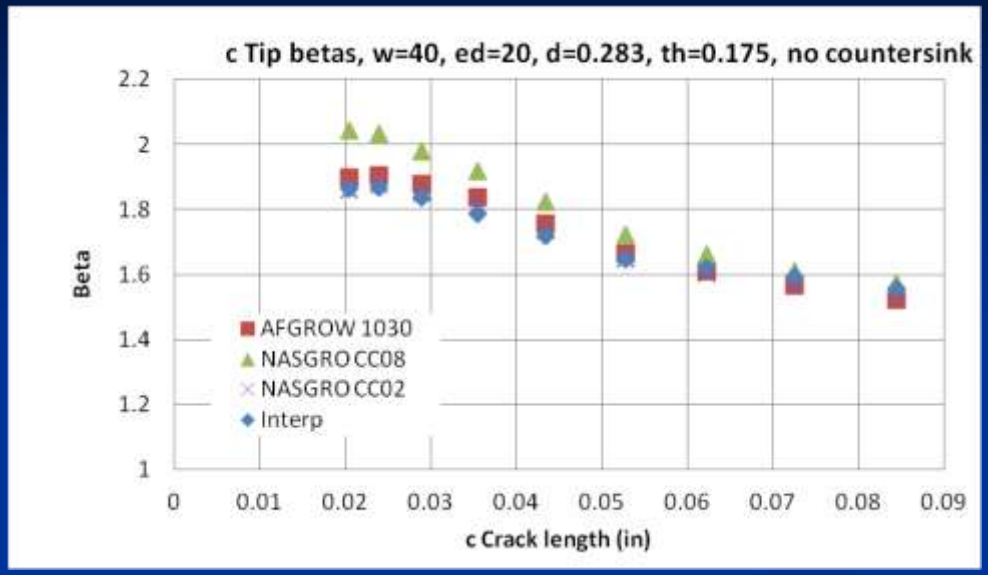
- Experimental validation results showed good agreement between experimentally calculated K1 value and FE derived K1 values
- Material properties may be reason for slight offset observed for longer crack lengths

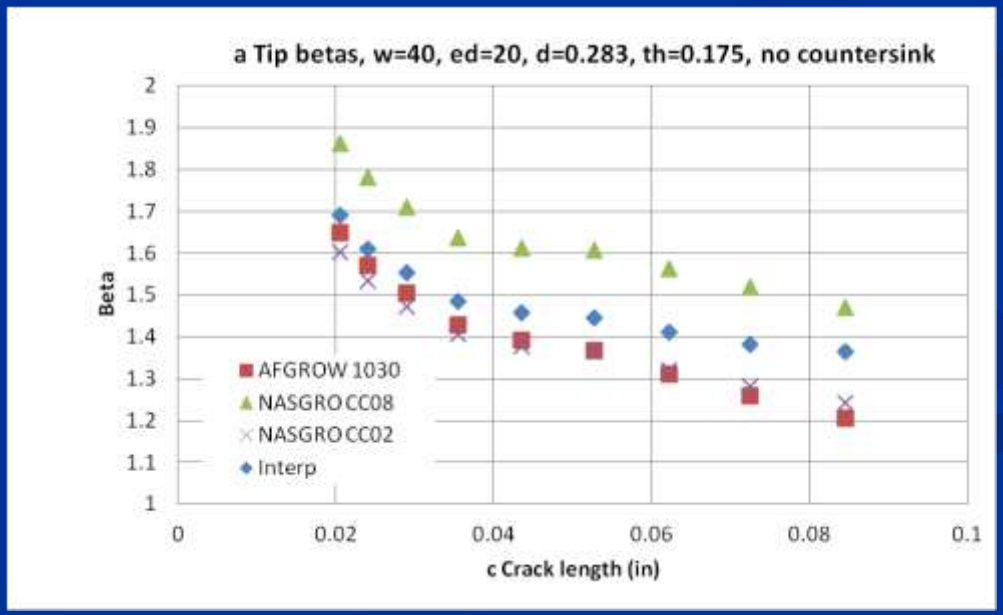




Validation & Error Analysis - Continued

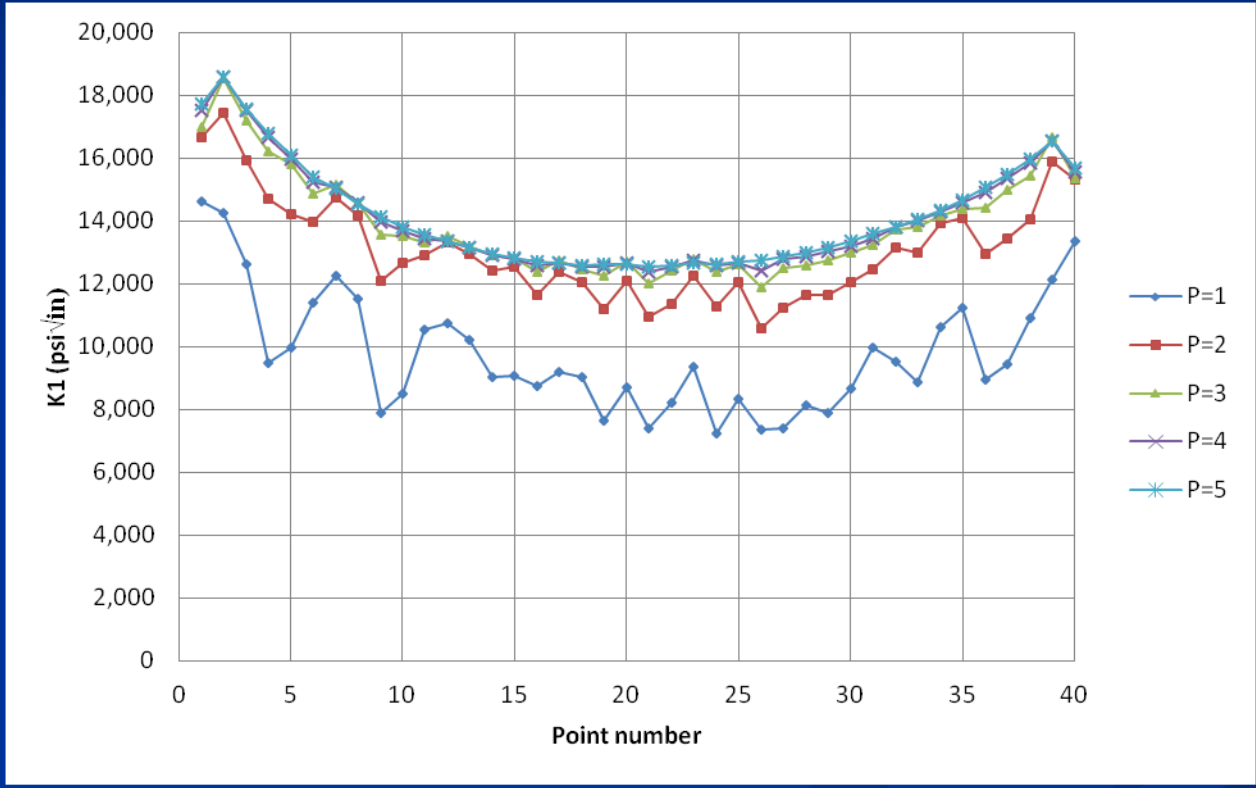
- Results compared to other published and industry standard solutions
- Results show good correlation





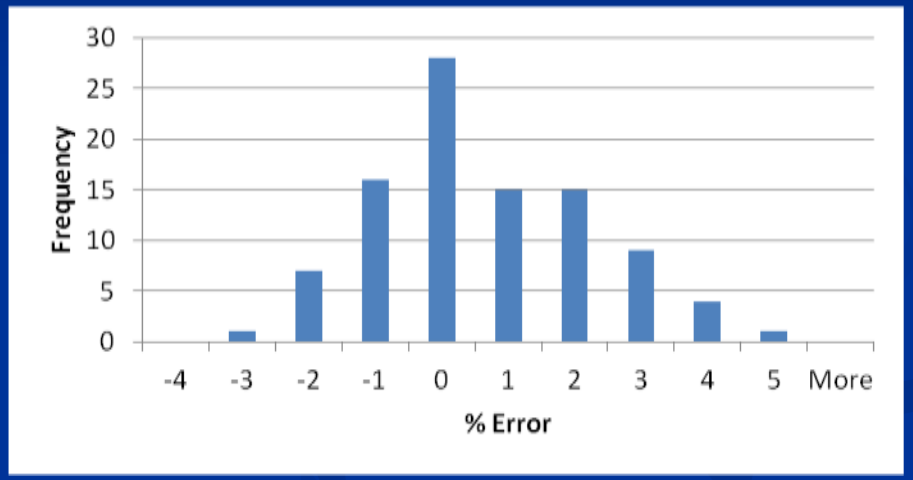
Validation & Error Analysis - Continued

- Convergence study used to minimize discretization and modeling error
- Global energy norm error was low (<1%) for all solutions



Validation & Error Analysis - Continued

- Interpolation error less than 3.3% for 95% of approximated solutions
- Some of this error may be due to variations in K1 extraction angle



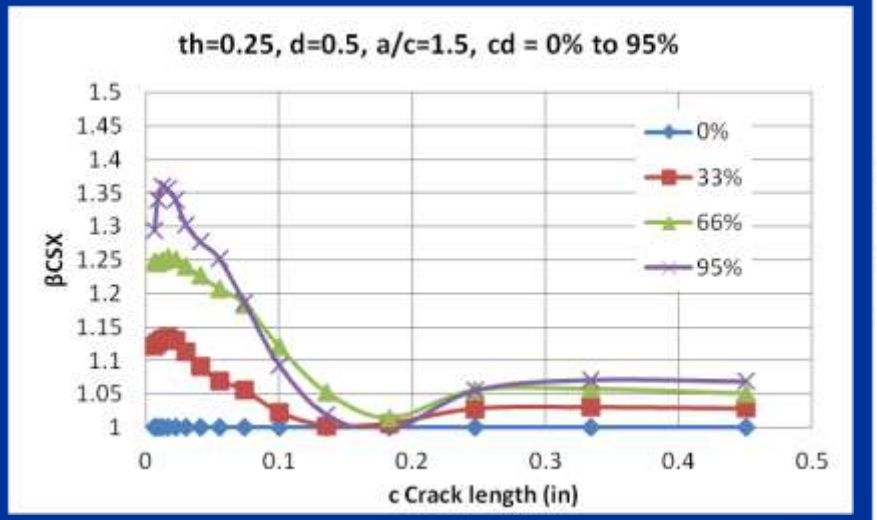
Validation & Error Analysis - Continued

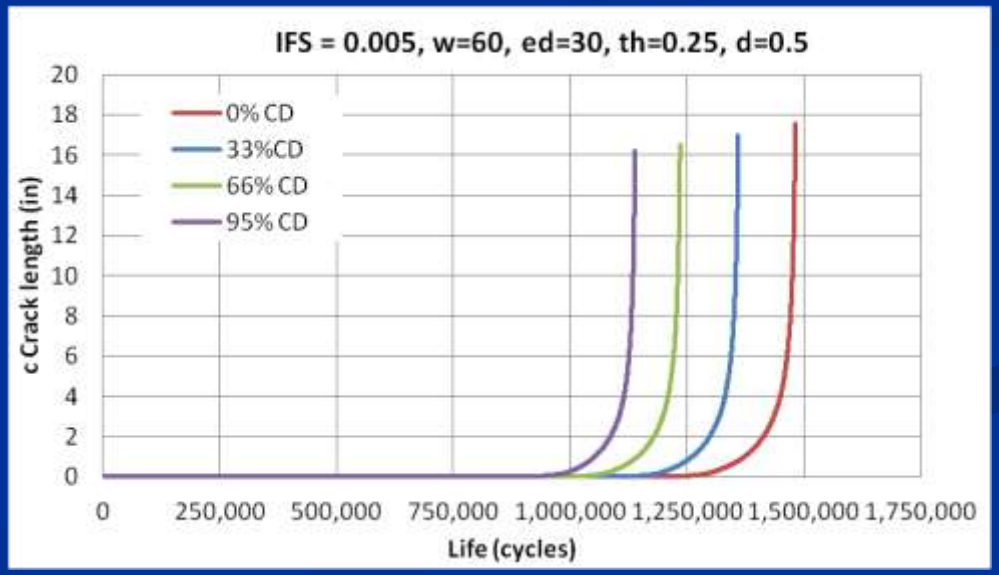
- Total or maximum real world error approximated at 7.3% for 95% of solutions
- Apparent error should be less than this as some errors may counteract each other

$$\|E_{SIFtot}\| \leq \|E_{interp}\| + \|E_{K1EXT}\| + \|E_{modeling}\| + \|E_{discretization}\|$$

Results, Discussion, Conclusion

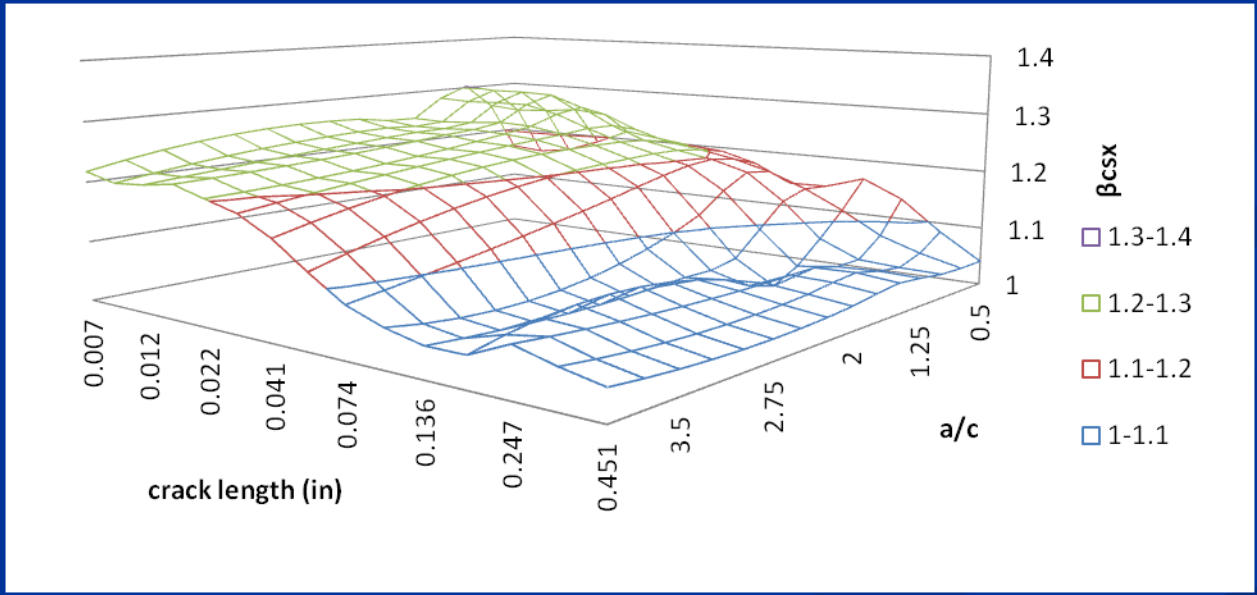
- How does adding a CS to a hole effect crack growth rates?
- For a CS hole in a wide plate, fatigue life can be reduced buy 23.2% if CS is deep
- These results will vary based on CS depth and other factors





Results, Discussion, Conclusion - Continued

- Countersunk hole K1 SIFs and beta estimates can now be quickly and approximated for the most common aerospace geometries
- A first for this type of research



Results, Discussion, Conclusion - Continued

- Elliptical crack shape approximation was shown to be good within the limits of the geometry tested (short cracks)
- Future work should included:
  - Evaluating pin/bearing and bending loads
  - Careful look at the solution space developed for this research to see how it may be further optimized
  - Possibly do some additional 'cleaned up' of SIF solutions

Questions?

Appendix Slides

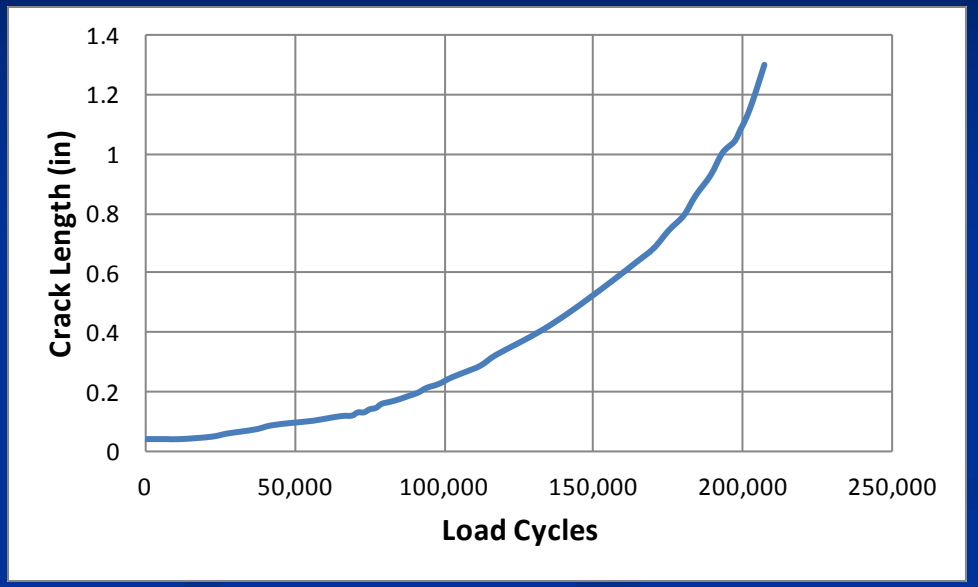
Solution Space Dimensions

- Five input dimensions
  - Plate thickness (th)
  - Hole diameter (d)
  - Countersink depth (cd)
  - c-tip Crack length (c)
  - a-tip Crack length (a) or faying surface crack length (ct)
- Normalized by th to reduce to four input dimensions
  - d/th
  - cd/th
  - c/th
  - a/th

$a_{i+1} = a_i + \Delta a$

LEFM Overview

- Linear elastic fracture mechanics (LEFM) theory:
  - Used to make fatigue life predictions
  - Assumes an initial flaw size and shape
  - Calculates the incremental crack growth, Δa, as a result of applying load cycles

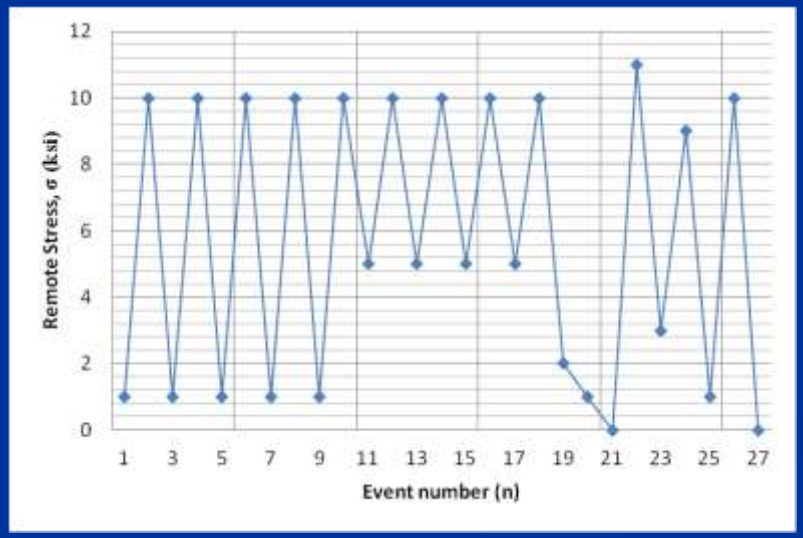


$$a_{i+1} = a_i + \Delta a$$

LEFM Overview 2

- LEFM theory suggests that the stress intensity (SIF) at the crack tip, K, can be calculated as a function of the remote stress, σ, √(πa) and the geometry factor, β.

$$\Delta K = \Delta\sigma\sqrt{\pi a}\,\beta$$



$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Literature Survey

- 1965: Richard E. Whaley, birefringent plastic coatings around countersunk holes to develop stress-concentration factors
  - Surface only
  - Fatigue tests showed higher stress in countersink
- 1978: Y. F. Cheng, stress freezing technique to measure the stress through the thickness
  - Highest stress for tension is at base of countersink

Literature Survey (Continued)

- 1983: J.C. Newman and I.S. Raju used FEA to evaluate cracks growing in 3-D bodies
  - Four geometries (no countersink)
  - Empirical SIF equations
  - First widely excepted research using FEA to calculate SIFs
- 1987: FRANC3D, program for automated crack growth research begins
- 1992: K. N. Shivakumar and J.C. Newman applied FEA to get stress-concentrations for straight-shank and countersunk holes in a large plate subjected to various loading conditions
  - No cracks
  - FORTRAN program still used today

Literature Survey (Continued)

- 1994: J.B. Young and K.K. Lee published stress concentration design equations for countersunk hole
  - Only tension loading, no crack
  - Coarse mesh used and equation was overly conservative
- 1994: P. W. Tan et al published set of SIF solutions for cracks growing from CS hole
  - Remote tension only
  - FEA
  - Three crack locations

Literature Survey (Continued)

- 1998: M. Gosz and B. Moran use FEM to extend set of SIF solutions for countersunk holes
  - Only tensile loading
- 1998: S.A. Fawaz used Three-dimensional virtual crack closure (3D VCCT) to evaluate large cracks
  - No countersink
  - Three types of loading
  - Used in AFGROW SIF database

Literature Survey (Continued)

- 1999: A. Rahman et al
  - Three types of loading
  - Faying surface only
  - Global, Intermediate, Local (GIL) meshing
  - J-integral using equivalent domain integral method (EDIM)
- 2002: C. Brooks et al use p-version FEM
  - Countersunk hole with three types of loading
  - Faying surface only

Literature Survey (Continued)

- 2002: G. P. Nikishkov and S. N. Atluri combine SGBEM and FEM to model 3-D crack crowth
- 2003: C. Y. Park combine FEM and lab testing to evaluate countersunk holes
  - Poly(methyl methacrylate) (PMMA)
  - V&V
  - h-version FEM
  - Three types of loading
  - Limited geometry
- 2004: J. Rungamornrat, Mark E. Mear use SGBEM to calculate SIFs for crack in 3-D bodies

Literature Survey (Continued)

- 2005: J. Suh expands on Park's 2003 work adding
  - P-version FEM
  - Additional testing
  - FRANC3D BEM
  - Limited geometries
- 2007: K. N. Shivakumar develop new stress concentration equation for countersunk holes
  - No crack

State-of-the-Art Computer Analysis Tools

- FEM
  - p-version (StressCheck and others)
  - h-version (ANSYS, ABAQUS,…)
- BEM
  - FADD3D (UT Austin)
  - FRANC3D (Cornell)
  - BEASY (Commercial)

Proposed Solution Approach

1. Calculate comprehensive set of SIF solutions for cracks emanating from countersunk holes
   - Primary computation with p-version FEM
   - Convergence study for error
   - Lab testing for validation
   - Literature and BEM comparison for verification

2. Develop easy to implement method for applying SIFs
   - Empirical equations?
   - Database?
   - Software?

Preliminary Study

- This study compares three analysis methods
  - Hole with no countersink
  - Hole with countersink, T-38 stress compounding applied to account for countersink
  - Hole with countersink, FEM used to account for countersink

Linear Elastic Fracture Mechanics (LEFM) and the FEM

- Must use FEM to calculate β(a) for different crack lengths.

  For mode I crack growth:

$$\beta = \frac{K_I}{\sigma\sqrt{\pi a}}$$

- K_I SIF is obtained from the finite element analysis (FEA)

LEFM and the FEM (Continued)

- LEFM is used to calculate crack growth rates
- Crack growth rate
  da/dN = f(ΔK)

a = crack length

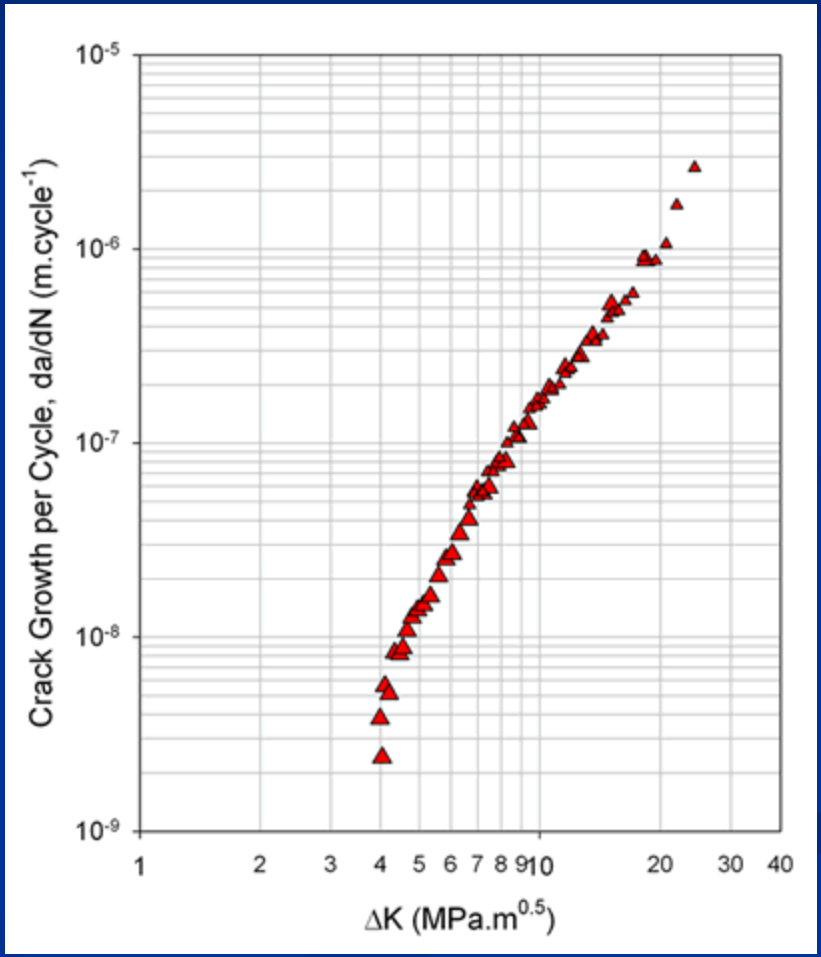
N = cycle number

$$\Delta K = \Delta\sigma\sqrt{\pi a}\,\beta(a)$$

σ = remote stress

K = stress intensity factor (SIF)

β(a) = geometry correction factor



Mathematical Model and Solution Approach

- Used StressCheck (SC) to model the crack
- SC is a "p" element FEA program.
- p elements are well suited for modeling cracks
- SC utilizes the contour integral method (CIM) to compute SIFs

CIM Overview

$$K_I = \sqrt{2\pi}\, A_1^{(1)}$$

$$K_{II} = \sqrt{2\pi}\, A_1^{(2)}$$

$$A_1^{(m)} \approx \int_\Gamma (w_m T_{FE} - u_{FE} T^{(Wm)}) ds$$

Where:

w_m = extraction function

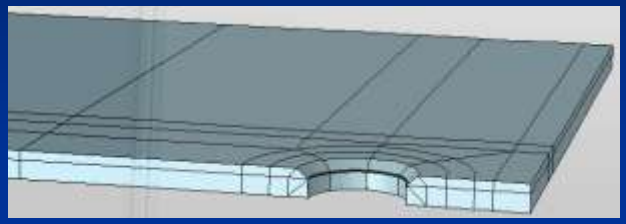
T_{FE} = traction vector computed from the FE solution u_{FE} along the circle Γ

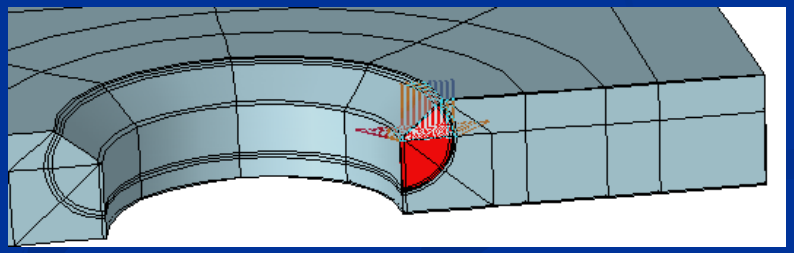
$T^{(Wm)}$ = traction vector computed from w_m along Γ

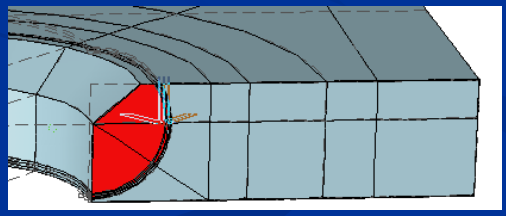
For more information about how SC applies the CIM, see the SC users manual.

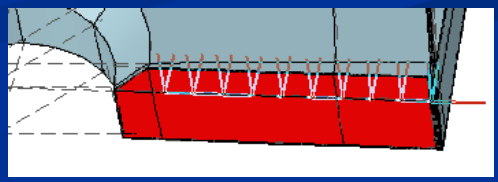
Finite Element Model

- Model symmetries
- Geometry came from T-38 fatigue critical location (FCL)
- Parametric and hand meshed
  - Pentahedron elements
  - Hexahedron elements



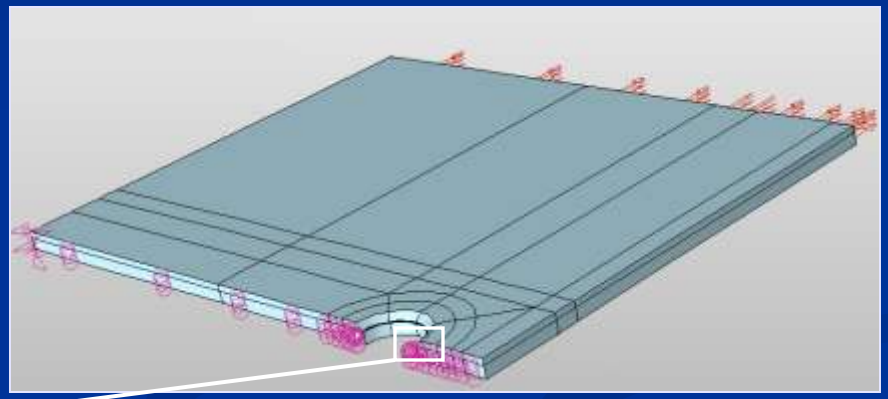


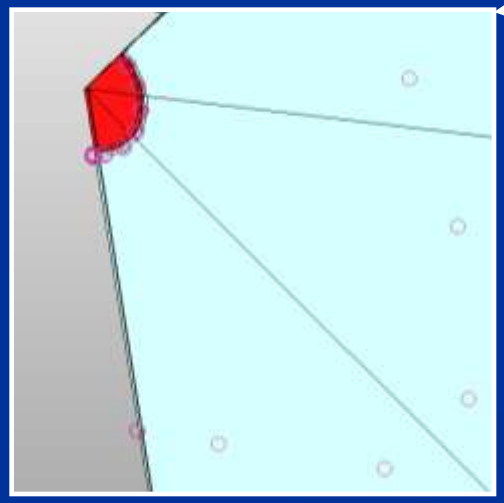




Finite Element Model (Continued)

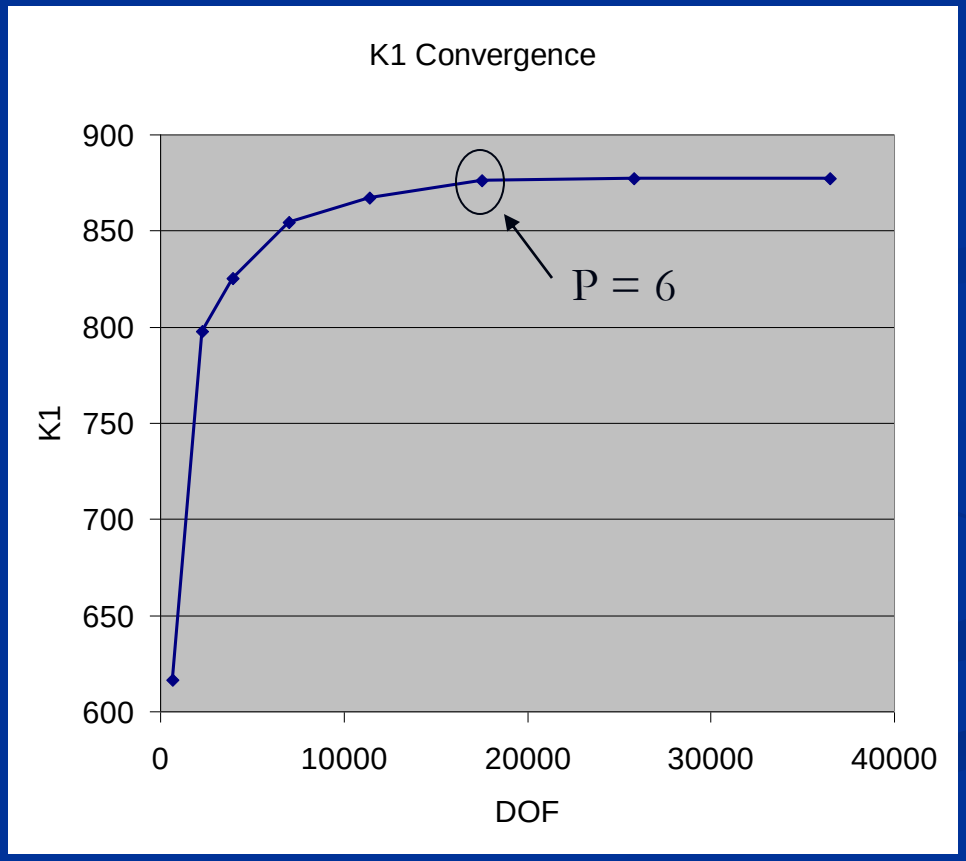
- Tension loading
- Boundary conditions
- Material properties
  - Al 7075-T6





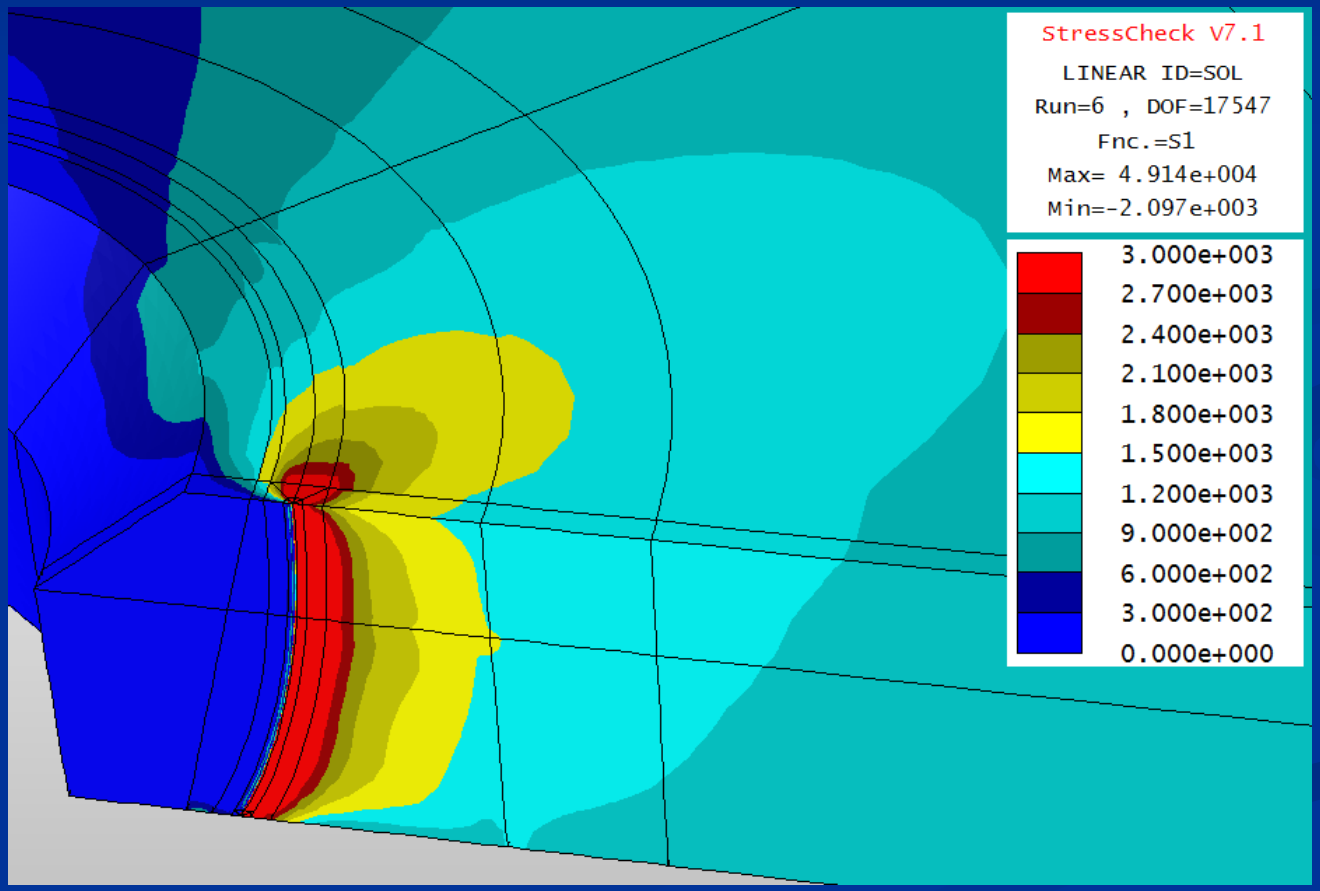
Finite Element Model (Continued)

K1 convergence study used for error estimation



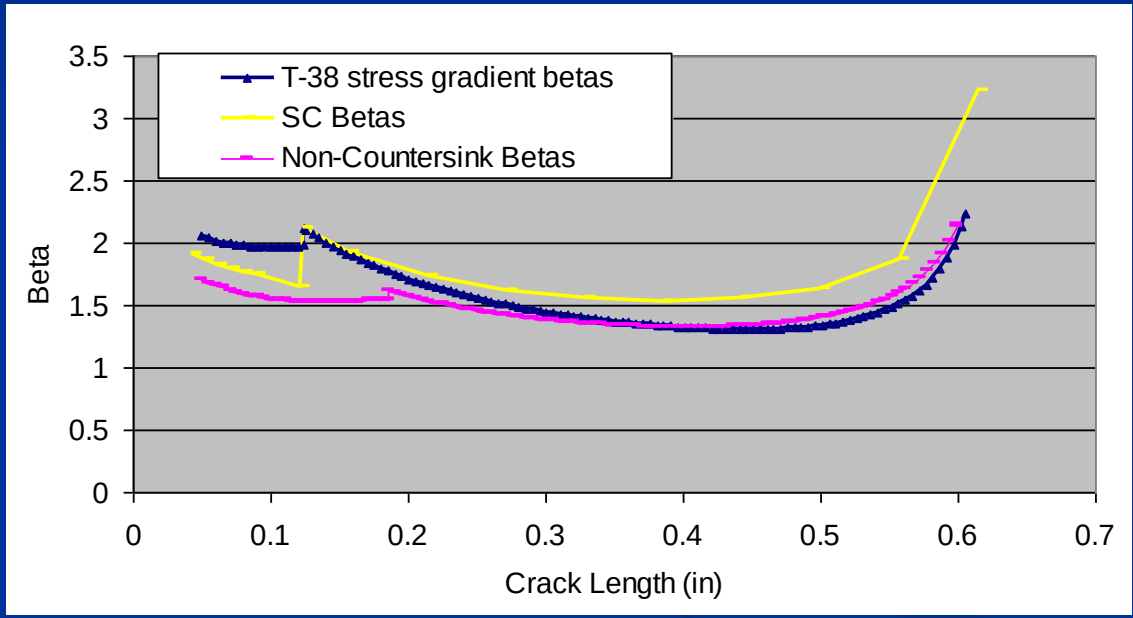
Finite Element Model (Continued)

- SC models showed good convergence at p = 6



Results and Discussion

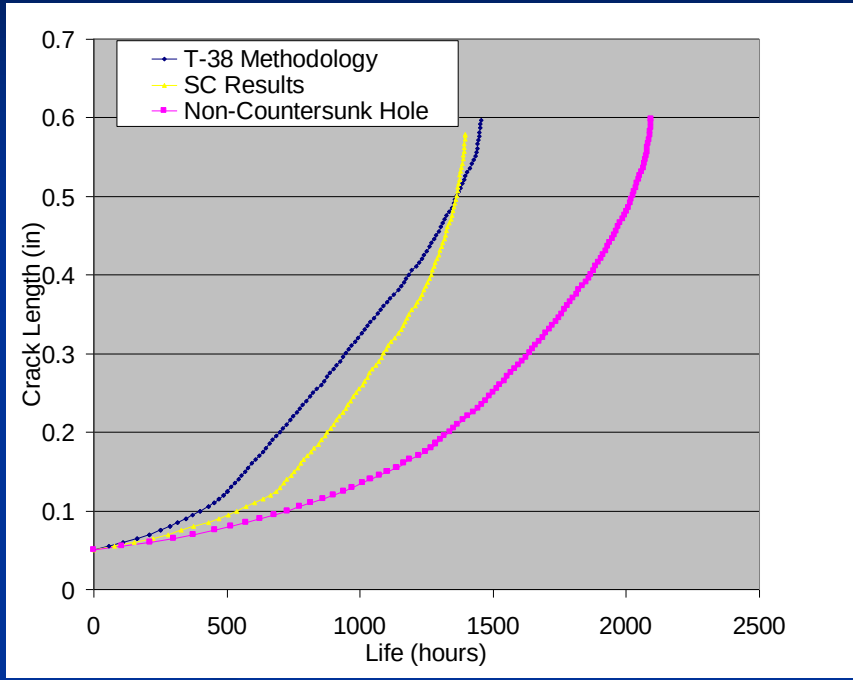
- SC was used to calculate beta values for 17 different crack lengths
- SC beta values were compared to T-38 stress compounded betas and non-countersunk hole betas



Results and Discussion (Continued)

- Crack growth results

Non-Countersunk hole results:
Life = 2,094



Using the traditional T-38 approach we get:
Life = 1,455

Using the SC betas we get:
Life = 1395

Example Problem Conclusions

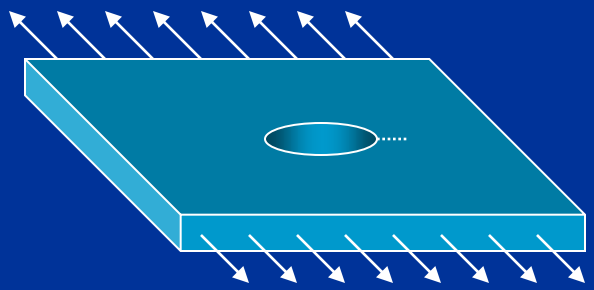
- Adding a countersink to the hole evaluated in this investigation significantly shortened the hole's inspection interval
- The legacy T-38 stress compounding approach for beta corrections may not be conservative
- StressCheck shows good K_I convergence at p = 6
- Min. detectible crack size
- Auto meshing should be evaluated for use with future StressCheck FEA models

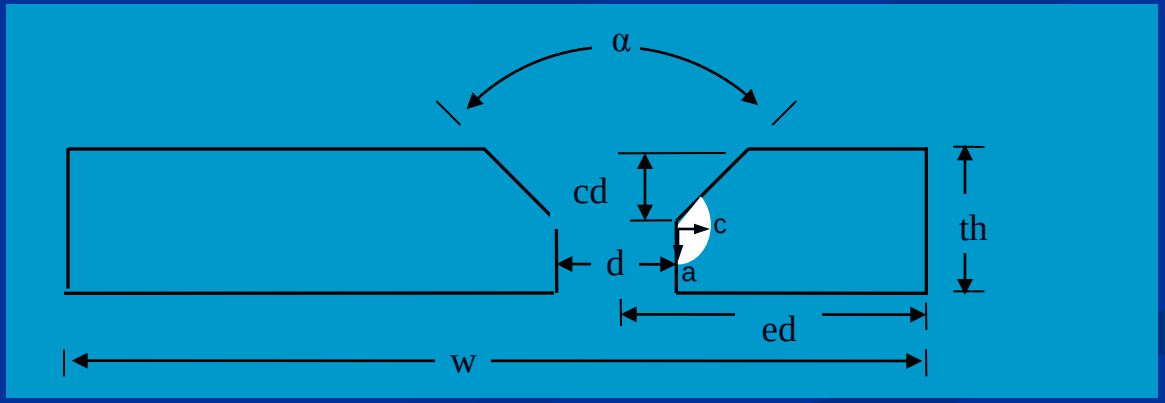
Next Steps

Geometry and Loading

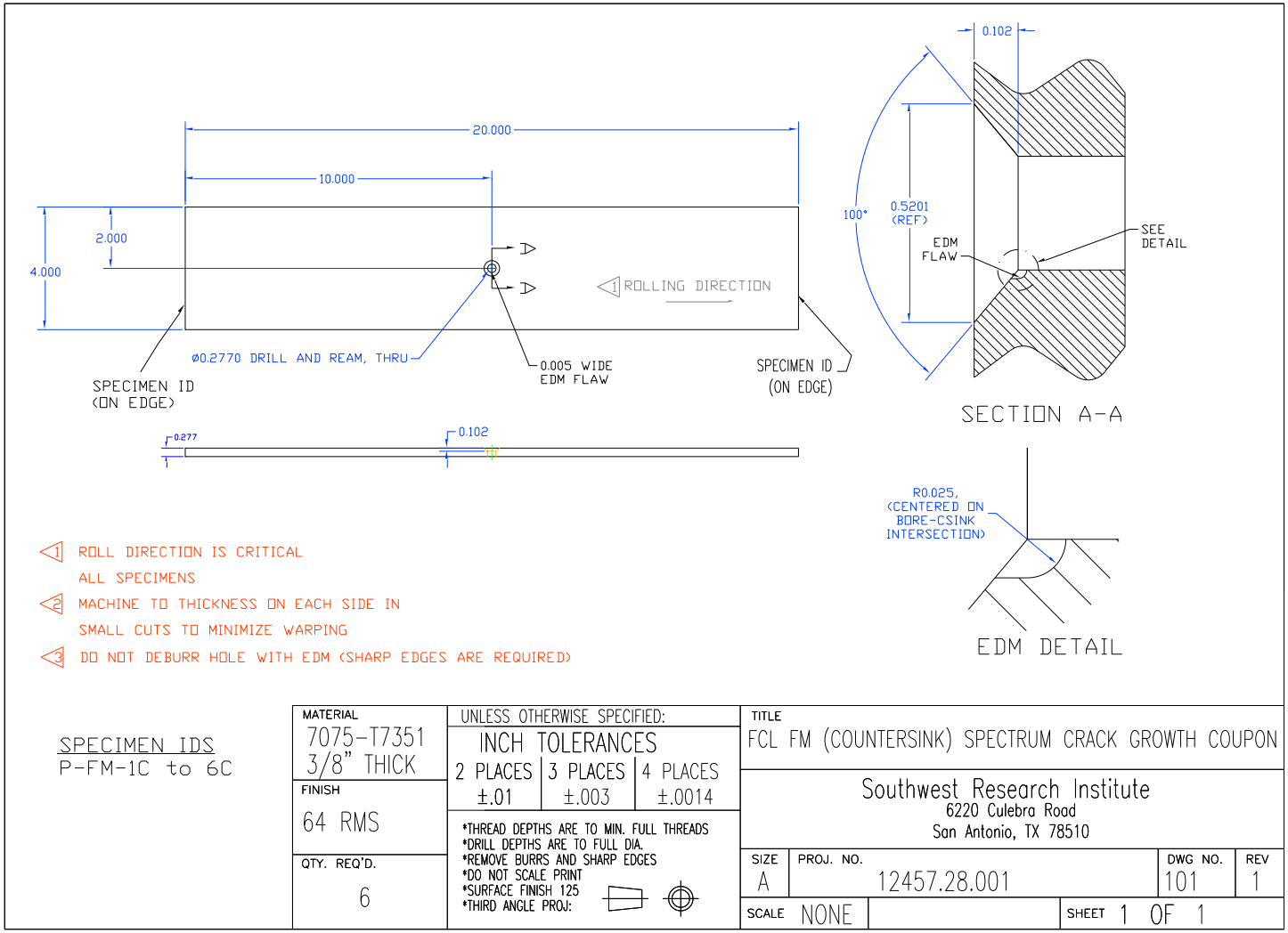
- Tensile loading
- Cracks emanating from countersink knee

Remote Tension



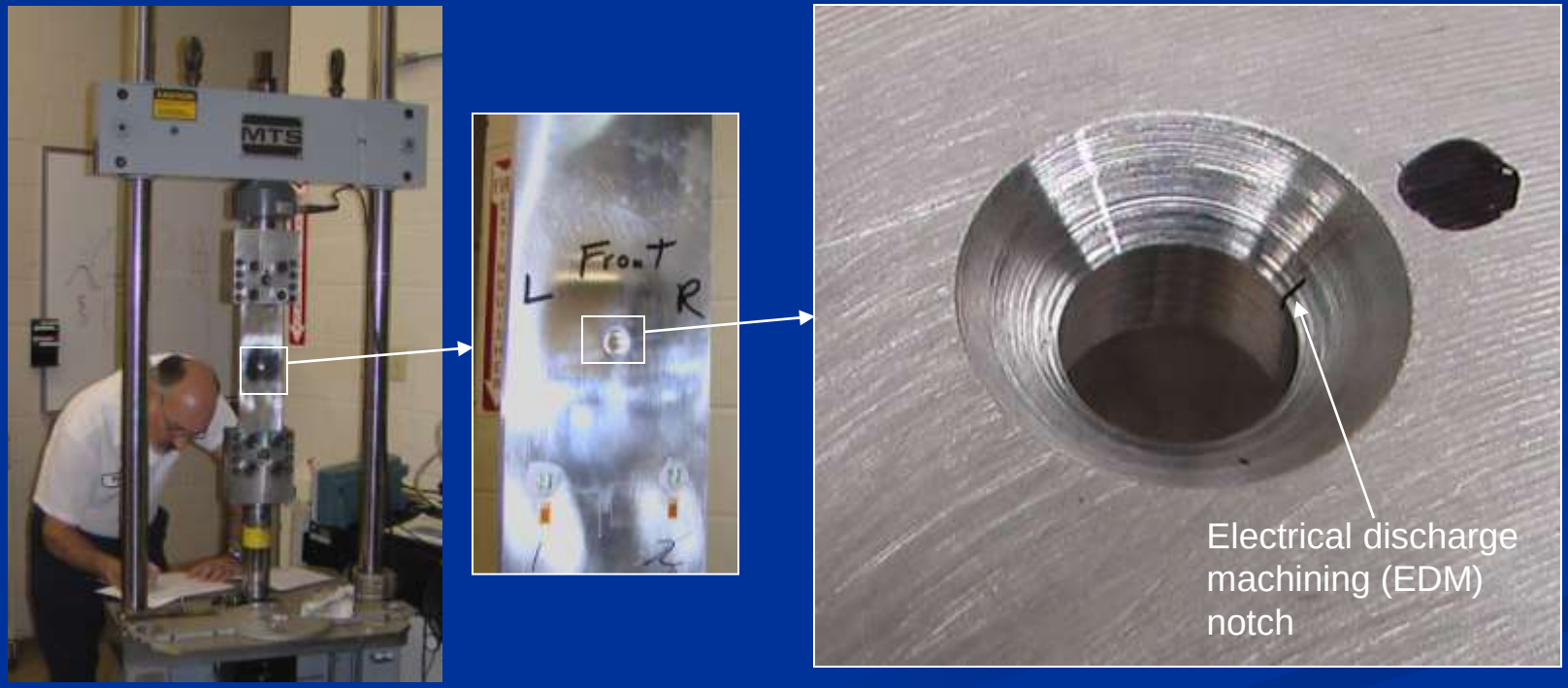


Validation Via Lab Testing – Specimen Geometry –



Validation Via Lab Testing – Test Setup –

- Back calculations will be applied to test data to experimentally determine SIFs for different crack lengths



Note: Validation will also be conducted via other published test results

SIF Solution Technique Verification

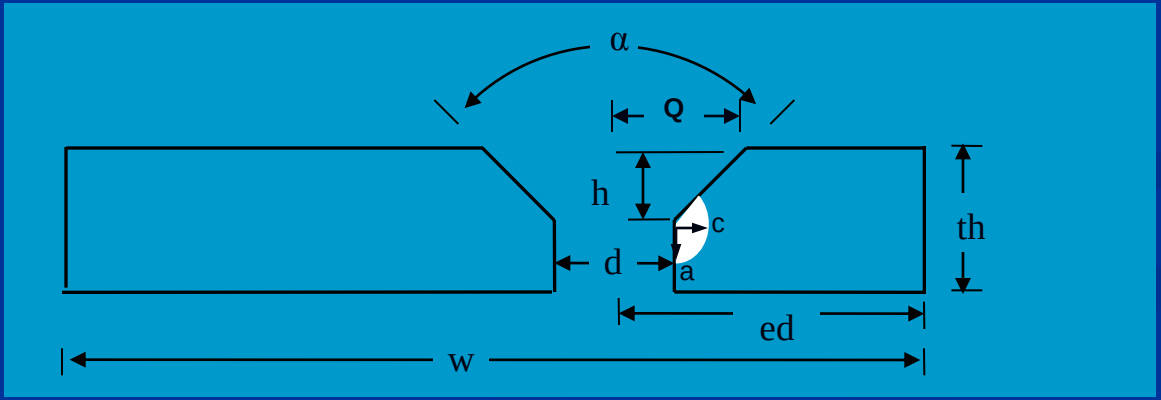
- Verify numerical solution technique by comparing solutions to other published material
  - Park 2003
  - Suh 2005

Solution Matrix

- After V&V process is successfully completed, run complete solution matrix

	Plate Geometry					Crack Geomety					
	CS Angle (α)	Finite Width Effects**	Hole radius to plate width ratio**	**CS Depth Effect**	**Hole radius to thickness ratio**	**Crack depth ratio**	**Crack aspect ratio**	Ellipse Placement**			
Dimensionless Ratio	N/A	2*(ed-Q)/w	r/w	**h/th**	**r/th**	**a/(t-h)**	**a/c**	Ell. Org Hight/th			
Range	N/A	0 to 1	0.005 to 0.2	**0 to 1**	**0.25 to 2**	**0 to 1**	**0.1 to 10**	0 to 1		Total SIF Solutions	Total Run Time (days)
Estimated Min. Permutations	1	5	5	**5**	**5**	**5**	**5**	1	=	3,125	22
Estimated Max. Permutations	1	10	10	**10**	**10**	**10**	**10**	10	=	1,000,000	6,944

** Indicate items that may be removed from scope



Note: Total Run time based on 10 minutes per solution estimate

7,000 to 12,000 elements per model

SIF Implementation Technique

Develop an easy to implement means of applying SIF solutions

- Empirical equation
- Computer program
- Other options?

Other Related Topics

- Hand meshing vs. auto meshing
- Poisson's ratio (0.3?)
- COM Enhancements

Conclusion

- The countersunk hole is a three-dimensional problem with many input variables
- Current simplification techniques for countersunk hole analysis are insufficient
- An accurate, easy to implement means to calculate SIFs for countersunk holes is needed