Progress Report No.1

DEVELOPMENT OF WEIGHT FUNCTIONS AND COMPUTER INTEGRATION PROCEDURES FOR CALCULATING STRESS INTENSITY FACTORS AROUND CRACKS SUBJECTED TO COMPLEX STRESS FIELDS

Prepared for

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Summary

The report consist the first part of the project described in the proposal associated with the Contract No. 554. The Progress Report No.1 consists of the theoretical background, the integration methods and the collection of weight functions as described in Tasks 2.1, 2.2 and 2.3. The development of the coded numerical routines is being carried out and it will be submitted in the next Report.

1. INTRODUCTION

Durability and strength evaluation of notched and cracked structural elements requires calculation of stress intensity factors for cracks located in regions characterized by complex stress fields. This is particularly true for cracks emanating form notches and other stress concentration regions that might be frequently found in mechanical and structural components. In the case of engine components complex stress distributions are often due to temperature effects. The existing stress intensity factor handbook solutions are not sufficient in such cases due to the fact that most of them have been derived for simple geometry and loading configurations. The variety of notch and crack configurations and the complexity of stress fields occurring in engineering practice require more efficient tools for calculating stress intensity factors than the large but nevertheless limited number of currently available ready made solutions, obtained for a few specific geometry and load combinations.

Therefore, it is proposed to develop a methodology for efficient calculation of stress intensity factors for cracks in complex stress fields by using the weight function approach.

2. STRESS INTENSITY FACTORS AND WEIGHT FUNCTIONS - TECHNICAL BACKGROUND

Most of the existing methods of calculating stress intensity factors require separate analysis for each load and geometry configuration. The weight function method developed by Bueckner [1] and Rice [2] simplifies considerably the determination of stress intensity factors. If the weight function is known for a given cracked body, the stress intensity factor due to any load system applied to the body can be determined by using the same weight function. Therefore there is no need to derive ready made stress intensity factor for each load system and resulting internal stress distribution. The stress intensity factor can be obtained by multiplying the weight function m(x,a) and the internal stress distribution σ(x) in the prospective crack plane and integrating the product along the crack length ‘a’.

The success of the weight function technique for calculating stress intensity factors lies in the possibility of using superposition. It can be shown [3] that the stress intensity factor for a crack body (Fig. 1) subjected to the external loading S is the same as stress intensity factor in geometrically identical body with the local stress field σ(x) applied to the crack faces. The local stress field σ(x) in the prospective crack plane is due to the external load S and it is determined for *uncracked* body by ignoring the presence of the crack.

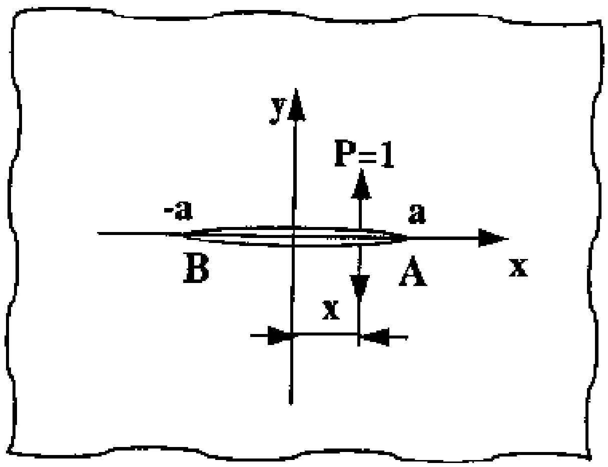
The unique feature of the weight function method is that once the weight function for a particular cracked body is determined, the stress intensity factor for any loading system applied to the body can be calculated by a simple integration of the product of the stress field, σ(x), and the weight function, m(x,a).

$$K = \int_0^a \sigma(x)\, m(x,a)\, dx \qquad (1)$$

The idea of using weight functions is illustrated in Fig.2, where the through thickness crack is used as an example. The weight function m(x,a) can be interpreted as the stress intensity factor that results from a pair of splitting forces applied to the crack face at position x. Since the stress intensity factors are linearly dependent on the applied loads, the contributions from multiple splitting forces applied along the crack surface can be superposed and the resultant stress intensity factor can be calculated as the sum of all individual load contributions. This results in the integral (1) of the product of the weight function m(x,a) and the stress field σ(x) for the case of continuously distributed stress field.

The Weight Function Approach to Calculation of Stress Intensity Factors

Weight Function - stress intensity factor, K, for a pair of splitting forces applied to crack surfaces



For a through crack in infinite plate:

$$m_A(x,a) = \frac{1}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}$$

$$m_B(x,a) = -\frac{1}{\sqrt{\pi a}}\sqrt{\frac{a-x}{a+x}}$$

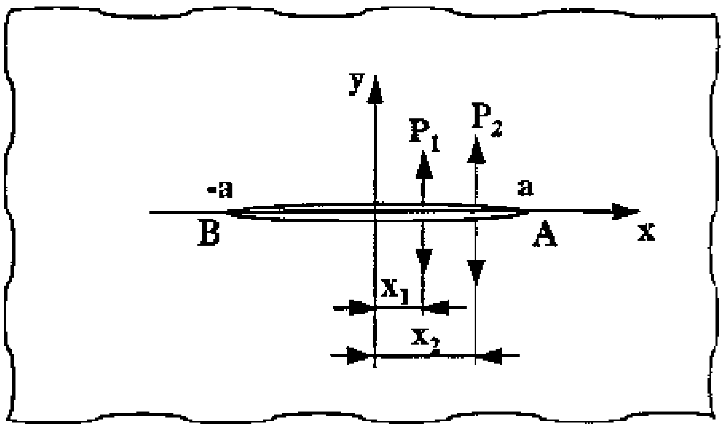
Stress intensity factors for crack crack tips A and B respectively:

$$K_A = m_A(x,a,P) = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}$$

$$K_B = m_B(x,a,P) = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a-x}{a+x}}$$

Fig. 2. a) Weight function and stress intensity factors for a through crack in an infinite plate.

The Weight Function Approach to Calculation of Stress Intensity Factors



In the case of two forces, by using superposition

$$K_A = m_A(x_1, a, P_1) + m_A(x_2, a, P_2)$$

$$K_A = \frac{P_1}{\sqrt{\pi a}}\sqrt{\frac{x_1 + a}{a - x_1}} + \frac{P_2}{\sqrt{\pi a}}\sqrt{\frac{x_2 + a}{a - x_2}}$$

In the case of multiple forces

$$K_A = m(a, x_1, P_1) + m(a, x_2, P_2) + \ldots + m(a, x_i, P_i)$$

For the stress distribution σ(x)

$$K_A = \int_{-a}^{a} \sigma(x) m_A(x, a) dx$$

$$K_B = \int_{-a}^{a} \sigma(x) m_B(x, a) dx$$

Fig.2. b) Weight function and stress intensity factors for a through crack in an infinite plate subjected to multiple loads or/and continuously distributed stress.

3. UNIVERSAL WEIGHT FUNCTIONS FOR ONE-DIMENSIONAL CRACKS

The weight function is dependent on the geometry only and in principle should be derived individually for each geometrical configuration. However, Glinka and Shen [4] have found that one general weight function expression can be used to approximate weight functions for a variety of geometrical configurations of cracked bodies with one dimensional cracks of Mode I.

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right] \qquad (2)$$

The system of coordinates and the notation for internal through and edge cracks are given in Fig. 3.

In order to determine the weight function m(x,a) of eq(2) for a particular cracked body it is sufficient to determine [5] the three parameters M_1, M_2, and M_3. Because the weight function of form (2) is the same for all cracks then the same integration routine can be used for calculating stress intensity factors from eq.(1). Moreover, only limited number of generic weight functions is needed to enable the determination of stress intensity factors for a large number of load and geometry configurations.

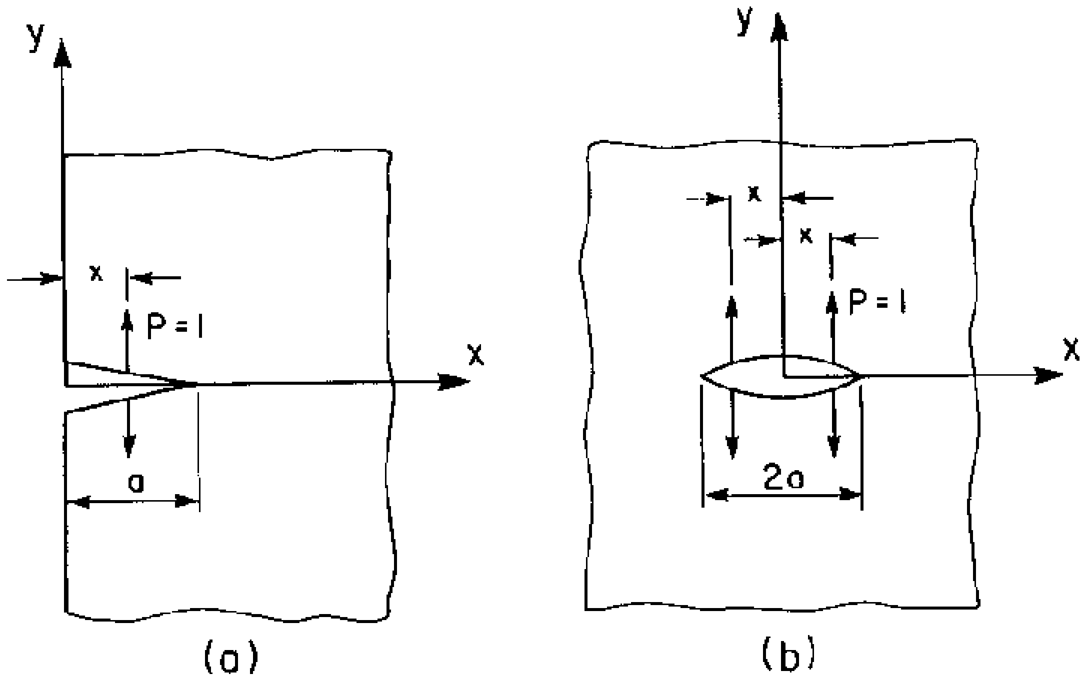


Figure 3. Single Edge Crack and Central Through Crack Under Symmetrical Loading

4. UNIVERSAL WEIGHT FUNCTIONS FOR TWO-DIMENSIONAL PART-THROUGH SURFACE AND CORNER CRACKS

In the case of 2-D cracks such as semi-elliptical and corner surface cracks in plates and cylinders the stress intensity factor changes along the crack front. However, in most practical cases the deepest point A (Fig. 4) and the surface point B are associated with the highest and the lowest value of the stress intensity factor along the crack front. Therefore, the weight functions for the points A and B have been derived [6] analogously to the universal weight function (2).

- For point A (Fig. 4)

$$m_A(x, a, a/c, a/t) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right] \quad (3)$$

- For point B (Fig. 4)

$$m_B(x, a, a/c, a/t) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right] \quad (4)$$

The weight functions $m_A(x,a)$ and $m_B(x,a)$ given above corresponding to the deepest and the surface point A and B respectively have been derived for one-dimensional stress fields (Fig. 4), dependent on one variable , x , only.

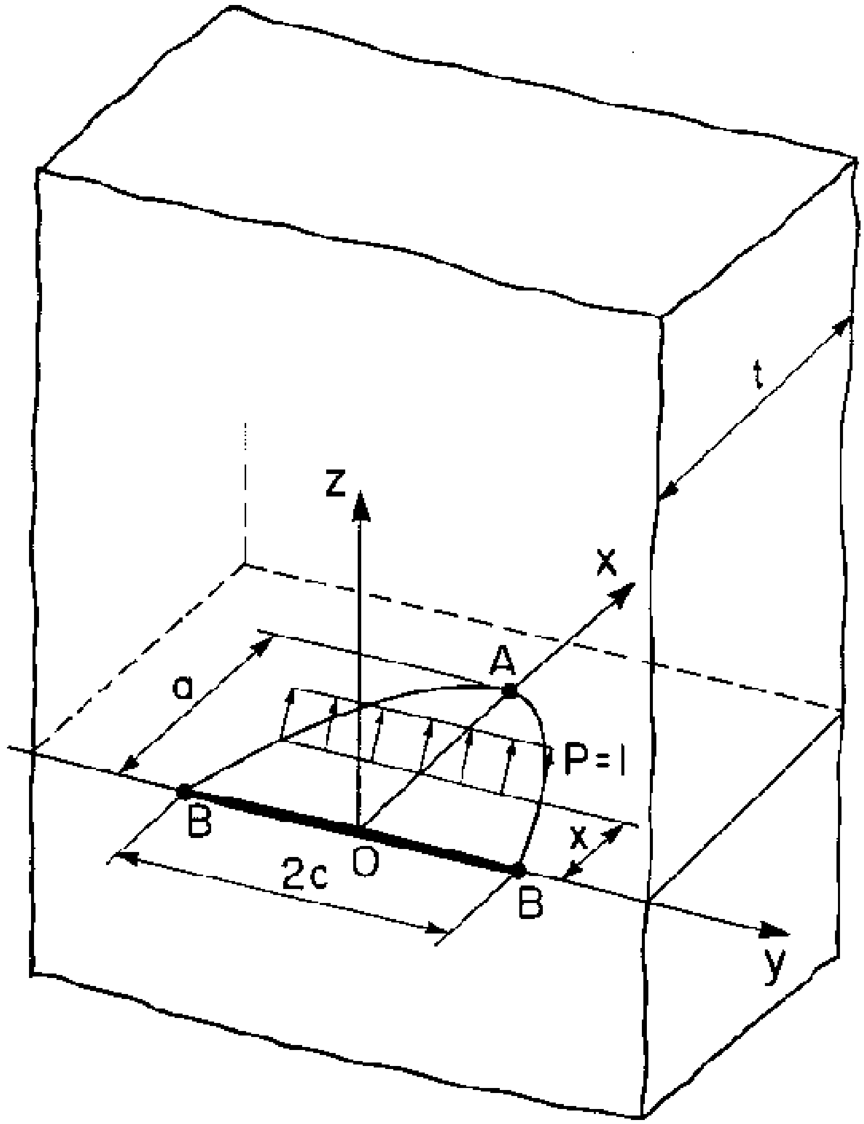


Fig. 4. Semi-elliptical surface crack in an infinite plate of finite thickness, t.

5. SEQUENCE OF STEPS FOR CALCULATING STRESS INTENSITY FACTORS USING WEIGHT FUNCTIONS

In order to calculate stress intensity factors using the weight function technique the following tasks need to be carried out:

- Determine stress distribution $\sigma(x)$ in the prospective crack plane using linear elastic analysis of uncracked body (Fig. 1a), i.e. perform the stress analysis ignoring the crack and determine the stress distribution $\sigma(x) = \sigma_0\, f(S,x)$;
  where: σ_0 - nominal stress, x - coordinate, S - external load

- Apply the "uncracked" stress distribution $\sigma(x)$ to the crack surfaces (Fig. 1b) as tractions

- Choose appropriate generic weight function (i.e. choose appropriate M_1, M_2, and M_3 parameters).

- Integrate the product of the stress function $\sigma(x)$ and the weight function m(x,a) over the entire crack length or crack surface, eq.(1).

6. DETERMINATION OF WEIGHT FUNCTIONS

The derivation of the weight function of eq.(2) for a one-dimensional crack is essentially reduced to the determination of parameters M_1, M_2, and $M_{3.}$ There are several methods available depending on the amount of information available as described in reference [5]. The most frequently used method based on two reference stress intensity factors is described below.

Supposing that two reference stress intensity solutions K_{r1} and K_{r2} for two different stress fields $\sigma_{r1}(x)$ and $\sigma_{r2}(x)$ respectively are known, a set of two equations can be written by using eqs.(1) and (2). The third equation necessary to determine parameters M_1, M_2, and

M_3 can be formulated based on the knowledge of the crack surface slope [5,6] at the crack center (central through cracks) or from the crack surface curvature at the crack mouth (edge cracks). The three pieces of information result in three independent equations which can be used for the determination of unknown parameters M_1, M_2, and M_3.

- **Central through cracks under symmetric loading** (Fig. 3a)

$$K_{r1} = \int_0^a \sigma_{r1}(x)\frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]dx \qquad (5)$$

$$K_{r2} = \int_0^a \sigma_{r2}(x)\frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]dx \qquad (6)$$

$$\left.\frac{\partial}{\partial x}\left\{\frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]\right\}\right|_{x=0} = 0 \qquad (7)$$

- **Edge cracks**

$$K_{r1} = \int_0^a \sigma_{r1}(x)\frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]dx \qquad (8)$$

$$K_{r2} = \int_0^a \sigma_{r2}(x)\frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]dx \qquad (9)$$

$$\left.\frac{\partial}{\partial x}\left\{\frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]\right\}\right|_{x=0} = 0 \qquad (10)$$

The two reference cases can be obtained from the literature or they can be derived using finite element method. Most often available are the solutions for the uniform and linear stress distributions induced by simple tension or bending load and those cases were predominantly used to derive the weight functions listed below.

- **Semi-elliptical surface and corner cracks**

The set of equations necessary for deriving M_{iA} parameters for the weight function corresponding to the deepest point A on the crack front (Fig. 3) was found to be identical to the set of equations derived for the edge crack.

$$K_{r1}^{A} = \int_0^a \sigma_{r1}(x) \frac{2}{\sqrt{2\pi(a-x)}} \left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}} \right] dx \qquad (11)$$

$$K_{r2}^{A} = \int_0^a \sigma_{r2}(x) \frac{2}{\sqrt{2\pi(a-x)}} \left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}} \right] dx \qquad (12)$$

$$\left.\frac{\partial}{\partial x}\left\{ \frac{2}{\sqrt{2\pi(a-x)}} \left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}} \right]\right\}\right|_{x=0} = 0 \qquad (13)$$

In the case of the surface point B the first two equations were the same as previously and the additional equation was derived [6] by satisfying the condition that the weight function must vanish for x=0.

$$K_{r1}^{B} = \int_0^a \sigma_{r1}(x) \frac{2}{\sqrt{\pi x}} \left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}} \right] dx \qquad (14)$$

$$K_{r2}^{B} = \int_0^a \sigma_{r2}(x) \frac{2}{\sqrt{\pi x}} \left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}} \right] dx \qquad (15)$$

$$0 = 1 + M_{1B} + M_{2B} + M_{3B} \qquad (16)$$

The three unknown parameters M_i can be determined by simultaneously solving one of the set of equations presented above.

7. NUMERICAL INTEGRATION OF THE WEIGHT FUNCTION AND CALCULATION OF STRESS INTENSITY FACTORS

The calculation of stress intensity factor from the weight function requires integration of of the product “s(x)m(x,a)” along the crack length according to eq.(1). The weight function can always be written in the general form of eq.(3) or eq.(4). However the stress distribution “s(x)” can take any form depending on the problem of interest. If the stress distribution is given in the form of a mathematical expression analytical integration can be performed and closed form integrals of eq.(1) are sometimes feasible. However, very often the stress distribution “s(x)” is obtained from finite element calculations and the results are given as a series of stress values corresponding to a range of point of coordinate “x”. Therefore, a numerical integration technique is needed for the integration of equation (1) and the calculation of stress intensity factors. Two methods of efficient integration of eq.(1) are described below.

7.1 Integration by using the centroids of areas under the weight function curve

The integration method using the area centroids is based on the following theorem:

If m(x,a) and □ (x) are monotonic and linear function respectively and both depend on variable x only, (Fig. 5), then the integral (1) can be calculated from expression (14), representing the product of the area S under the curve m(x,a) and the value of the function σ(X) corresponding to the coordinate x=X of the centroid C.

$$K = S * \sigma(X) \qquad (14)$$

The weight functions, m(x,a), are monotonic and non-linear. The stress functions, σ(x), are usually nonlinear as well. Therefore, in order to apply the theorem above to the integral (1), the integration interval is divided into “n” sub-intervals in such a way that, the stress function σ(x) is approximated by the secant line drawn between the end points of each sub-interval Fig. (6). Thus, the stress function σ(x) over the sub-interval “i”, may be written in the form of Eq. (15),

$$\sigma_i(x) = A_i x + B_i \quad \text{for} \quad (x_i - 1) \le x \le x_i \qquad (15)$$

where:

$$A_i = \frac{\sigma(x_i) - \sigma(x_{i-1})}{x_i - x_{i-1}} \quad \text{and} \quad B_i = \sigma(x_i) - A_i x_i \qquad (15a)$$

After substitution of expression (15) into eq. (1) and summation over all sub-intervals the following expression for the stress intensity factor can be derived.

$$K = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} (A_i x + B_i) m(x,a) dx \qquad (16)$$

Each integral in eq.(16) can be computed by using the simplified (Fig. 5) integration method given in the form of Eq.(14). Thus, the stress intensity factor, K, can be finally written in the form of expression (17).

$$K = \sum_{i}^{n} S_i * \sigma(X_i), \qquad \text{where } i = 1,2,\ldots n \qquad (17)$$

In order to calculate the stress intensity factor given in the form of Eq. (17), it is necessary to calculate the areas S_i under the weight function curve m(x,a), and the co-ordinates of their centroids, X_i. Both the areas S_i and the centroid co-ordinates X_i for each sub-interval can be calculated once in a general form based on the generalised weight function (2). However, they appear to be too lengthy for an easy hand calculation. Fortunately, further simplification of the integration routine is possible due to the fact that the weight functions are smooth within their ranges of integration. Therefore the procedure can be reversed (Fig. 7) by calculating first the areas S_i^* under the stress function $\sigma(x)$ and the coordinates X_i^* of their centroid. Then the appropriate values of the weight function $m(X^*,a)$ can be calculated from expression (2). It is worth noting that in the case of the piece-wise approximation of the stress function, $\sigma(x)$, the areas S_i^* and the coordinates of their centroids, X_i^*, can be easily calculated from relations (18) and (19) respectively.

$$S_i^* = \frac{1}{2}\left[\sigma(x_i) + \sigma(x_{i-1})\right](x_i - x_{i-1}) \tag{18}$$

$$X_i^* = x_i - \frac{(x_i - x_{i-1})\left[2\sigma(x_{i-1}) + \sigma(x_i)\right]}{3\left[\sigma(x_i) + \sigma(x_{i-1})\right]} \tag{19}$$

Finally the stress intensity factor, K, is calculated from expression (20).

$$K = \sum_{i=1}^{n} S_i^* m\left(X_i^*, a\right) \tag{20}$$

Thus the numerical procedure for calculating the stress intensity factor using the integration method described above requires the calculation of appropriate parameters using equations (2) and (18-20). The method described above is recommended for quick approximate calculations with the help of a hand calculator.

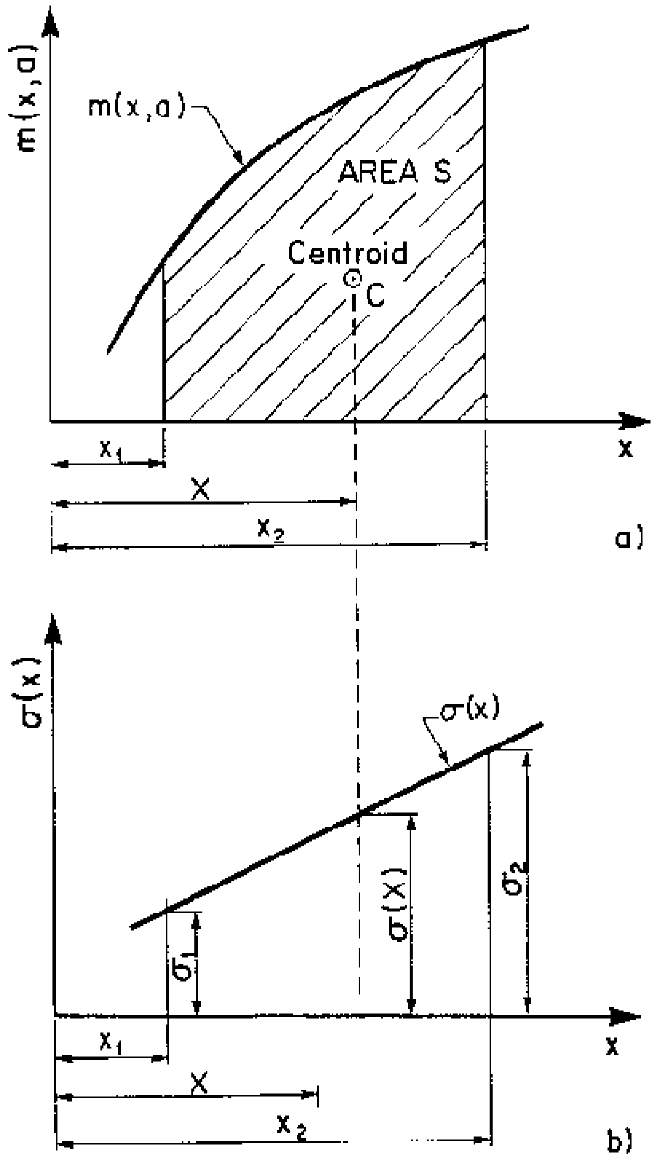


Fig. 5.Graphical representation of simplified integration of the product of two one-dimensional functions, a) monotonic non-linear weight function m(x,a), b) linear stress function s(x).

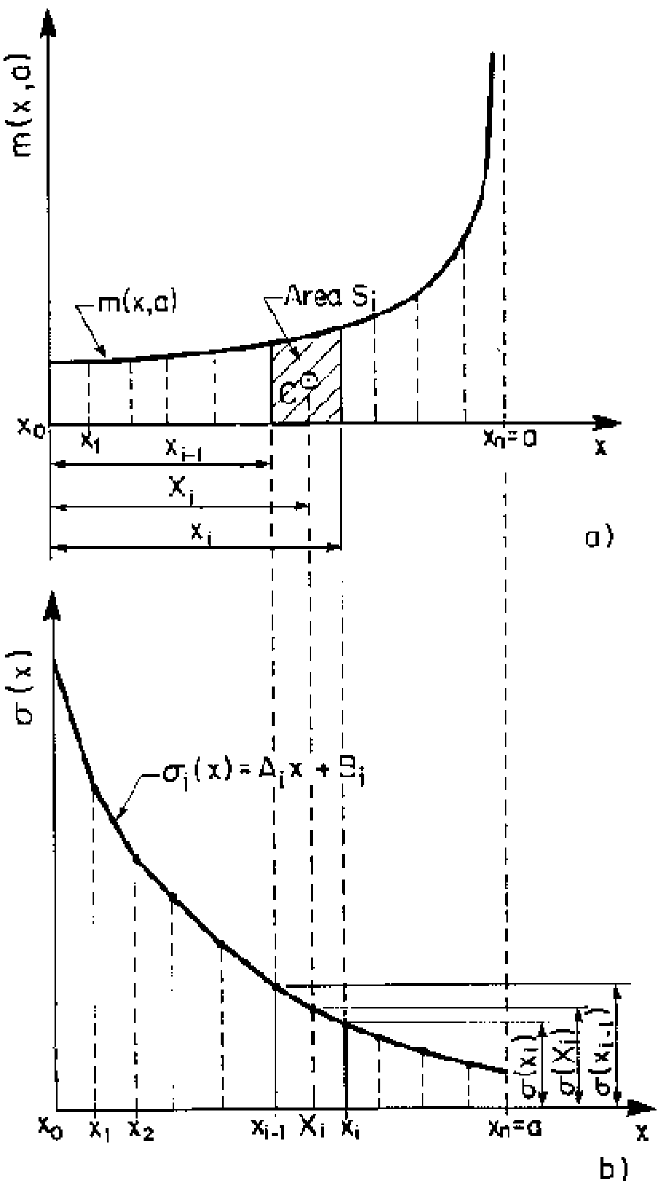


Fig. 6. Application of the simplified integration method to the case of non-linear stress distribution; a) weight function m(x,a), b) linearized stress function.

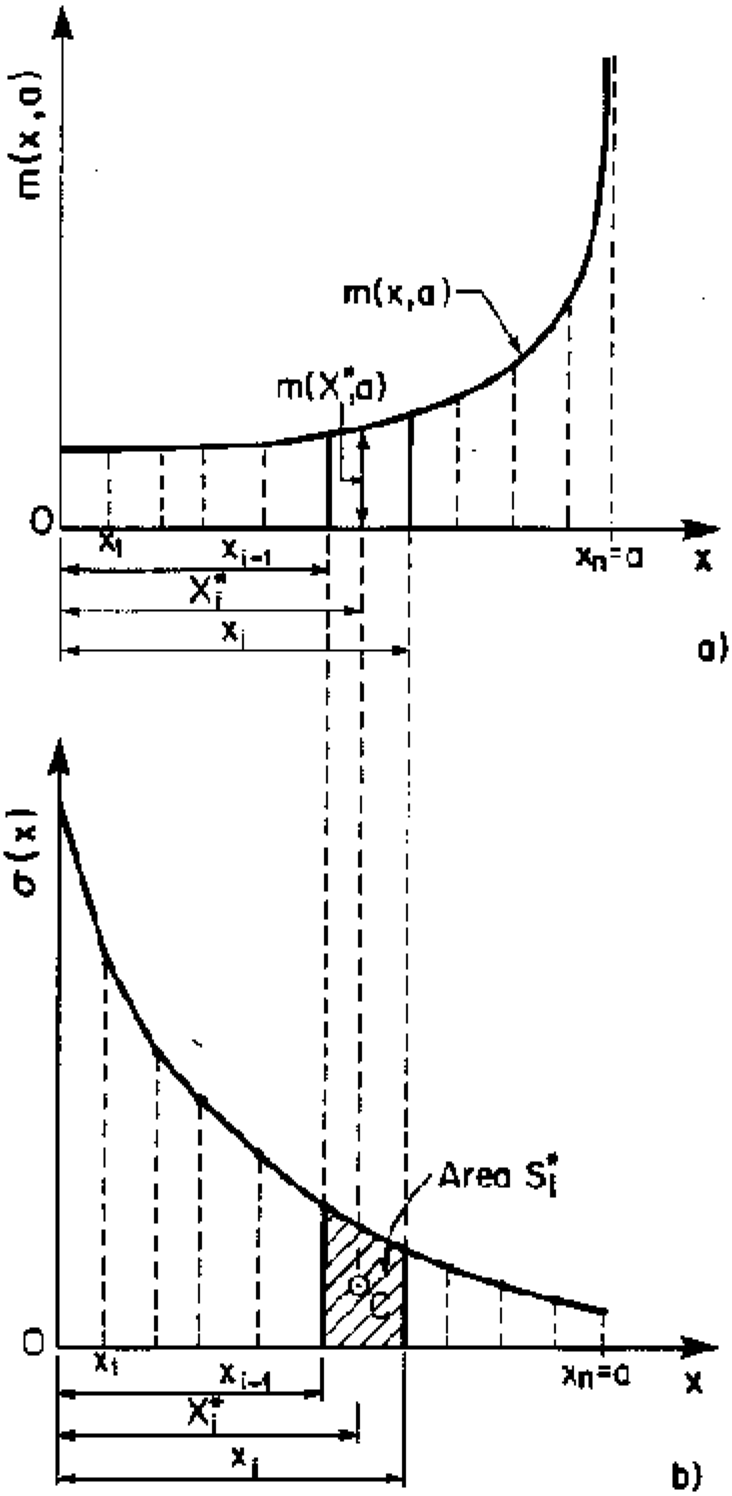


Fig. 7. Integration method based on the sub-areas and centroids of the linearized piecewise stress function, a) weight function m(x,a), b) linearized piecewise stress function.

7.2 Analytical integration of the linearized piecewise stress distribution and the weight function

The integration technique described in Section 7.1 is convenient for hand calculator calculation when the stress distribution can be approximated by a few linear segments. However, the segments adjacent to the crack tip can not be large because the weight function tends to infinity near the crack tip and if the stress is the highest near the crack tip the method described above might be sometimes inaccurate. Moreover, in the case of stress distributions characterized by high gradients accurate approximation of the stress distribution requires relatively large number of linear pieces and the integration has to be carried out with the help of a computer. However, the computer integration routine can also be significantly simplified because closed form solution to the integral (16) can be derived analytically for each linear piece (Fig. 7) of the stress function s(x).

The stress function s(x) over the linear segment "i" can be given in the form of the linear equation (15). Thus the contribution to the stress intensity factor associated with the stress segment "i" can be calculated from eq.(1) after substituting appropriate expressions for the stress and the weight function. The solutions given below have been derived for the deepest and for the surface point of semi-elliptical crack using the weight function (3) and (4) respectively. The solution to one-dimensional cracks is the same as for the deepest point of semi-elliptical cracks.

- Deepest point A (Fig. 4)

$$K_i^A = \int_{x_{i-1}}^{x_i} \left(A_i x + B_i\right) \frac{2}{\sqrt{2\pi a\left(1-\frac{x}{a}\right)}} \left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right] dx \qquad (21)$$

- Surface point B (Fig.4)

$$K_i^B = \int_{x_{i-1}}^{x_i} \left(A_i x + B_i\right) \frac{2}{\sqrt{2\pi x}} \left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right] dx \qquad (22)$$

The closed form expressions resulting from the integration of eq.(21) and eq.(22) are given below.

- Deepest point A (Fig. 4)

$$K_i^A = \sqrt{\frac{2}{\pi a}}\left[\alpha_i\left(C_{i1} + M_{1A}C_{i2} + M_{2A}C_{i3} + M_{3A}C_{i4}\right) + \beta_i\left(C_{i3} + M_{1A}C_{i4} + M_{2A}C_{i5} + M_{3A}C_{i6}\right)\right] \quad (23)$$

where:

$\alpha_i = B_i + aA_i$ and $\beta_i = -aA_i$

$$C_{i1} = 2a\left[\left(1-\frac{x_{i-1}}{a}\right)^{\frac{1}{2}} - \left(1-\frac{x_i}{a}\right)^{\frac{1}{2}}\right] \qquad C_{12} = a\left[\left(1-\frac{x_{i-1}}{a}\right)^{1} - \left(1-\frac{x_i}{a}\right)^{1}\right]$$

$$C_{i3} = \frac{2a}{3}\left[\left(1-\frac{x_{i-1}}{a}\right)^{\frac{3}{2}} - \left(1-\frac{x_i}{a}\right)^{\frac{3}{2}}\right] \qquad C_{14} = \frac{a}{2}\left[\left(1-\frac{x_{i-1}}{a}\right)^{2} - \left(1-\frac{x_i}{a}\right)^{2}\right]$$

$$C_{i5} = \frac{2a}{5}\left[\left(1-\frac{x_{i-1}}{a}\right)^{\frac{5}{2}} - \left(1-\frac{x_i}{a}\right)^{\frac{5}{2}}\right] \qquad C_{16} = \frac{a}{3}\left[\left(1-\frac{x_{i-1}}{a}\right)^{3} - \left(1-\frac{x_i}{a}\right)^{3}\right]$$

Surface point B (Fig.4)

$$K_i^B = \sqrt{\frac{2}{\pi a}}\left[\begin{array}{l}(\alpha_i + \beta_i)(D_{i1} + M_{1B}D_{i2} + M_{2B}D_{i3} + M_{3B}D_{i4}) \\ \qquad -\beta_i(D_{i3} + M_{1B}D_{i4} + M_{2B}D_{i5} + M_{3B}D_{i6})\end{array}\right] \quad (24)$$

where:

$\alpha_i = B_i + aA_i$ and $\beta_i = -aA_i$

$$D_{i1} = 2a\left[\left(\frac{x_i}{a}\right)^{\frac{1}{2}} - \left(\frac{x_{i-1}}{a}\right)^{\frac{1}{2}}\right] \qquad D_{12} = a\left[\left(\frac{x_i}{a}\right)^{1} - \left(\frac{x_{i-1}}{a}\right)^{1}\right]$$

$$D_{i3} = \frac{2a}{3}\left[\left(\frac{x_i}{a}\right)^{\frac{3}{2}} - \left(\frac{x_{i-1}}{a}\right)^{\frac{3}{2}}\right] \qquad D_{14} = \frac{a}{2}\left[\left(\frac{x_i}{a}\right)^{2} - \left(\frac{x_{i-1}}{a}\right)^{2}\right]$$

$$D_{i5} = \frac{2a}{5}\left[\left(\frac{x_i}{a}\right)^{\frac{5}{2}} - \left(\frac{x_{i-1}}{a}\right)^{\frac{5}{2}}\right] \qquad D_{16} = \frac{a}{3}\left[\left(\frac{x_i}{a}\right)^{3} - \left(\frac{x_{i-1}}{a}\right)^{3}\right]$$

Equations (23) and (24) can be used for calculating stress intensity contributions due to each linear piece of the stress distribution function by substituting appropriate values for a, x_{i-1}, x_i, A_i and B_i. The stress intensity factor K can be finally calculated as the sum of allcontributions K_i associated with the linear pieces within the range of 0 = x = a.

$$K = \sum_{i}^{n} K_i \qquad (25)$$

Thus the integration can be reduced to the substitution of appropriate parameters into equations (23 -24) and summation according to eq(25). This makes it possible to develop very efficient numerical integration routine which is important in the case of lengthy fatigue crack growth analyses.

8. WEIGHT FUNCTIONS FOR CRACKS IN PLATES and DISKS

The weight functions are given in the form of expressions describing the M_i parameters as functions of crack dimensions and geometry of the cracked body. Given are the range of application, the accuracy and the source of the reference stress intensity factors for each set of parameters M_i including the generic geometry of the crack body.

8.1 Through crack in an infinite plate subjected to symmetrical loading

WEIGHT FUNCTION (Fig. 8a)

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

M_1= 0.0698747

M_2= -0.0904839

M_3= 0.427203

ACCURACY: Max. error less than 1% when compared to exact solution

REFERENCES:

weight function:
Glinka G., Shen G., 1991, "Universal Features of Weight Functions for Cracks in Mode I "*Engng. Fract. Mech"*, Vol. 40, No. 6, pp. 1135-1146.

reference data:
Tada H., Paris P. C., Irwin G.R., 1985, *The Stress Analysis of Cracks Handbook*, 2nd ed., Paris Productions Inc., St. Louis, Missouri, pp. 5.11-5.11a.

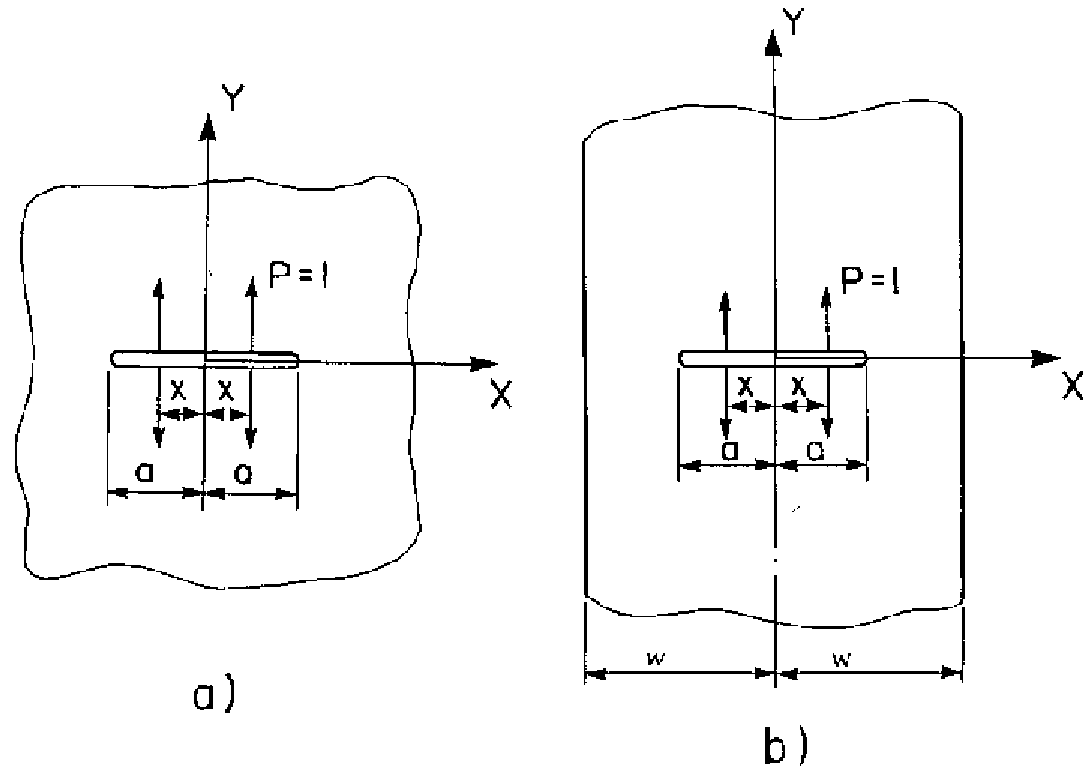


Figure 8. Central through-crack in an infinite plate (a) and finite-width plate (b).

8.2 *Central through crack in a finite width plate subjected to symmetric loading*

WEIGHT FUNCTION (Fig. 8b)

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_1 = 0.06987 + 0.40117\left(\frac{a}{w}\right) - 5.5407\left(\frac{a}{w}\right)^2 + 50.0886\left(\frac{a}{w}\right)^3 - 200.699\left(\frac{a}{w}\right)^4 + 395.552\left(\frac{a}{w}\right)^5 - 377.939\left(\frac{a}{w}\right)^6 + 140.218\left(\frac{a}{w}\right)^7$$

$$M_2 = 0.09049 - 2.14886\left(\frac{a}{w}\right) + 22.5325\left(\frac{a}{w}\right)^2 - 89.6553\left(\frac{a}{w}\right)^3 + 210.599\left(\frac{a}{w}\right)^4 - 239.445\left(\frac{a}{w}\right)^5 + 111.128\left(\frac{a}{w}\right)^6$$

$$M_3 = 0.427216 + 2.56001\left(\frac{a}{w}\right) - 29.6349\left(\frac{a}{w}\right)^2 + 138.40\left(\frac{a}{w}\right)^3 - 347.255\left(\frac{a}{w}\right)^4 + 457.128\left(\frac{a}{w}\right)^5 - 295.882\left(\frac{a}{w}\right)^6 + 68.1575\left(\frac{a}{w}\right)^7$$

RANGE OF APPLICATION: $0 < a/w < 0.9$.

ACCURACY: Better than 1% when compared to the reference solution.

REFERENCES:

- weight function:
Moftakhar A., Glinka G., 1992, “Calculation of Stress Intensity Factors by Efficient Integration of Weight Functions,“ *Engng. Fract. Mech.*, Vol. 43, No. 5, pp. 749-756.

Glinka G., Shen G., 1991, “Universal Features of Weight Functions for Cracks in Mode I, “*Engng. Fract. Mech.*, Vol. 40, No. 6, pp. 1135-1146.

- reference data:
Tada H., Paris P. C., Irwin G.R., 1985, *The Stress Analysis of Cracks Handbook*, 2nd ed., Paris Productions Inc., St. Louis, Missouri, pp. 2.1-2.2, 2.34.

Sih G., 1973, *Handbook of Stress Intensity Factors*, Institute of Fracture and Solid Mechanics, Lehigh University, Bethlehem, P.A.

8.3 Edge crack in a semi-infinite plate

WEIGHT FUNCTION (Fig. 9a)

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

M1= 0.0719768

M2= 0.246984

M3= 0.514465

ACCURACY: Max. error less than 1% when compared to the reference solution

REFERENCES:

- weight function:

Glinka G., Shen G., 1991, “Universal Features of Weight Functions for Cracks in Mode I,“ *Engng. Fract. Mech.*, Vol. 40, No. 6, pp. 1135-1146.

- reference data:

Tada H., Paris P. C., Irwin G.R., 1985, *The Stress Analysis of Cracks Handbook*, 2nd ed., Paris Productions Inc., St. Louis, Missouri, pp. 8.3-8.3a.

Sih G., 1973, *Handbook of Stress Intensity Factors*, Institute of Fracture and Solid Mechanics, Lehigh University, Bethlehem, P.A.

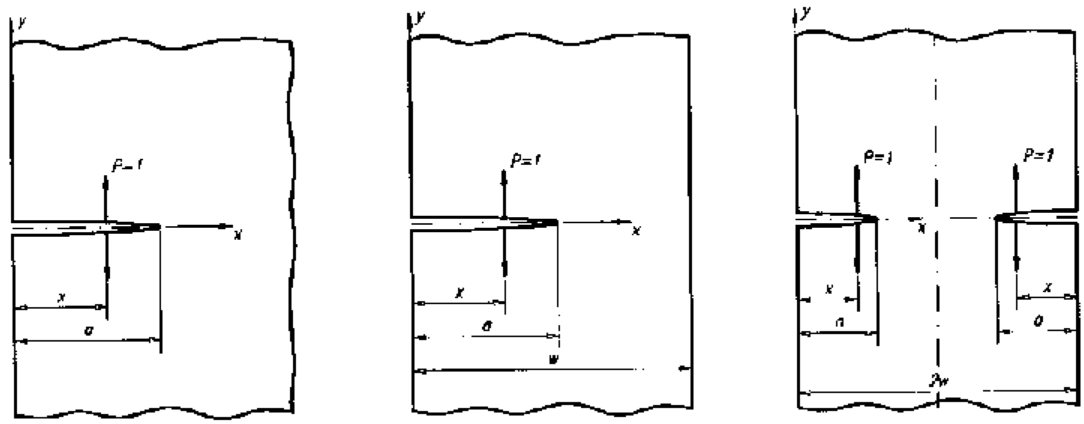


Fig 9: Edge crack solutions in semi-infinite and finite-width plates.

8.4 Edge crack in a finite width plate

WEIGHT FUNCTION (Fig.9b)

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_1 = 0.0719768 - 1.51346\left(\frac{a}{w}\right) - 61.1001\left(\frac{a}{w}\right)^2 + 1554.95\left(\frac{a}{w}\right)^3 - 14583.8\left(\frac{a}{w}\right)^4 + 71590.7\left(\frac{a}{w}\right)^5$$
$$- 205384\left(\frac{a}{w}\right)^6 + +356469\left(\frac{a}{w}\right)^7 - 368270\left(\frac{a}{w}\right)^8 + 208233\left(\frac{a}{w}\right)^9 - 49544\left(\frac{a}{w}\right)^{10}$$

$$M_2 = 0.246984 + 6.47583\left(\frac{a}{w}\right) + 176.456\left(\frac{a}{w}\right)^2 - 4058.76\left(\frac{a}{w}\right)^3 + 37303.8\left(\frac{a}{w}\right)^4 - 181755\left(\frac{a}{w}\right)^5$$
$$+ 520551\left(\frac{a}{w}\right)^6 - 904370\left(\frac{a}{w}\right)^7 + 936863\left(\frac{a}{w}\right)^8 - 531940\left(\frac{a}{w}\right)^9 + 127291\left(\frac{a}{w}\right)^{10}$$

$$M_3 = 0.529659 - 22.3235\left(\frac{a}{w}\right) + 532.074\left(\frac{a}{w}\right)^2 - 5479.53\left(\frac{a}{w}\right)^3 + 28592.2\left(\frac{a}{w}\right)^4$$
$$- 81388.6\left(\frac{a}{w}\right)^5 + 128746\left(\frac{a}{w}\right)^6 - 106246\left(\frac{a}{w}\right)^7 + 35780.7\left(\frac{a}{w}\right)^8$$

RANGE OF APPLICATION: $0 < a/w < 0.9$.

ACCURACY: Better than 1% when compared to the reference solution.

REFERENCES:

- weight function:

Moftakhar A., Glinka G., 1992, "Calculation of Stress Intensity Factors by Efficient Integration of Weight Functions," *Engng. Fract. Mech.*, Vol. 43, No. 5, pp. 749-756.

Glinka G., Shen G., 1991, "Universal Features of Weight Functions for Cracks in Mode I," *Engng. Fract. Mech.*, Vol. 40, No. 6, pp. 1135-1146.

- reference data:

Tada H., Paris P. C., Irwin G.R., 1985, *The Stress Analysis of Cracks Handbook*, 2nd ed., Paris Productions Inc., St. Louis, Missouri, p. 2.27.

Kaya A. C.., Erdogan F., 1980, "Stress Intensity Factors and COD in an Orthotropic Strip, "*Int. J. Fracture*, Vol. 16, pp. 171-190.

8.5 Two symmetrical edge cracks in a finite width plate subjected to symmetrical loading

WEIGHT FUNCTION (Fig.9c)

$$m(x,a)=\frac{2}{\sqrt{2\pi(a-x)}}\left[1+M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+M_2\left(1-\frac{x}{a}\right)^{1}+M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_1=0.08502-0.02230\left(\frac{a}{w}\right)-1.41028\left(\frac{a}{w}\right)^3+4.64559\left(\frac{a}{w}\right)^3+19.6924\left(\frac{a}{w}\right)^4$$
$$-148.266\left(\frac{a}{w}\right)^5+336.837\left(\frac{a}{w}\right)^6-336.591\left(\frac{a}{w}\right)^7+127.009\left(\frac{a}{w}\right)^8$$

$$M_2=0.2234-0.6146\left(\frac{a}{w}\right)+11.1687\left(\frac{a}{w}\right)^2-56.5326\left(\frac{a}{w}\right)^3+151.937\left(\frac{a}{w}\right)^4$$
$$-182.634\left(\frac{a}{w}\right)^5+86.4731\left(\frac{a}{w}\right)^6$$

$$M_3=0.4983+0.7512\left(\frac{a}{w}\right)-10.5597\left(\frac{a}{w}\right)^2+47.9251\left(\frac{a}{w}\right)^3-115.933\left(\frac{a}{w}\right)^4$$
$$+131.976\left(\frac{a}{w}\right)^5-59.8893\left(\frac{a}{w}\right)^6$$

RANGE OF APPLICATION: $0 < a/w < 0.9$.

ACCURACY: Better than 1% when compared to the reference solution.

REFERENCES:

- weight function:

Moftakhar A., Glinka G., 1992, "Calculation of Stress Intensity Factors by Efficient Integration of Weight Functions," *Engng. Fract. Mech.*, Vol. 43, No. 5, pp. 749-756.

Glinka G., Shen G., 1991, "Universal Features of Weight Functions for Cracks in Mode I, "*Engng. Fract. Mech.*, Vol. 40, No. 6, pp. 1135-1146.

- reference data:

Tada H., Paris P. C., Irwin G.R., 1985, *The Stress Analysis of Cracks Handbook*, 2nd ed., Paris Productions Inc., St. Louis, Missouri, p. 2.31.

8.6 Embedded penny-shape crack in an infinite body subjected to symmetric loading

WEIGHT FUNCTION (Fig. 10) - **Point A**

$$m_A(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{1/2} + M_{2A}\left(1-\frac{x}{a}\right) + M_{3A}\left(1-\frac{x}{a}\right)^{3/2}\right]$$

PARAMETERS

$$M_{1A} = -0.646714$$
$$M_{2A} = 0.303783$$
$$M_{3A} = 0.527654$$

WEIGHT FUNCTION (Fig. 10) - **Point B**

$$m_B(x,a) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{1/2} + M_{2B}\left(\frac{x}{a}\right) + M_{3B}\left(\frac{x}{a}\right)^{3/2}\right]$$

PARAMETERS

$$M_{1B} = -1$$
$$M_{2B} = 0$$
$$M_{3B} = 0$$

ACCURACY: Better than 1% when compared to the reference solution.

REFERENCES:

- weight function:

Glinka G., Shen G., 1991, "Universal Features of Weight Functions for Cracks in Mode I ,"*Engng. Fract. Mech.*, Vol. 40, No. 6, pp. 1135-1146.

- reference data:

Gallin L. A., 1953, *Contact Problems in the Theory of Elasticity*, (in Russian), translation: North Carolina State College Publications (1961).

Tada H., Paris P. C., Irwin G.R., 1985, *The Stress Analysis of Cracks Handbook*, 2nd ed., Paris Productions Inc., St. Louis, Missouri, pp. 24.2-24.3.

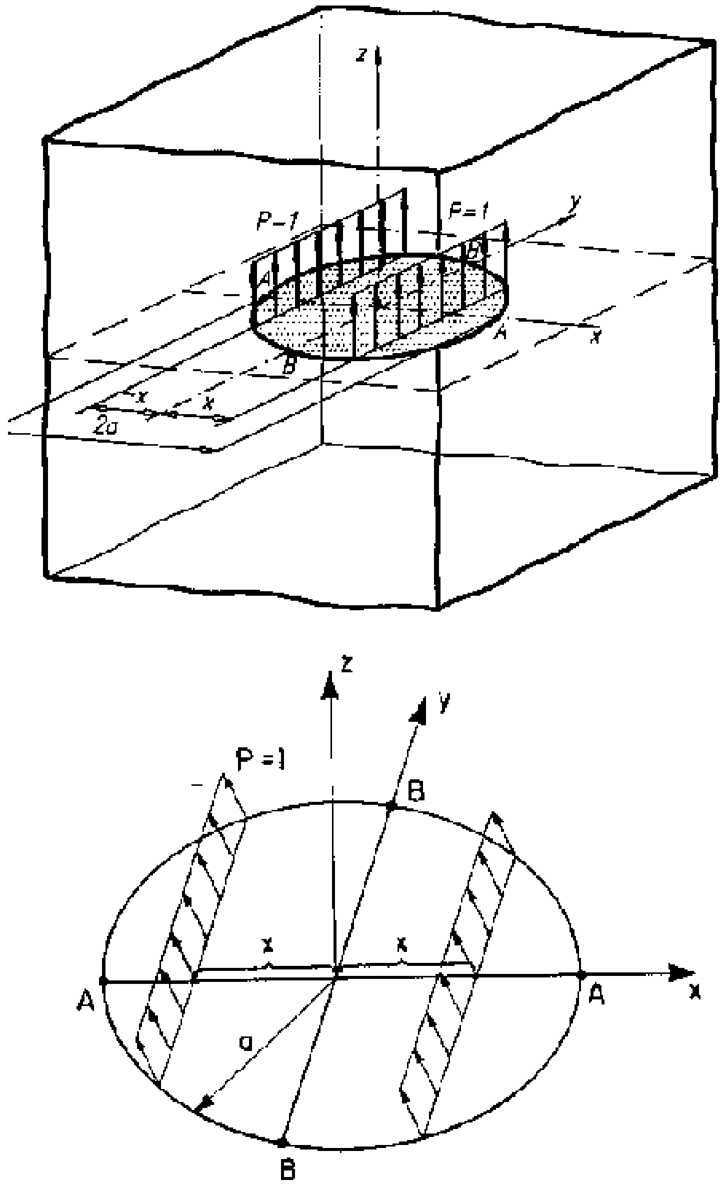


Fig. 10. Embedded penny-shape crack in an infinite body subjected to symmetrical loading

8.7 Shallow semi-elliptical surface crack in a finite thickness plate (a/c < 1)

WEIGHT FUNCTION (Fig.11) - **Deepest Point A**

$$m_A(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1A} = \frac{2\pi}{\sqrt{2Q}}(2Y_0 - 3Y_1) - \frac{24}{5}$$

$$M_{2A} = 3$$

$$M_{3A} = \frac{6\pi}{\sqrt{2Q}}(2Y_1 - Y_0) + \frac{8}{5}$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

$$Y_0 = A_0 + A_1\left(\frac{a}{t}\right)^2 + A_2\left(\frac{a}{t}\right)^4 + A_3\left(\frac{a}{t}\right)^6$$

$$A_0 = 1.0929 + 0.2581\left(\frac{a}{c}\right) - 0.7703\left(\frac{a}{c}\right)^2 + 0.4394\left(\frac{a}{c}\right)^3$$

$$A_1 = 0.456 - 3.045\left(\frac{a}{c}\right) - 2.007\left(\frac{a}{c}\right)^2 + \frac{1}{0.147 + \left(\frac{a}{c}\right)^{0.688}}$$

$$A_2 = 0.995 - \frac{1}{0.027 + \left(\frac{a}{c}\right)} + 22.0\left[1 - \left(\frac{a}{c}\right)\right]^{9.953}$$

$$A_3 = -1.459 + \frac{1}{0.014 + \left(\frac{a}{c}\right)} - 24.211\left[1 - \left(\frac{a}{c}\right)\right]^{8.071}$$

and

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right)^2 + B_2\left(\frac{a}{t}\right)^4 + B_3\left(\frac{a}{t}\right)^6$$

$$B_0 = 0.4537 + 0.1231\left(\frac{a}{c}\right) - 0.7412\left(\frac{a}{c}\right)^2 + 0.460\left(\frac{a}{c}\right)^3$$

$$B_1 = -1.652 + 1.665\left(\frac{a}{c}\right) - 0.534\left(\frac{a}{c}\right)^2 + \frac{1}{0.198 + \left(\frac{a}{c}\right)^{0.846}}$$

$$B_2 = 3.148 - 3.126\left(\frac{a}{c}\right) - \frac{1}{0.041 + \left(\frac{a}{c}\right)} + 17.259\left[1 - \left(\frac{a}{c}\right)\right]^{9.286}$$

$$B_3 = -4.228 + 3.643\left(\frac{a}{c}\right) + \frac{1}{0.020 + \left(\frac{a}{c}\right)} - 21.924\left[1 - \left(\frac{a}{c}\right)\right]^{9.203}$$

WEIGHT FUNCTION (Fig.11) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^1 + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1B} = \frac{3\pi}{\sqrt{Q}}(5F_1 - 3F_0) - 8$$

$$M_{2B} = \frac{15\pi}{\sqrt{Q}}(2F_0 - 3F_1) + 15$$

$$M_{3B} = \frac{3\pi}{\sqrt{Q}}(10F_1 - 7F_0) - 8$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

$$F_0 = \left[C_0 + C_1\left(\frac{a}{t}\right)^2 + C_2\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{a}{c}}$$

$$C_0 = 1.29782 - 0.1548\left(\frac{a}{c}\right) - 0.0185\left(\frac{a}{c}\right)^2$$

$$C_1 = 1.5083 - 1.3219\left(\frac{a}{c}\right) + 0.5128\left(\frac{a}{c}\right)^2$$

$$C_2 = -1.101 - \frac{0.879}{0.157 + \left(\frac{a}{c}\right)}$$

and

$$F_1 = \left[D_0 + D_1\left(\frac{a}{t}\right)^2 + D_2\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{a}{c}}$$

$$D_0 = 1.2687 - 1.0642\left(\frac{a}{c}\right) + 1.4646\left(\frac{a}{c}\right)^2 - 0.7250\left(\frac{a}{c}\right)^3$$

$$D_1 = 1.1207 - 1.2289\left(\frac{a}{c}\right) + 0.5876\left(\frac{a}{c}\right)^2$$

$$D_2 = 0.190 - 0.608\left(\frac{a}{c}\right) + \frac{0.199}{0.035 + \left(\frac{a}{c}\right)}$$

RANGE OF APPLICATION: 0 = a/t =0.8 and 0 = a/c = 1.0

ACCURACY: Better than 3% when compared to the FEM data.

REFERENCES:

- weight function:

Wang X., Lambert S. B., 1995, “Stress Intensity Factors for Low Aspect Ratio Semi-Elliptical Surface Cracks in Finite-Thickness Plates Subjected to Nonuniform Stresses,“ *Engng. Fract. Mech.,* Vol. 51, No. 4, pp. 517-532.

Shen G., Glinka G., 1991, “Weight Functions for a Surface Semi-Elliptical Crack in a Finite Thickness Plate,“ *Theor. Appl. Fract. Mech.*, Vol. 15, No. 2, pp. 247-255 (this paper contains the older version of the weight function).

Shen G., Plumtree A., Glinka G., 1991, "Weight Function for the Surface Point of Semi-Elliptical Surface Crack in a Finite Thickness Plate," *Engng. Fract. Mech.*, Vol. 40, No. 1, pp. 167-176 (this paper contains the older version of the weight function).

- reference data:

Shiratori M., Miyoshi T., Tanikawa K., 1986, "Analysis of Stress Intensity Factors for Surface Cracks Subjected to Arbitrarily Distributed Surface Stress (2nd Report, Analysis and Application of Influence Coefficients for Flat Plates with a Semielliptical Surface Crack)," *Trans. JSME*, Vol. 52, pp. 390-398.

Gross B., Srawley J. E., 1965, "Stress Intensity Factor for Single-Edge-Notch Specimens in Bending or Combined Bending and Tension by Boundary Collocation of a Stress Function," *NASA TN, D-2603.*

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Raju I. S., Newman J. C., 1979, "Stress Intensity Factors for a Wide Range of Semi-Elliptical Surface Cracks in Finite Thickness Plate, "*Engng. Fracture Mech.*, Vol. 11, pp. 817-829.

Newman J. C., Raju I. S., 1981, "An Empirical Stress Intensity Factor Equation for the Surface Crack, "*Engng. Fracture Mech.*, Vol. 15, pp. 185-192.

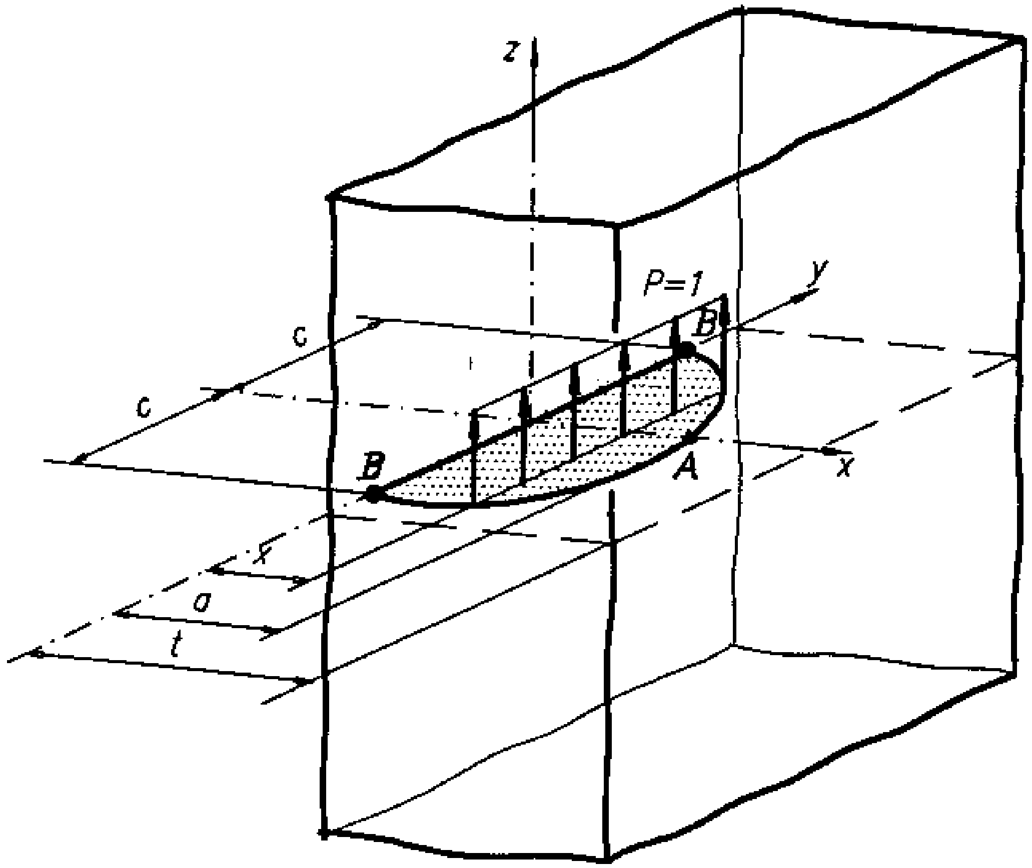

Fig. 11. Shallow semi-elliptical surface crack in a finite thickness plate.

8.8 Deep semi-elliptical surface crack in a finite thickness plate (a/c > 1)

WEIGHT FUNCTION (Fig. 12) - **Deepest Point A**

$$m_A(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1A} = \frac{2\pi}{\sqrt{2Q}}(2Y_0 - 3Y_1) - \frac{24}{5}$$

$$M_{2A} = 3$$

$$M_{3A} = \frac{6\pi}{\sqrt{2Q}}(2Y_1 - Y_0) + \frac{8}{5}$$

where:

$$Q = \begin{cases} 1 + 1.464\left(\frac{a}{c}\right)^{1.65} & \text{for } 0 \le \frac{a}{c} \le 1.0 \\ \left[1 + 1.464\left(\frac{c}{a}\right)^{1.65}\right]\left(\frac{a}{c}\right)^{2} & \text{for } \frac{a}{c} > 1.0 \end{cases}$$

$$Y_0 = A_0 + A_1\left(\frac{a}{t}\right)^2 + A_2\left(\frac{a}{t}\right)^4$$

$$A_0 = 1.13047 - 0.12945\left(\frac{a}{c}\right) + 0.03526\left(\frac{a}{c}\right)^2$$

$$A_1 = 1.08461 - 1.01106\left(\frac{a}{c}\right) + 0.2454\left(\frac{a}{c}\right)^2$$

$$A_2 = 0.7855 + 0.5517\left(\frac{a}{c}\right) - 0.0934\left(\frac{a}{c}\right)^2$$

and

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right)^2 + B_2\left(\frac{a}{t}\right)^4$$

$$B_0 = 0.5044 - 0.2609\left(\frac{a}{c}\right) + 0.0529\left(\frac{a}{c}\right)^2$$

$$B_1 = 0.7259 - 0.6352\left(\frac{a}{c}\right) + 0.1492\left(\frac{a}{c}\right)^2$$

$$B_2 = -0.6459 + 0.4177\left(\frac{a}{c}\right) - 0.0731\left(\frac{a}{c}\right)^2$$

WEIGHT FUNCTION (Fig. 12) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1B}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(1-\frac{x}{a}\right)^{1} + M_{3B}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1B} = \frac{3\pi}{\sqrt{Q}}\left(5F_1 - 3F_0\right) - 8$$

$$M_{2B} = \frac{15\pi}{\sqrt{Q}}\left(2F_0 - 3F_1\right) + 15$$

$$M_{3B} = \frac{3\pi}{\sqrt{Q}}\left(10F_1 - 7F_0\right) - 8$$

where:

$$Q = \begin{cases} 1 + 1.464\left(\frac{a}{c}\right)^{1.65} & \text{for } 0 \le \frac{a}{c} \le 1.0 \\ \left[1 + 1.464\left(\frac{c}{a}\right)^{1.65}\right]\left(\frac{a}{c}\right)^{2} & \text{for } \frac{a}{c} > 1.0 \end{cases}$$

$$F_0 = \left[C_0 + C_1\left(\frac{a}{t}\right)^2 + C_2\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{a}{c}}$$

$$C_0 = 1.33469 - 0.29091\left(\frac{a}{c}\right) + 0.08125\left(\frac{a}{c}\right)^2$$

$$C_1 = 1.757673 - 1.5275\left(\frac{a}{c}\right) + 0.37185\left(\frac{a}{c}\right)^2$$

$$C_2 = 0.08429 + 0.4423\left(\frac{a}{c}\right) - 0.1894\left(\frac{a}{c}\right)^2$$

and

$$F_1 = \left[D_0 + D_1\left(\frac{a}{t}\right)^2 + D_2\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{a}{c}}$$

$$D_0 = 1.11855 - 0.2065\left(\frac{a}{c}\right) + 0.0781\left(\frac{a}{c}\right)^2$$

$$D_1 = 1.15312 - 0.98743\left(\frac{a}{c}\right) + 0.23315\left(\frac{a}{c}\right)^2$$

$$D_2 = 0.2246 - 0.4784\left(\frac{a}{c}\right) + 0.1864\left(\frac{a}{c}\right)^2$$

RANGE OF APPLICATION: 0 = a/t = 0.8 and 0.6 = a\c = 2.0

ACCURACY: Better than 2% when compared to the reference FEM data.

REFERENCES:

- weight function:
Wang X., 1995, "Weight Functions for High Aspect Ratio Semi-Elliptical Surface Cracks in Finite-Thickness Plates", University of Waterloo, to be published.

- reference data:
Shiratori M., Miyoshi T., Tanikawa K., 1986, "Analysis of Stress Intensity Factors for Surface Cracks Subjected to Arbitrarily Distributed Surface Stress (2nd Report, Analysis and Application of Influence Coefficients for Flat Plates with a Semielliptical Surface Crack)," *Trans. JSME*, Vol. 52, pp. 390-398.

Murakami Y., et al., 1992, *Stress Intensity Factors Handbook*, Vol. 3, Pergamon Press, pp. 588-590.

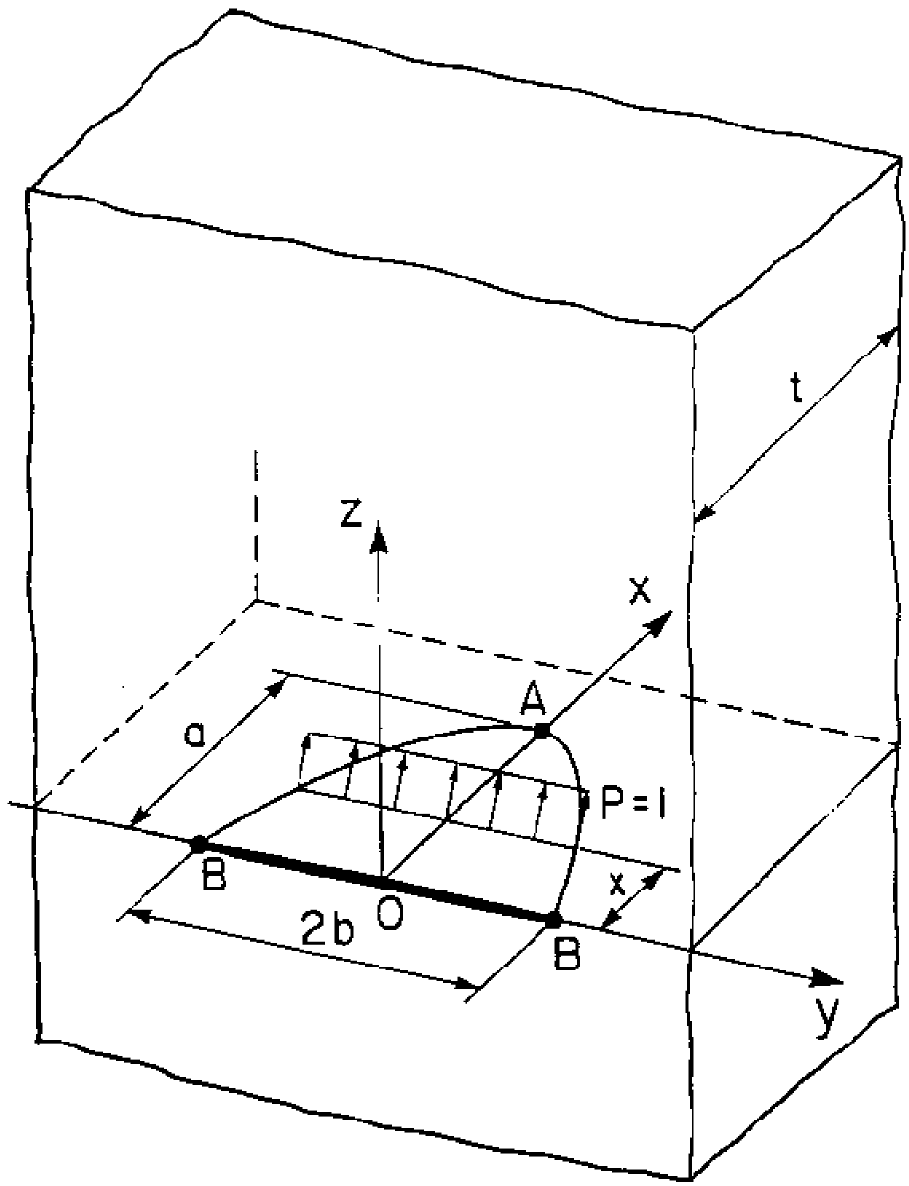

Fig. 12. Deep semi-elliptical surface crack in a finite thickness plate

8.9 Quarter-elliptical corner crack in a finite thickness plate

WEIGHT FUNCTION (Fig. 13) - **Deepest Point A**

$$m_A(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1A} = \frac{2\pi}{\sqrt{2Q}}(2Y_0 - 3Y_1) - \frac{24}{5}$$

$$M_{2A} = 3$$

$$M_{3A} = \frac{6\pi}{\sqrt{2Q}}(2Y_1 - Y_0) + \frac{8}{5}$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

$$Y_0 = A_0 + A_1\left(\frac{a}{t}\right) + A_2\left(\frac{a}{t}\right)^2 + A_3\left(\frac{a}{t}\right)^3 + A_4\left(\frac{a}{t}\right)^4$$

$$A_0 = 1.041 - 0.016\left(\frac{a}{c}\right) + 0.186\left(\frac{a}{c}\right)^2 - 0.111\left(\frac{a}{c}\right)^3$$

$$A_1 = -0.599 + 1.953\left(\frac{a}{c}\right) - 1.310\left(\frac{a}{c}\right)^2 - 0.028\left(\frac{a}{c}\right)^3$$

$$A_2 = 4.972 - 13.216\left(\frac{a}{c}\right) + 6.747\left(\frac{a}{c}\right)^2 + 1.918\left(\frac{a}{c}\right)^3$$

$$A_3 = -1.293 + 1.857\left(\frac{a}{c}\right) + 12.906\left(\frac{a}{c}\right)^2 - 13.441\left(\frac{a}{c}\right)^3$$

$$A_4 = -0.572 + 3.073\left(\frac{a}{c}\right) - 10.797\left(\frac{a}{c}\right)^2 + 8.393\left(\frac{a}{c}\right)^3$$

and

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right) + B_2\left(\frac{a}{t}\right)^2 + B_3\left(\frac{a}{t}\right)^3 + B_4\left(\frac{a}{t}\right)^4$$

$$B_0 = 0.500 - 0.323\left(\frac{a}{c}\right) + 0.213\left(\frac{a}{c}\right)^2 - 0.052\left(\frac{a}{c}\right)^3$$

$$B_1 = -0.507 + 1.373\left(\frac{a}{c}\right) - 0.740\left(\frac{a}{c}\right)^2 - 0.184\left(\frac{a}{c}\right)^3$$

$$B_2 = 3.468 - 9.028\left(\frac{a}{c}\right) + 6.349\left(\frac{a}{c}\right)^2 - 0.135\left(\frac{a}{c}\right)^3$$

$$B_3 = -1.359 + 1.731\left(\frac{a}{c}\right) + 5.357\left(\frac{a}{c}\right)^2 - 6.370\left(\frac{a}{c}\right)^3$$

$$B_4 = -0.162 + 2.977\left(\frac{a}{c}\right) - 8.250\left(\frac{a}{c}\right)^2 + 5.804\left(\frac{a}{c}\right)^3$$

WEIGHT FUNCTION (Fig. 13) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1B} = \frac{3\pi}{\sqrt{Q}}\left(5F_1 - 3F_0\right) - 8$$

$$M_{2B} = \frac{15\pi}{\sqrt{Q}}\left(2F_0 - 3F_1\right) + 15$$

$$M_{3B} = \frac{3\pi}{\sqrt{Q}}\left(10F_1 - 7F_0\right) - 8$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

$$F_0 = \left[C_0 + C_1\left(\frac{a}{t}\right) + C_2\left(\frac{a}{t}\right)^2 + C_3\left(\frac{a}{t}\right)^3 + C_4\left(\frac{a}{t}\right)^4\right]\left(\frac{a}{c}\right)$$

$$C_0 = 3.340 - 4.495\left(\frac{a}{c}\right) + 3.016\left(\frac{a}{c}\right)^2 - 0.7278\left(\frac{a}{c}\right)^3$$

$$C_1 = 0.2318 - 0.2261\left(\frac{a}{c}\right) - 1.658\left(\frac{a}{c}\right)^2 + 1.504\left(\frac{a}{c}\right)^3$$

$$C_2 = 22.95 - 100.9\left(\frac{a}{c}\right) + 152.2\left(\frac{a}{c}\right)^2 - 72.92\left(\frac{a}{c}\right)^3$$

$$C_3 = -39.16 + 194.1\left(\frac{a}{c}\right) - 302.0\left(\frac{a}{c}\right)^2 + 145.9\left(\frac{a}{c}\right)^3$$

$$C_4 = 30.80 - 142.9\left(\frac{a}{c}\right) + 212.6\left(\frac{a}{c}\right)^2 - 99.92\left(\frac{a}{c}\right)^3$$

and

$$F_1 = \left[D_0 + D_1\left(\frac{a}{t}\right) + D_2\left(\frac{a}{t}\right)^2 + D_3\left(\frac{a}{t}\right)^3 + D_4\left(\frac{a}{t}\right)^4\right]\left(\frac{a}{c}\right)$$

$$D_0 = 2.831 - 3.840\left(\frac{a}{c}\right) + 2.477\left(\frac{a}{c}\right)^2 - 0.511\left(\frac{a}{c}\right)^3$$

$$D_1 = 4.600 - 20.498\left(\frac{a}{c}\right) + 29.001\left(\frac{a}{c}\right)^2 - 13.226\left(\frac{a}{c}\right)^3$$

$$D_2 = -4.019 + 15.057\left(\frac{a}{c}\right) - 12.624\left(\frac{a}{c}\right)^2 + 2.677\left(\frac{a}{c}\right)^3$$

$$D_3 = 9.682 - 15.932\left(\frac{a}{c}\right) - 8.848\left(\frac{a}{c}\right)^2 + 13.910\left(\frac{a}{c}\right)^3$$

$$D_4 = -1.141 - 9.176\left(\frac{a}{c}\right) + 30.228\left(\frac{a}{c}\right)^2 - 19.195\left(\frac{a}{c}\right)^3$$

RANGE OF APPLICATION: 0 = a\t =0.8 and 0.2= a\c = 1.0

ACCURACY: Better than 1.5% when compared to the FEM data.

REFERENCES:

- weight function:

Zheng X. J., Glinka G., Dubey R. N., 1994, "Stress Intensity Factors and Weight Functions for a Corner Crack in a Finite Thickness Plate," *Engng. Fracture Mech.*,(to be published, 1996)

- reference data:

Shiratori M., Miyoshi T., 1985, "Analysis of Stress Intensity Factors for Surface Cracks Subjected to Arbitrarily Distributed Surface Stress (Analysis and Application of Data Base of Influence Coefficients K_{ij})," *Proc. Third Conf. Fract. Mech.*, Soc. Materials Science Japan, pp 82-86.

Murakami Y., et al., 1992, *Stress Intensity Factors Handbook*, Vol. 3, Pergamon Press, pp. 591-597.

Raju I. S., Newman J. C., 1988, "Stress Intensity Factors for Corner Cracks in Rectangular Bars," in *Fracture Mechanics: Nineteenth Symposium, ASTM STP 969*, T.A.Cruse, ed., pp. 43-55.

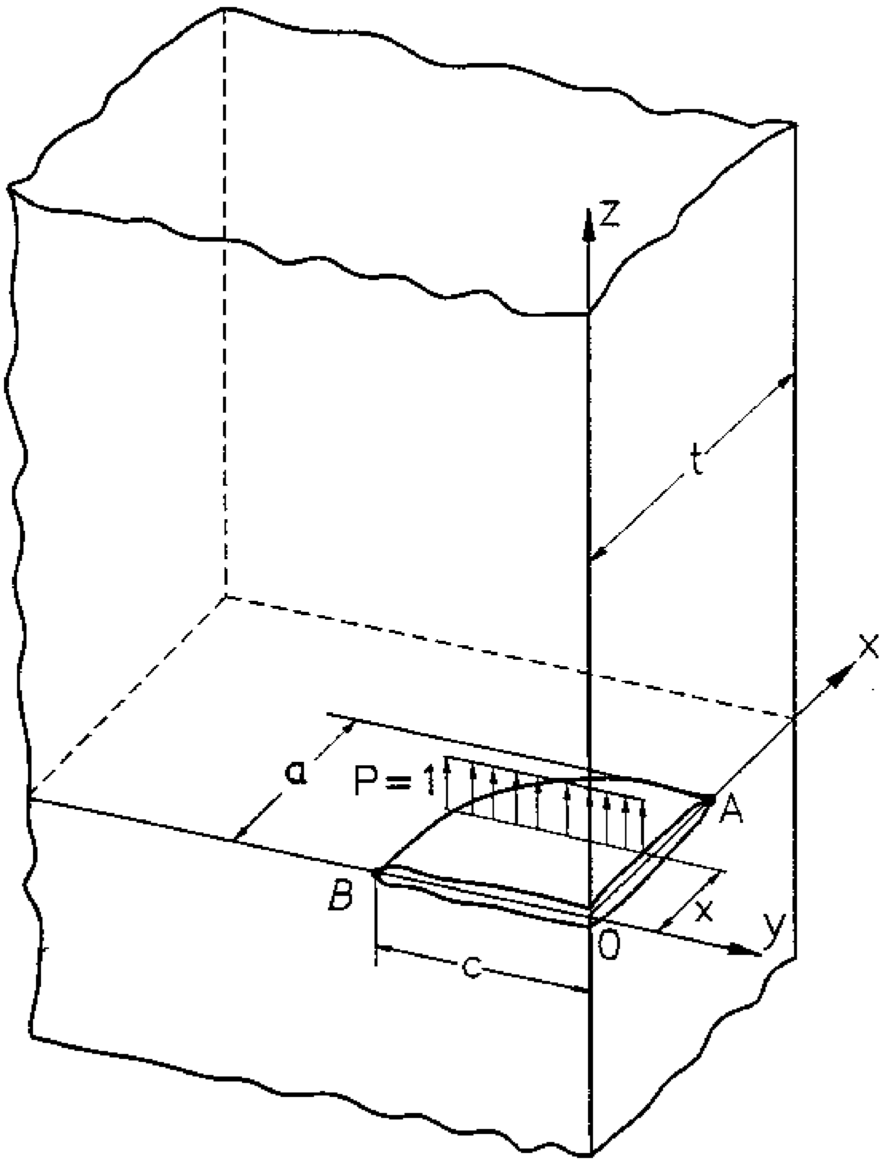

Fig. 13. Quarter-elliptical corner crack in a finite thickness plate

8.10 Radial edge crack in a circular disk

WEIGHT FUNCTION (Fig. 14)

$$m(x,a)=\frac{2}{\sqrt{2\pi(a-x)}}\left[1+M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+M_2\left(1-\frac{x}{a}\right)^{1}+M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_1=\left[\begin{array}{l}-0.04732+0.49586\left(\frac{a}{D}\right)-1.94141\left(\frac{a}{D}\right)^2+3.96175\left(\frac{a}{D}\right)^3\\-4.56109\left(\frac{a}{D}\right)^4+2.83026\left(\frac{a}{D}\right)^5-0.74080\left(\frac{a}{D}\right)^6\end{array}\right]\exp\left[9.87969\left(\frac{a}{D}\right)\right]$$

$$M_2=\exp\left[\begin{array}{l}-0.58602+8.48276\left(\frac{a}{D}\right)-30.90993\left(\frac{a}{D}\right)^2+112.21153\left(\frac{a}{D}\right)^3\\-280.25303\left(\frac{a}{D}\right)^4+428.48183\left(\frac{a}{D}\right)^5-356.66155\left(\frac{a}{D}\right)^6+125.34267\left(\frac{a}{D}\right)^7\end{array}\right]$$

$$M_3=\exp\left[\begin{array}{l}-1.09836+3.06605\left(\frac{a}{D}\right)+16.85709\left(\frac{a}{D}\right)^2-48.14897\left(\frac{a}{D}\right)^3\\+54.61627\left(\frac{a}{w}\right)^4+6.91042\left(\frac{a}{D}\right)^5-61.35817\left(\frac{a}{D}\right)^6+36.1270\left(\frac{a}{D}\right)^7\end{array}\right]$$

RANGE OF APPLICATION: $0 < a/D < 0.9$.

ACCURACY: Better than 1.5% compared to the reference solution.

REFERENCES:

- weight function:
Kiciak A., 1995, University of Waterloo, to be published.

Glinka G., Shen G., 1991, "Universal Features of Weight Functions for Cracks in Mode I, "*Engng. Fract. Mech.*, Vol. 40, No. 6, pp. 1135-1146 - this paper contains the original version of the weight function with coefficients M_i given in tabular form.

- reference data:
Wu X. R., Carlsson J. A., 1991, *Weight Functions and Stress Intensity Factor Solutions,* Pergamon Press.

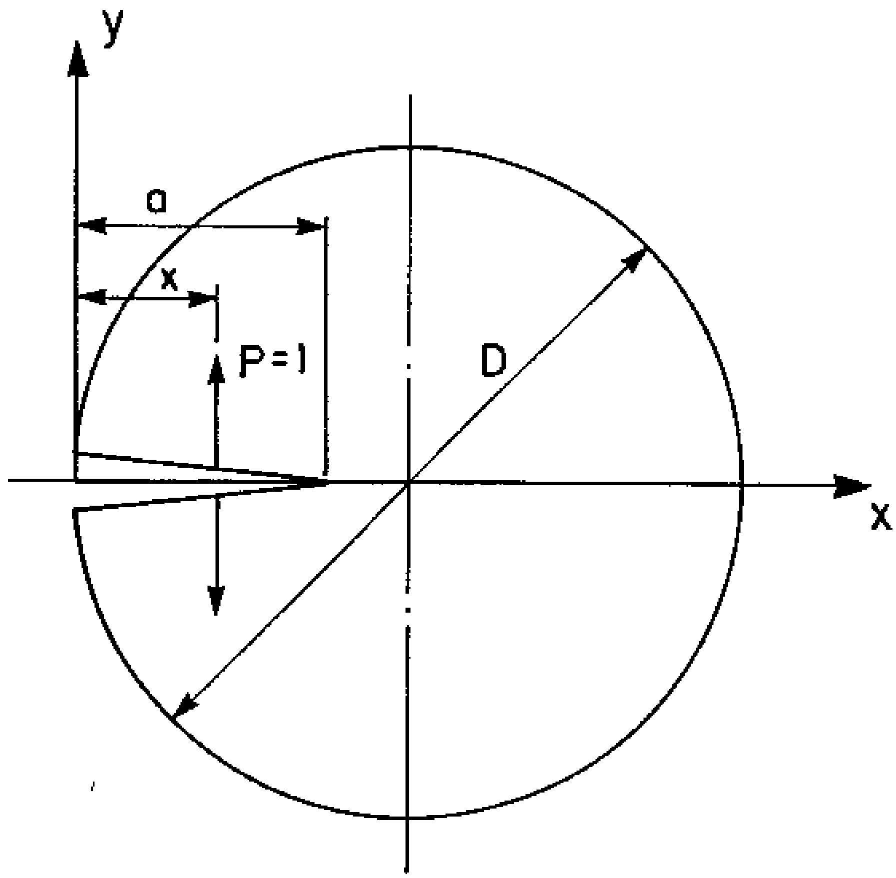

Fig. 14. Radial edge crack in a circular disk

8.11 Edge crack in a finite thickness plate with the right angle corner

WEIGHT FUNCTION (Fig. 15)

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right) + M_2\left(1-\frac{x}{a}\right)^2\right]$$

PARAMETERS

$$\mathbf{M_1} = \mathbf{0.6643 - 12.7438}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{1.5} + \mathbf{397.8081}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{3} - \mathbf{3285.181}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{4.5} + \mathbf{14162.587}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{6}$$
$$- \mathbf{30127.158}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{7.5} + \mathbf{258119.535}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{9}$$

$$\mathbf{M_2} = \mathbf{0.1117 + 3.857}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{1.5} - \mathbf{30127.158}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{3} + \mathbf{285.4393}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{4.5} - \mathbf{647.6118}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{6}$$
$$+ \mathbf{934.4538}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{7.5} - \mathbf{596.8319}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{9}$$

RANGE OF APPLICATION: 0 = a\t = 0.5, a = 90^o.

ACCURACY: Better than 3% compared to the reference data.

REFERENCES:

- weight function:

Niu X., Glinka G., 1990, “Weight Functions for Edge and Surface Semi-Elliptical Crack in Flat Plates and Plates with Corners,“ *Engng. Fract. Mech.*, Vol. 36, No. 3, pp. 459-475.

- reference data:

Hasebe N., Ueda M., 1981, “A Crack Originating from an Angular Corner of a Semi-Infinite Plate with a Step,“ *Bull. JSME*, Vol. 24, pp. 483-488.

Hasebe N., Matsuura S., 1984, “Stress Analysis of a Strip with a Step and a Crack,“ *Engng. Fract. Mech.*, Vol. 20, pp. 447-462.

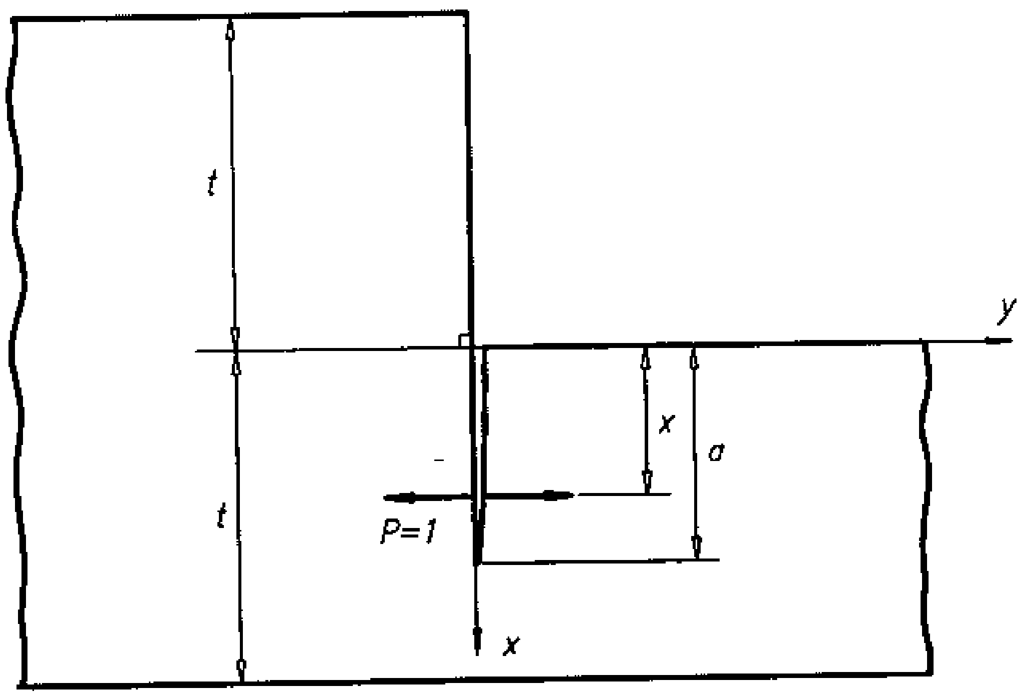

Fig. 15. Edge crack in a finite thickness plate with the right angle corner

8.12 Edge crack in a finite thickness plate with an angular corner

WEIGHT FUNCTION (Fig. 16)

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right) + M_2\left(1-\frac{x}{a}\right)^2\right]\cdot F_\alpha$$

PARAMETERS

$$\mathbf{M_1} = \mathbf{0.6643 - 12.7438}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{1.5} + \mathbf{397.8081}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{3} - \mathbf{3285.181}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{4.5} + \mathbf{14162.587}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{6}$$
$$-\mathbf{30127.158}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{7.5} + \mathbf{258119.535}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{9}$$

$$\mathbf{M_2} = \mathbf{0.1117 + 3.857}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{1.5} - \mathbf{30127.158}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{3} + \mathbf{285.4393}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{4.5} - \mathbf{647.6118}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{6}$$
$$+\mathbf{934.4538}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{7.5} - \mathbf{596.8319}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{9}$$

$$\mathbf{F_\alpha} = \mathbf{1} + \left(\frac{\mathbf{6}}{\pi}\alpha - \mathbf{2}\right)\left[-\mathbf{0.0355 + 3.3324}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{1/2} - \mathbf{21.5999}\left(\frac{\mathbf{a}}{\mathbf{t}}\right) + \mathbf{58.8513}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{3/2} - \mathbf{81.6246}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{2}\right.$$
$$\left. + \mathbf{56.9396}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{5/2} - \mathbf{15.8784}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{3}\right]$$

RANGE OF APPLICATION: 0 = a/t = 0.5 and $\pi/6$ = a = $\pi/3$ for **a** in radians.

ACCURACY: unknown.

REFERENCES:

- weight function:

Niu X., Glinka G., 1990, “Weight Functions for Edge and Surface Semi-Elliptical Crack in Flat Plates and Plates with Corners,“ *Engng. Fract. Mech.*, Vol. 36, No. 3, pp. 459-475.

Niu X., Glinka G., 1987, “The Weld Profile Effect on Stress Intensity Factors in Weldments,” *Int. J. Fract.*, Vol. 35, pp. 3-20.

- reference data:

Hasebe N., Ueda M., 1981, “A Cracks Originating from an Angular Corner of a Semi-Infinite Plate with a Step,“ *Bull. JSME*, Vol. 24, pp. 483-488.

Hasebe N., Matsuura S., 1984, “StressAnalysis of a Strip with a Step and a Crack,“ *Engng. Fract. Mech.*, Vol. 20, pp. 447-462.

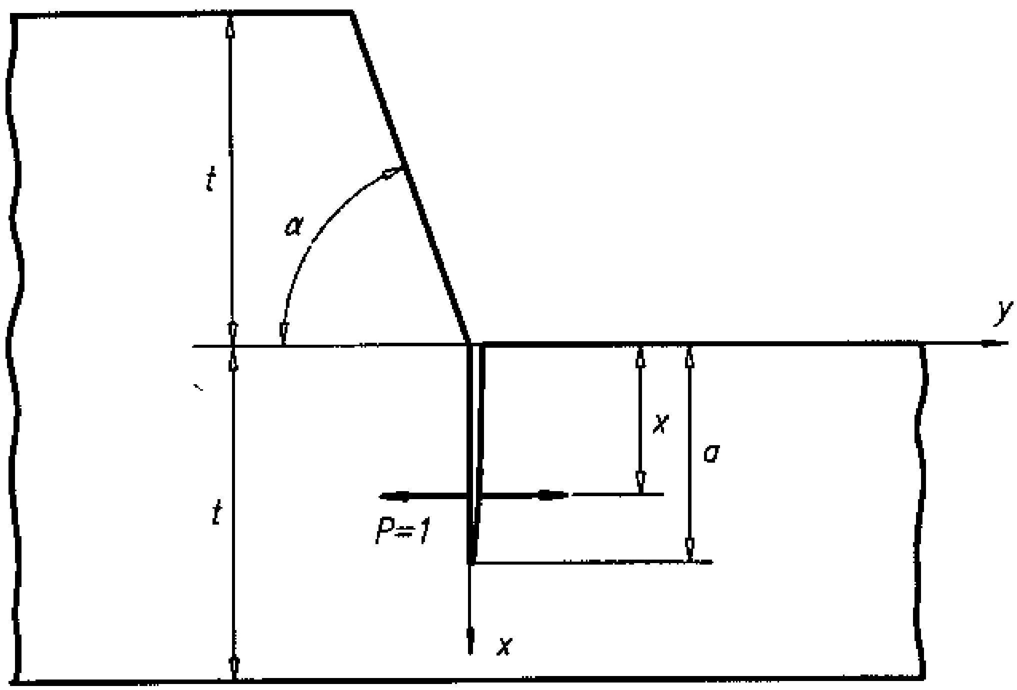

Fig. 16. Edge crack in a finite thickness plate with an angular corner

9. WEIGHT FUNCTIONS FOR CRACKS IN CYLINDERS

The weight functions are given in the form of expressions describing the M_i parameters as functions of crack dimensions and geometry of the cracked body. Given are the range of application, the accuracy and the source of the reference stress intensity factors for each set of parameters M_i including the generic geometry of the crack body. The geometry of a cylinder is characterized by the ratio of the external to internal radius, R_o/R_i.

9.1 Internal circumferential edge crack in a thick cylinder subjected to axisymmetric loading (R_o/R_i=2.0)

WEIGHT FUNCTION (Fig. 17)

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_1 = \frac{2\pi}{\sqrt{2}}\left(\frac{3Y_1}{\left(a/t\right)} - Y_0\right) - \frac{24}{5}$$

$$M_2 = 3$$

$$M_3 = \frac{6\pi}{\sqrt{2}}\left(Y_0 - \frac{2Y_1}{\left(a/t\right)}\right) - \frac{296}{15}$$

where:

$t = R_0 - R_i$ (cylinder wall thickness)

$$\mathbf{Y_0} = \alpha + \mathbf{exp}\left[\mathbf{A_0} + \mathbf{A_1}\left(\frac{\mathbf{a}}{\mathbf{t}}\right) + \mathbf{A_2}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{\mathbf{2}} + \mathbf{A_3}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{\mathbf{3}} + \mathbf{A_4}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{\mathbf{4}} + \mathbf{A_5}\left(\frac{\mathbf{a}}{\mathbf{t}}\right)^{\mathbf{5}}\right]$$

$\alpha = \mathbf{1.033505}$

$\mathbf{A_0} = -\mathbf{2.41397}$

$\mathbf{A_1} = -\mathbf{15.34436}$

$\mathbf{A_2} = \mathbf{46.42214}$

$\mathbf{A_3} = -\mathbf{22.49592}$

$\mathbf{A_4} = -\mathbf{31.75198}$

$\mathbf{A_5} = \mathbf{26.94214}$

RANGE OF APPLICATION: $R_0/R_i = 2$ and $0 = a/t = 0.8$

ACCURACY: Better than 5% when compared to the FEM data.

REFERENCES:

- weight function:

Liebster T. D., Glinka G., Burns D. J., Mettu S. R., 1994, "Calculating Stress Intensity Factors for Internal Circumferential Cracks by Using Weight Functions," *ASME PVP-Vol. 281, High Pressure Technology,* J. A. Kapp, ed., pp.1-6.

- reference data:

Mettu S. R., Forman R. G., 1993, "Analysis of Circumferential Cracks in Cylinders Using the Weight Function Method," *Fracture Mechanics: 23rd Symposium,ASTM STP 1189,* R. Chona, ed., American Society for Testing and Mateials, pp. 417-440.

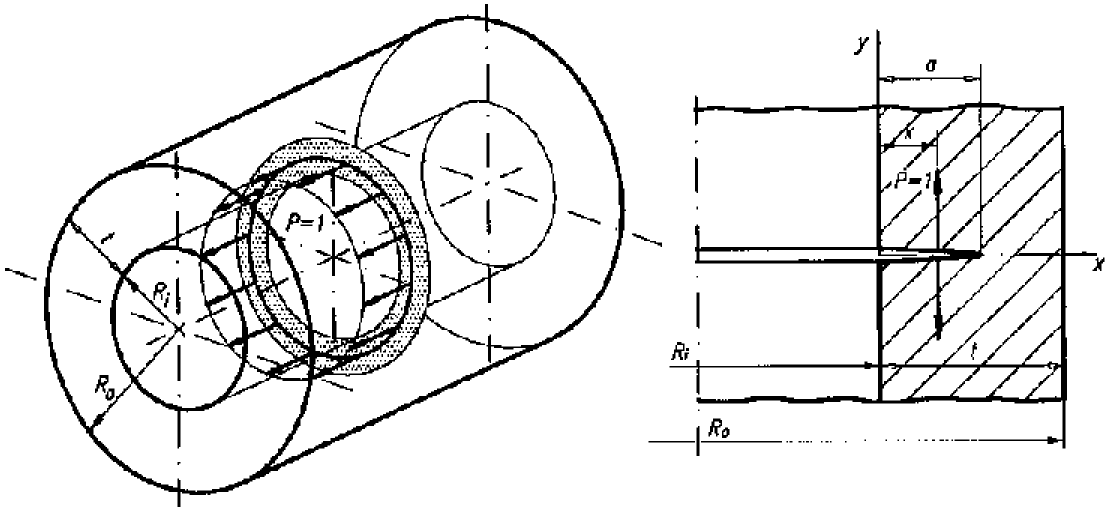

Figure 17. Internal circumferential edge crack in a thick cylinder subjected to axisymmetric loading.

9.2 Two internal symmetrical axial edge cracks in a thick cylinder subjected to symmetrical loading (R_o/R_i=2.0)

WEIGHT FUNCTION (Fig. 18)

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_2\left(1-\frac{x}{a}\right)^{1} + M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_1 = \frac{6\pi}{\sqrt{2}}\left(-\frac{13Y_1}{s} + \frac{14Y_2}{s^2} + 2Y_0\right) - \frac{48}{5}$$

$$M_2 = \frac{105\pi}{\sqrt{2}}\left(\frac{3Y_1}{s} - \frac{3Y_2}{s^2} - \frac{Y_0}{2}\right) + 21$$

$$M_3 = \frac{12\pi}{\sqrt{2}}\left(-\frac{22Y_1}{s} + \frac{21Y_2}{s^2} + 4Y_0\right) - \frac{64}{5}$$

where:

s = a/(R_0 - R_i), and

for $0 \le s \le 0.1$

$$Y_0 = 1.11198 - 0.978998s + 360.49s^2 - 42949.2s^3 + 26.1945\cdot 10^5 s^4 - 9.5658\cdot 10^7 s^5 + 2.19014\cdot 10^9 s^6 - 3.1481\cdot 10^{10} s^7 + 2.74822\cdot 10^{11} s^8 - 1.32888\cdot 10^{12} s^9 + 2.72711\cdot 10^{12} s^{10}$$

$$Y_1 = 0.000291 + 0.66207s + 0.224015s^2$$

$$Y_2 = 8.01761\cdot 10^{-6} - 7.74379\cdot 10^{-4} s + 0.534838s^2$$

for $0.1 \le s \le 0.8$

$$Y_0 = 1.071 + 0.424314s + 1.20826s^2 + 5.11629s^3 - 9.74362s^4 + 6.08975s^5$$

$$Y_1 = -0.009535 + 0.866583s - 1.35905s^2 + 5.89469s^3 - 7.68059s^4 + 4.25993s^5$$

$$Y_2 = -0.007826 + 0.161374s - 0.63532s^2 + 3.75368s^3 - 4.85997s^4 + 2.85235s^5$$

RANGE OF APPLICATION: R_0/R_i = 2.0 and 0.1 = s = 0.8

ACCURACY: Better than 0.2% when compared to the FEM data.

REFERENCES:

- weight function:
Shen. G., Glinka G. ,1993, "Stress Intensity Factors for Internal Edge and Semi-Elliptical Cracks in Hollow Cylinders," *ASME PVP-Vol. 263, High Pressure - Codes, Analysis, and Applications,* A. Khare, ed., pp.73-79.

Glinka G., Shen G., 1991, "Universal Features of Weight Functions for Cracks in Mode I, "*Engng. Fract. Mech.*, Vol. 40, No. 6, pp. 1135-1146.

- reference data:
Andrasic C. P., Parker A. P., 1984, "Dimensionless Stress Intensity Factors for Cracked Thick Cylinder Under Polynomial Crack Face Loading," *Engng. Fract. Mech.,* Vol. 19, 1984, pp. 187-193.

Murakami Y., et al., 1986, *Stress Intensity Factor Handbook,* Vol. 2, Pergamon Press, pp. 309-317.

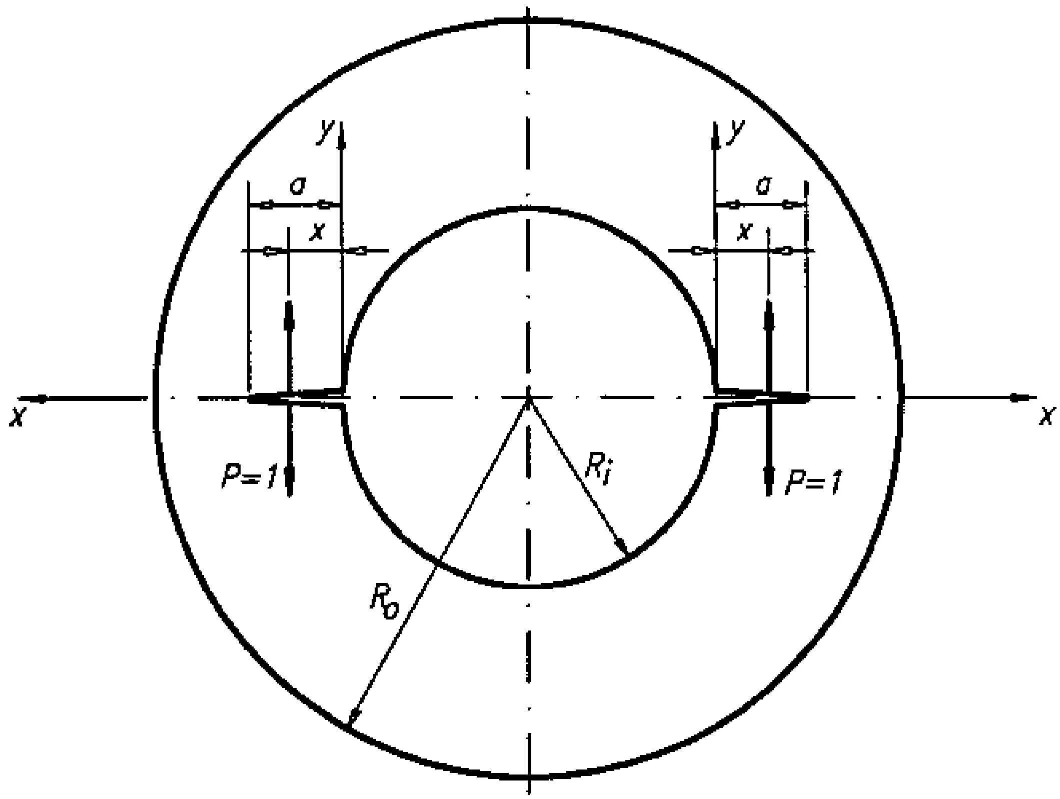

Fig. 18. Two internal symmetrical axial edge cracks in a thick cylinder subjected to symmetrical loading

9.3 External axial edge crack in a thick cylinder (R_o/R_i=2.0)

WEIGHT FUNCTION (Fig. 19)

$$m(x,a)=\frac{2}{\sqrt{2\pi(a-x)}}\left[1+M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+M_2\left(1-\frac{x}{a}\right)^{1}+M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_1=\frac{6\pi}{\sqrt{2}}\left(-\frac{13Y_1}{s}+\frac{14Y_2}{s^2}+2Y_0\right)-\frac{48}{5}$$

$$M_2=\frac{105\pi}{\sqrt{2}}\left(\frac{3Y_1}{s}-\frac{3Y_2}{s^2}-\frac{Y_0}{2}\right)+21$$

$$M_3=\frac{12\pi}{\sqrt{2}}\left(-\frac{22Y_1}{s}+\frac{21Y_2}{s^2}+4Y_0\right)-\frac{64}{5}$$

where:

$s=a/(R_o-R_i)$, and

for **0 = s = 0.1**

$$Y_0=1.11201+1.67679s+7.38413s^2-3543.83s^3+1.0534\cdot10^5s^4-7.33\cdot10^5s^5$$
$$-1.01865\cdot10^6s^6+1.68865\cdot10^8s^7-6.52672\cdot10^8s^8$$

$$Y_1=0.907986\cdot10^{-4}+0.676241s+0.310302s^2$$

$$Y_2=8.69582\cdot10^{-6}-9.8064\cdot10^{-4}s+0.54827s^2$$

for **0.1 = s = 0.8**

$$Y_0=1.64-12.2614s+121.488s^2-574.313s^3+1542.3s^4-2349.04s^5$$
$$+1895.84s^6-628.971s^7$$

$$Y_1=0.0243751+0.200288s+3.42782s^2-8.87021s^3+12.4676s^4-5.89425s^5$$

$$Y_2=-0.00172167+0.0261316s+0.42169s^2+0.230129s^3+0.342948s^4$$

RANGE OF APPLICATION: R_0/R_i = 2.0 and 0.1 = s = 0.8

ACCURACY: Better than 0.2% when compared to the FEM data.

REFERENCES:

- weight function:

Shen. G., Liebster T. D., Glinka G. ,1993, "Calculation of Stress Intensity Factors for Cracks in Pipes," *Proc. of the 12th Int. Conf. on Offshore Mech. and Arctic Engng..*, *ASME*,M. Salama et al., ed., Vol. III-B, *Materials Engineering*, pp.847-854.

Glinka G., Shen G., 1991, "Universal Features of Weight Functions for Cracks in Mode I, "*Engng. Fract. Mech.*, Vol. 40, No. 6, pp. 1135-1146.

- reference data:

Andrasic C. P., Parker A. P., 1984, "Dimensionless Stress Intensity Factors for Cracked Thick Cylinder Under Polynomial Crack Face Loading," *Engng. Fract. Mech.*, Vol. 19, 1984, pp. 187-193.

Murakami Y., et al., 1986, *Stress Intensity Factor Handbook*, Vol. 2, Pergamon Press, pp. 309-317.

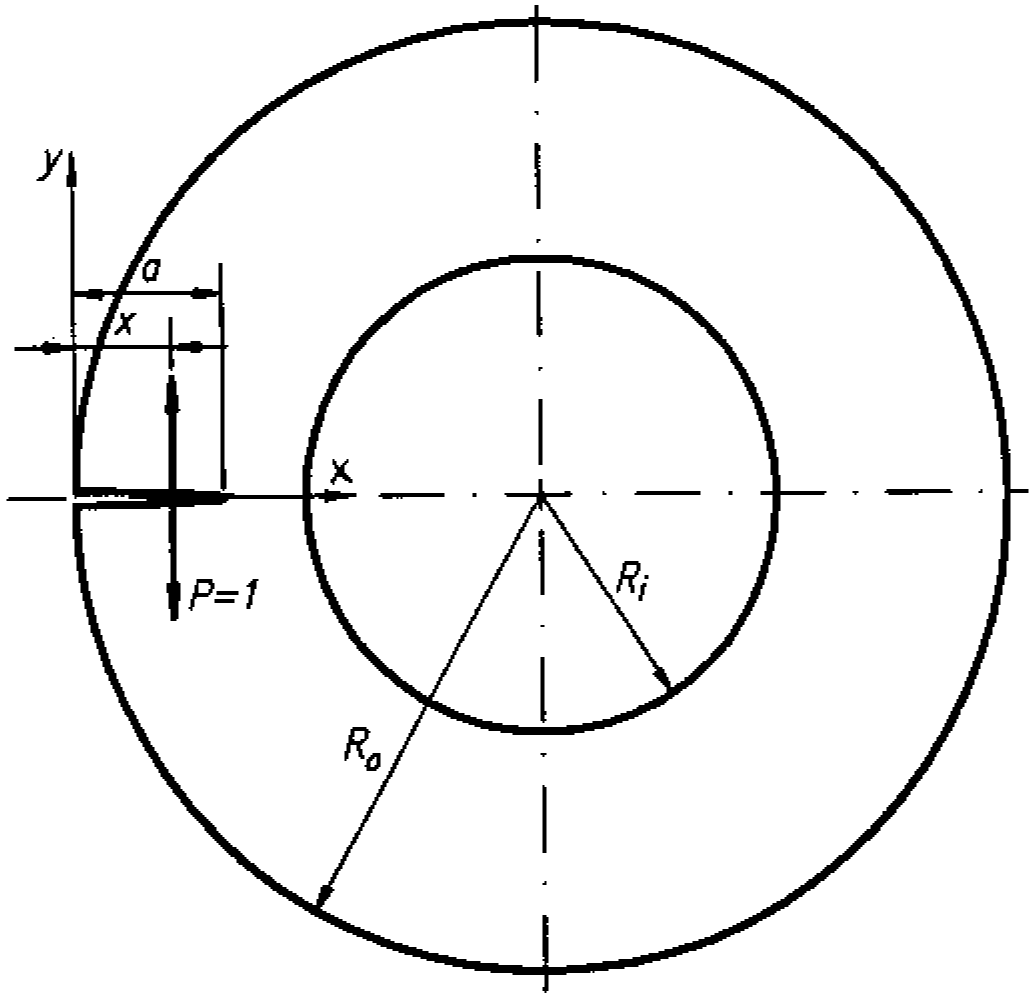

Fig. 19. External axial edge crack in a thick cylinder

9.4 Internal circumferential semi-elliptical surface crack in a thick cylinder (R_o/R_i=1.1)

WEIGHT FUNCTION (Fig. 20) - **Deepest Point A**

$$m_A(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{1/2} + M_{2A}\left(1-\frac{x}{a}\right) + M_{3A}\left(1-\frac{x}{a}\right)^{3/2}\right]$$

PARAMETERS

$$\mathbf{M_{1A}} = \frac{\mathbf{2\pi}}{\sqrt{\mathbf{2Q}}}\left(\mathbf{2Y_0} - \mathbf{3Y_1}\right) - \frac{\mathbf{24}}{\mathbf{5}}$$

$$\mathbf{M_{2A}} = \mathbf{3}$$

$$\mathbf{M_{3A}} = \frac{\mathbf{6\pi}}{\sqrt{\mathbf{2Q}}}\left(\mathbf{2Y_1} - \mathbf{Y_0}\right) + \frac{\mathbf{8}}{\mathbf{5}}$$

where:

$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$, and $t = R_o - R_i$

$$Y_0 = A_0 + A_1\left(\frac{a}{t}\right) + A_2\left(\frac{a}{t}\right)^2$$

$$A_0 = 1.1378 - 0.4259\left(\frac{a}{c}\right) + 0.1299\left(\frac{a}{c}\right)^2$$

$$A_1 = -0.1223 + 1.502\left(\frac{a}{c}\right) - 0.8013\left(\frac{a}{c}\right)^2$$

$$A_2 = 2.0572 - 9.4342\left(\frac{a}{c}\right) + 13.8825\left(\frac{a}{c}\right)^2 - 6.5333\left(\frac{a}{c}\right)^3$$

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right) + B_2\left(\frac{a}{t}\right)^2$$

$$B_0 = 0.5116 - 0.373\left(\frac{a}{c}\right) + 0.1129\left(\frac{a}{c}\right)^2$$

$$B_1 = -0.3724 + 2.3922\left(\frac{a}{c}\right) - 3.3694\left(\frac{a}{c}\right)^2 + 1.5589\left(\frac{a}{c}\right)^3$$

$$B_2 = 1.3148 - 5.771\left(\frac{a}{c}\right) + 7.9406\left(\frac{a}{c}\right)^2 - 3.6193\left(\frac{a}{c}\right)^3$$

WEIGHT FUNCTION (Fig. 20) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{1/2} + M_{2B}\left(\frac{x}{a}\right) + M_{3B}\left(\frac{x}{a}\right)^{3/2}\right]$$

PARAMETERS

$$\mathbf{M_{1B}} = \frac{\mathbf{3\pi}}{\sqrt{\mathbf{Q}}}\left(\mathbf{5F_1 - 3F_0}\right) - \mathbf{8}$$

$$\mathbf{M_{2B}} = \frac{\mathbf{15\pi}}{\sqrt{\mathbf{Q}}}\left(\mathbf{2F_0 - 3F_1}\right) + \mathbf{15}$$

$$\mathbf{M_{3B}} = \frac{\mathbf{3\pi}}{\sqrt{\mathbf{Q}}}\left(\mathbf{10F_1 - 7F_0}\right) - \mathbf{8}$$

where:

$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$ and $t = R_o - R_i$

$$F_0 = C_0\left(\frac{a}{c}\right)^{C_1}$$

$$C_0 = 0.9242 + 0.6172\left(\frac{a}{t}\right) - 0.1379\left(\frac{a}{t}\right)^2$$

$$C_1 = 0.5437 - 2.1302\left(\frac{a}{t}\right) + 8.0279\left(\frac{a}{t}\right)^2 - 11.8896\left(\frac{a}{t}\right)^3 + 5.8708\left(\frac{a}{t}\right)^4$$

and

$$F_1 = D_0\left(\frac{a}{c}\right)^{D_1}$$

$$D_0 = 0.7631 + 0.4891\left(\frac{a}{t}\right) - 0.1164\left(\frac{a}{t}\right)^2$$

$$D_1 = 0.6287 - 3.88279\left(\frac{a}{t}\right) + 15.5542\left(\frac{a}{t}\right)^2 - 24.1589\left(\frac{a}{t}\right)^3 + 12.6441\left(\frac{a}{t}\right)^4$$

RANGE OF APPLICATION: $R_0/R_i = 1.1$, $0.1 = a/t = 0.8$, and $0.2 = a/c = 1.0$

ACCURACY: unknown.

REFERENCES:

- weight function:

Shen. G., Glinka G. ,1993, “Stress Intensity Factors for Internal Edge and Semi-Elliptical Cracks in Hollow Cylinders,“ *ASME PVP-Vol. 263, High Pressure - Codes, Analysis, and Applications,* A. Khare, ed., pp.73-79.

- reference data:

Shiratori M., 1989, “Analysis of Stress Intensity Factors for Surface Cracks in Pipes by an Influence Function Method,“ *ASME PVP-Vol. 167,* pp. 45-50.

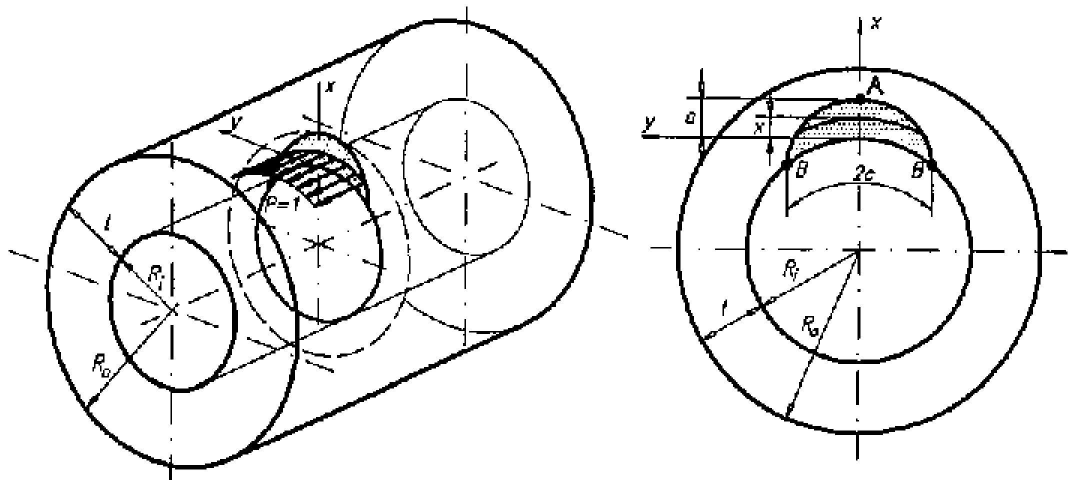

Figure 20. Internal circumferential semi-elliptical surface crack in a thick cylinder

9.5 Internal axial semi-elliptical surface crack in a thick cylinder (R_o/R_i=2.0)

WEIGHT FUNCTION (Fig. 21) - **Deepest Point A**

$$m_A(x,a)=\frac{2}{\sqrt{2\pi(a-x)}}\left[1+M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+M_{2A}\left(1-\frac{x}{a}\right)^{1}+M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1A}=\frac{2\pi}{\sqrt{2Q}}\left(-Y_0+3Y_1\right)-\frac{24}{5}$$

$$M_{2A}=3$$

$$M_{3A}=\frac{6\pi}{\sqrt{2Q}}\left(Y_0-2Y_1\right)+\frac{8}{5}$$

where:

$$Q=1+1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t=R_o - R_i$$

$$Y_0=A_0+A_1\left(\frac{a}{t}\right)+A_2\left(\frac{a}{t}\right)^2+A_3\left(\frac{a}{t}\right)^4$$

$$A_0=1.12-0.207\left(\frac{a}{c}\right)-0.153\left(\frac{a}{c}\right)^2+1.305\left(\frac{a}{c}\right)^3-2.007\left(\frac{a}{c}\right)^4+0.933\left(\frac{a}{c}\right)^5$$

$$A_1=-0.111-7.205\left(\frac{a}{c}\right)+36.455\left(\frac{a}{c}\right)^2-83.649\left(\frac{a}{c}\right)^3+89741\left(\frac{a}{c}\right)^4-35.219\left(\frac{a}{c}\right)^5$$

$$A_2=1.498+20.265\left(\frac{a}{c}\right)-132.935\left(\frac{a}{c}\right)^2+323.535\left(\frac{a}{c}\right)^3-343.920\left(\frac{a}{c}\right)^4+131.532\left(\frac{a}{c}\right)^5$$

$$A_3=-0.140+18.828\left(\frac{a}{c}\right)-85.243\left(\frac{a}{c}\right)^2+118.941\left(\frac{a}{c}\right)^3-52.084\left(\frac{a}{c}\right)^4$$

and

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right) + B_2\left(\frac{a}{t}\right)^2 + B_3\left(\frac{a}{t}\right)^3$$

$$B_0 = 0.687 - 0.377\left(\frac{a}{c}\right) + 3.617\left(\frac{a}{c}\right)^2 - 10.671\left(\frac{a}{c}\right)^3 + 12.482\left(\frac{a}{c}\right)^4 - 5.015\left(\frac{a}{c}\right)^5$$

$$B_1 = -0.163 - 1.496\left(\frac{a}{c}\right) - 7.725\left(\frac{a}{c}\right)^2 + 41.963\left(\frac{a}{c}\right)^3 + -56.282\left(\frac{a}{c}\right)^4 + 23.554\left(\frac{a}{c}\right)^5$$

$$B_2 = 0.821 + 7.481\left(\frac{a}{c}\right) - 33.313\left(\frac{a}{c}\right)^2 + 54.993\left(\frac{a}{c}\right)^3 - 44.961\left(\frac{a}{c}\right)^4 + 15.196\left(\frac{a}{c}\right)^5$$

$$B_3 = -0.087 + 3.742\left(\frac{a}{c}\right) - 20.172\left(\frac{a}{c}\right)^2 + 33.425\left(\frac{a}{c}\right)^3 - 16.841\left(\frac{a}{c}\right)^4$$

WEIGHT FUNCTION (Fig. 21) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1B} = \frac{3\pi}{\sqrt{Q}}\left(2F_0 - 5F_1\right) - 8$$

$$M_{2B} = \frac{15\pi}{\sqrt{Q}}\left(3F_1 - F_0\right) + 15$$

$$M_{3B} = \frac{3\pi}{\sqrt{Q}}\left(3F_0 - 10F_1\right) - 8$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t = R_o - R_i$$

$$F_0 = \left[C_0 + C_1\left(\frac{a}{t}\right) + C_2\left(\frac{a}{t}\right)^2 + C_3\left(\frac{a}{t}\right)^4\right]\left(\frac{a}{c}\right)$$

$$C_0 = 5.923 - 20.55\left(\frac{a}{c}\right) + 36.937\left(\frac{a}{c}\right)^2 - 31.634\left(\frac{a}{c}\right)^3 + 10.37\left(\frac{a}{c}\right)^4$$

$$C_1 = -3.607 + 11.686\left(\frac{a}{c}\right) - 14.138\left(\frac{a}{c}\right)^2 + 9.935\left(\frac{a}{c}\right)^3 - 3.774\left(\frac{a}{c}\right)^4$$

$$C_2 = 19.14 - 72.902\left(\frac{a}{c}\right) + 112.643\left(\frac{a}{c}\right)^2 - 86.904\left(\frac{a}{c}\right)^3 + 28.149\left(\frac{a}{c}\right)^4$$

$$C_3 = 9.586 - 64.389\left(\frac{a}{c}\right) + 151.449\left(\frac{a}{c}\right)^2 - 144.822\left(\frac{a}{c}\right)^3 + 48.124\left(\frac{a}{c}\right)^4$$

and

$$F_1 = \left[D_0 + D_1\left(\frac{a}{t}\right) + D_2\left(\frac{a}{t}\right)^2 + D_3\left(\frac{a}{t}\right)^4\right]\left(\frac{a}{c}\right)$$

$$D_0 = 0.687 - 1.821\left(\frac{a}{c}\right) + 2.718\left(\frac{a}{c}\right)^2 - 1.981\left(\frac{a}{c}\right)^3 + 0.567\left(\frac{a}{c}\right)^4$$

$$D_1 = -1.797 + 9.399\left(\frac{a}{c}\right) - 19.195\left(\frac{a}{c}\right)^2 + 17.881\left(\frac{a}{c}\right)^3 - 6.244\left(\frac{a}{c}\right)^4$$

$$D_2 = 8.504 - 42.608\left(\frac{a}{c}\right) + 87.828\left(\frac{a}{c}\right)^2 - 82.735\left(\frac{a}{c}\right)^3 + 29.055\left(\frac{a}{c}\right)^4$$

$$D_3 = -0.666 + 6.54\left(\frac{a}{c}\right) - 21.603\left(\frac{a}{c}\right)^2 + 27.036\left(\frac{a}{c}\right)^3 - 11.236\left(\frac{a}{c}\right)^4$$

RANGE OF APPLICATION: $R_0/R_i = 2.0$, $0 = a/t = 0.8$, $a/c = 0$ and $0.2 = a/c = 1.0$

ACCURACY: Better than 3% when compared to the FEM data.

REFERENCES:

- weight function:

Zheng X. J., Glinka G., 1995, "Weight Functions and Stress Intensity Factors for Longitudinal Semi-Elliptical Cracks in Thick-Wall Cylinders," *J. Press. Vess.Technol.*, ASME, vol. 117, 1995, pp 383 - 389

- reference data:

Mettu S. R., Raju I. S., Forman R. G., 1988, “Stress Intensity Factors for Part-Through Surface Cracks in Hollow Cylinders,“ NASA Technical Report, No. JSC 25685 LESC 30124.

Andrasic C. P., Parker A. P., 1984, “Dimensionless Stress Intensity Factors for Cracked Thick Cylinder Under Polynomial Crack Face Loading,“ *Engng. Fract. Mech.*, Vol. 19, 1984, pp. 187-193.

Murakami Y., et al., 1986, *Stress Intensity Factor Handbook*, Vol. 2, Pergamon Press, pp. 309-317.

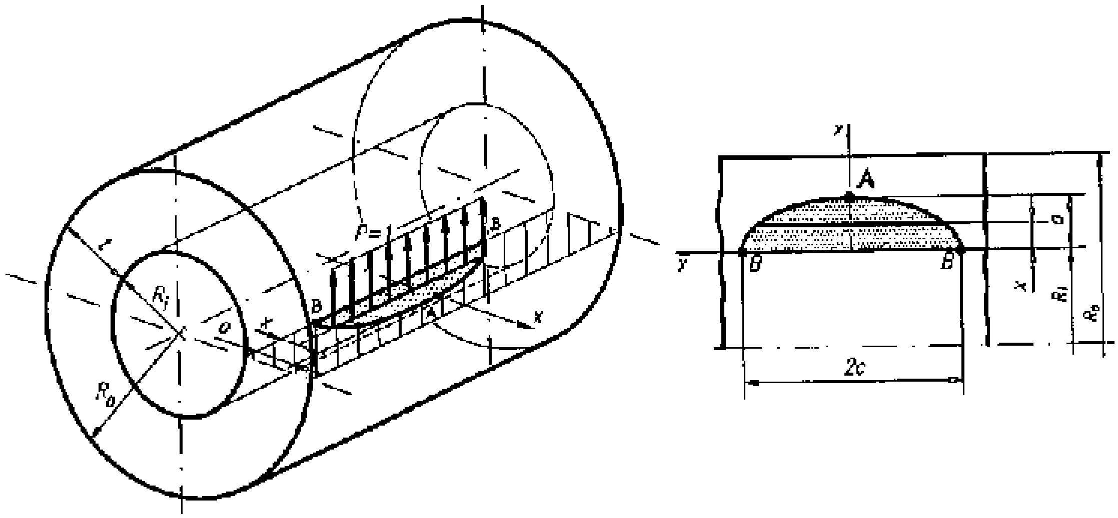

Figure 21. Internal axial semi-elliptical surface crack in a thick cylinder (R_o/R_i=2.0).

9.6 Internal axial semi-elliptical surface crack in a thick cylinder (R_o/R_i=1.5)

WEIGHT FUNCTION (Fig. 21) - **Deepest Point A**

$$m_A(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1A} = \frac{2\pi}{\sqrt{2Q}}(-Y_0 + 3Y_1) - \frac{24}{5}$$

$$M_{2A} = 3$$

$$M_{3A} = \frac{6\pi}{\sqrt{2Q}}(Y_0 - 2Y_1) + \frac{8}{5}$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t = R_o - R_i$$

$$Y_0 = A_0 + A_1\left(\frac{a}{t}\right) + A_2\left(\frac{a}{t}\right)^2 + A_3\left(\frac{a}{t}\right)^4$$

$$A_0 = 1.044 + 0.07\exp\left[-5.051\left(\frac{a}{c}\right)\right]$$

$$A_1 = -0.111 + 0.665\exp\left[-3.393\left(\frac{a}{c}\right)\right]$$

$$A_2 = 1.498 + 1.161\exp\left[-3.386\left(\frac{a}{c}\right)\right]$$

$$A_3 = -0.140 + 1.46\exp\left[-4.165\left(\frac{a}{c}\right)\right]$$

and

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right) + B_2\left(\frac{a}{t}\right)^2 + B_3\left(\frac{a}{t}\right)^3$$

$$B_0 = 2.825 - 2.16\exp\left[-0.035\left(\frac{a}{c}\right)\right]$$

$$B_1 = -0.225 + 0.265\exp\left[-5.574\left(\frac{a}{c}\right)\right]$$

$$B_2 = 0.307 + 0.753\exp\left[-4.025\left(\frac{a}{c}\right)\right]$$

$$B_3 = 1.398 - 1.284\exp\left[0.079\left(\frac{a}{c}\right)\right]$$

WEIGHT FUNCTION (Fig. 21) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1B} = \frac{3\pi}{\sqrt{Q}}\left(2F_0 - 5F_1\right) - 8$$

$$M_{2B} = \frac{15\pi}{\sqrt{Q}}\left(3F_1 - F_0\right) + 15$$

$$M_{3B} = \frac{3\pi}{\sqrt{Q}}\left(3F_0 - 10F_1\right) - 8$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \qquad \text{and} \qquad t = R_o - R_i$$

$$F_0 = \left[C_0 + C_1\left(\frac{a}{t}\right) + C_2\left(\frac{a}{t}\right)^2 + C_3\left(\frac{a}{t}\right)^4\right]\left(\frac{a}{c}\right)$$

$$C_0 = 0.972 + 5.163\exp\left[-5.061\left(\frac{a}{c}\right) + 1.568\left(\frac{a}{c}\right)^2\right]$$

$$C_1 = -0.199 - 10.239\exp\left[-46.053\left(\frac{a}{c}\right) - 4.009\left(\frac{a}{c}\right)^2\right]$$

$$C_2 = 0.119 + 8.784\exp\left[-4.081\left(\frac{a}{c}\right) + 1.092\left(\frac{a}{c}\right)^2\right]$$

$$C_3 = -0.104 + 28.33\exp\left[-9.959\left(\frac{a}{c}\right) - 9.817\left(\frac{a}{c}\right)^2\right]$$

and

$$F_1 = \left[D_0 + D_1\left(\frac{a}{t}\right) + D_2\left(\frac{a}{t}\right)^2 + D_3\left(\frac{a}{t}\right)^4\right]\left(\frac{a}{c}\right)$$

$$D_0 = 1.033 - 4.842\left(\frac{a}{c}\right) + 9.708\left(\frac{a}{c}\right)^2 - 8.397\left(\frac{a}{c}\right)^3 + 2.690\left(\frac{a}{c}\right)^4$$

$$D_1 = -3.448 + 24.231\left(\frac{a}{c}\right) - 50.221\left(\frac{a}{c}\right)^2 + 42.498\left(\frac{a}{c}\right)^3 - 13.099\left(\frac{a}{c}\right)^4$$

$$D_2 = 6.535 - 30.622\left(\frac{a}{c}\right) + 45.644\left(\frac{a}{c}\right)^2 - 25.05\left(\frac{a}{c}\right)^3 + 3.636\left(\frac{a}{c}\right)^4$$

$$D_3 = 2.243 - 21.677\left(\frac{a}{c}\right) + 65.546\left(\frac{a}{c}\right)^2 - 76.555\left(\frac{a}{c}\right)^3 + 30.433\left(\frac{a}{c}\right)^4$$

RANGE OF APPLICATION: $R_0/R_i = 1.5$, $0 = a/t = 0.8$, $a/c = 0$ and $0.2 = a/c = 1.0$

ACCURACY: Better than 3% when compared to the FEM data.

REFERENCES:

- weight function:

Zheng X. J., Kiciak A., Glinka G., 1996, “Determination of Weight Functions and Stress Intensity Factors for Semi-Elliptical Cracks in Thick-wall Cylinders,“ *Nucl. Engng. Design*, to be published.

- reference data:

Mettu S. R., Raju I. S., Forman R. G., 1988, "Stress Intensity Factors for Part-Through Surface Cracks in Hollow Cylinders," NASA Technical Report, No. JSC 25685 LESC 30124.

Andrasic C. P., Parker A. P., 1984, "Dimensionless Stress Intensity Factors for Cracked Thick Cylinder Under Polynomial Crack Face Loading," *Engng. Fract. Mech.*, Vol. 19, 1984, pp. 187-193.

Murakami Y., et al., 1986, *Stress Intensity Factor Handbook*, Vol. 2, Pergamon Press, pp. 309-317.

9.7 Internal axial semi-elliptical surface crack in a thick cylinder (R_o/R_i=1.25)

WEIGHT FUNCTION (Fig. 21) - **Deepest Point A**

$$m_A(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1A} = \frac{2\pi}{\sqrt{2Q}}\left(-Y_0 + 3Y_1\right) - \frac{24}{5}$$

$$M_{2A} = 3$$

$$M_{3A} = \frac{6\pi}{\sqrt{2Q}}\left(Y_0 - 2Y_1\right) + \frac{8}{5}$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t = R_o - R_i$$

$$Y_0 = A_0 + A_1\left(\frac{a}{t}\right) + A_2\left(\frac{a}{t}\right)^2 + A_3\left(\frac{a}{t}\right)^4$$

$$A_0 = 1.010 + 0.0998\exp\left[-13.15\left(\frac{a}{c}\right)\right]$$

$$A_1 = 0.055 + 0.366\exp\left[-31.17\left(\frac{a}{c}\right)\right]$$

$$A_2 = -0.057 + 3.269\exp\left[-3.859\left(\frac{a}{c}\right)\right]$$

$$A_3 = -0.149 + 0.061\exp\left[1.354\left(\frac{a}{c}\right)\right]$$

and

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right) + B_2\left(\frac{a}{t}\right)^2 + B_3\left(\frac{a}{t}\right)^3$$

$$B_0 = 6.594 - 5.944\exp\left[-0.012\left(\frac{a}{c}\right)\right]$$

$$B_1 = -0.136 + 0.436\exp\left[-8.663\left(\frac{a}{c}\right)\right]$$

$$B_2 = 0.269 + 0.787\exp\left[-4.562\left(\frac{a}{c}\right)\right]$$

$$B_3 = 1.552 - 1.538\exp\left[0.0434\left(\frac{a}{c}\right)\right]$$

WEIGHT FUNCTION (Fig. 21) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1B} = \frac{3\pi}{\sqrt{Q}}\left(2F_0 - 5F_1\right) - 8$$

$$M_{2B} = \frac{15\pi}{\sqrt{Q}}\left(3F_1 - F_0\right) + 15$$

$$M_{3B} = \frac{3\pi}{\sqrt{Q}}\left(3F_0 - 10F_1\right) - 8$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t = R_o - R_i$$

$$F_0 = \left[C_0 + C_1\left(\frac{a}{t}\right) + C_2\left(\frac{a}{t}\right)^2 + C_3\left(\frac{a}{t}\right)^4\right]\left(\frac{a}{c}\right)$$

$$C_0 = 5.566 - 19.583\left(\frac{a}{c}\right) + 37.335\left(\frac{a}{c}\right)^2 - 33.705\left(\frac{a}{c}\right)^3 + 11.507\left(\frac{a}{c}\right)^4$$

$$C_1 = -1.75 + 9.514\left(\frac{a}{c}\right) - 16.618\left(\frac{a}{c}\right)^2 + 10.44\left(\frac{a}{c}\right)^3 - 1.616\left(\frac{a}{c}\right)^4$$

$$C_2 = 12.497 - 49.067\left(\frac{a}{c}\right) + 72.59\left(\frac{a}{c}\right)^2 - 45.216\left(\frac{a}{c}\right)^3 + 9.55\left(\frac{a}{c}\right)^4$$

$$C_3 = 3.468 - 29.49\left(\frac{a}{c}\right) + 83.789\left(\frac{a}{c}\right)^2 - 93.289\left(\frac{a}{c}\right)^3 + 35.507\left(\frac{a}{c}\right)^4$$

and

$$F_1 = \left[D_0 + D_1\left(\frac{a}{t}\right) + D_2\left(\frac{a}{t}\right)^2 + D_3\left(\frac{a}{t}\right)^4\right]\left(\frac{a}{c}\right)$$

$$D_0 = 0.486 - 0.879\left(\frac{a}{c}\right) + 1.161\left(\frac{a}{c}\right)^2 - 0.793\left(\frac{a}{c}\right)^3 + 0.212\left(\frac{a}{c}\right)^4$$

$$D_1 = -0.533 + 2.626\left(\frac{a}{c}\right) - 3.412\left(\frac{a}{c}\right)^2 + 0.999\left(\frac{a}{c}\right)^3 + 0.333\left(\frac{a}{c}\right)^4$$

$$D_2 = 4.116 - 15.985\left(\frac{a}{c}\right) + 22.358\left(\frac{a}{c}\right)^2 - 12.235\left(\frac{a}{c}\right)^3 + 1.826\left(\frac{a}{c}\right)^4$$

$$D_3 = 0.569 - 6.605\left(\frac{a}{c}\right) + 21.548\left(\frac{a}{c}\right)^2 - 26.37\left(\frac{a}{c}\right)^3 + 10.853\left(\frac{a}{c}\right)^4$$

RANGE OF APPLICATION: R_0/R_i = 1.25, 0 = a/t = 0.8, and a/c = 0 and 0.2 = a/c = 1.0

ACCURACY: Better than 3% when compared to the FEM data.

REFERENCES:

- weight function:

Zheng X. J., Glinka G., Dubey R. N., 1995, “Calculation of Stress Intensity Factors for Semielliptical Cracks in a Thick-Wall Cylinder,“ *Int. J.Pres.Ves. & Piping*, Vol. 62, pp. 249-258.

- reference data:

Mettu S. R., Raju I. S., Forman R. G., 1988, "Stress Intensity Factors for Part-Through Surface Cracks in Hollow Cylinders," NASA Technical Report, No. JSC 25685 LESC 30124.

Andrasic C. P., Parker A. P., 1984, "Dimensionless Stress Intensity Factors for Cracked Thick Cylinder Under Polynomial Crack Face Loading," *Engng. Fract. Mech.*, Vol. 19, 1984, pp. 187-193.

Murakami Y., et al., 1986, *Stress Intensity Factor Handbook*, Vol. 2, Pergamon Press, pp. 309-317.

9.8 Internal axial semi-elliptical surface crack in a thick cylinder (R_o/R_i=1.1)

WEIGHT FUNCTION (Fig. 21) - **Deepest Point A**

$$m_A(x,a)=\frac{2}{\sqrt{2\pi(a-x)}}\left[1+M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+M_{2A}\left(1-\frac{x}{a}\right)^{1}+M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1A}=\frac{2\pi}{\sqrt{2Q}}(2Y_0+3Y_1)-\frac{24}{5}$$

$$M_{2A}=3$$

$$M_{3A}=\frac{6\pi}{\sqrt{2Q}}(2Y_1-Y_0)+\frac{8}{5}$$

where:

$$Q=1+1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t=R_o-R_i$$

$$Y_0=A_0+A_1\left(\frac{a}{t}\right)^2+A_2\left(\frac{a}{t}\right)^4$$

$$A_0=1.1449-0.6699\left(\frac{a}{c}\right)+1.0464\left(\frac{a}{c}\right)^2-0.5202\left(\frac{a}{c}\right)^3$$

$$A_1=3.84-10.531\left(\frac{a}{c}\right)+6.931\left(\frac{a}{c}\right)^2$$

$$A_2=-8.519+20.456\left(\frac{a}{c}\right)-13.027\left(\frac{a}{c}\right)^2+\frac{1}{0.061+\left(\frac{a}{c}\right)^{0.983}}$$

and

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right)^2 + B_2\left(\frac{a}{t}\right)^4$$

$$B_0 = 0.4732 - 0.4967\left(\frac{a}{c}\right) + 0.7576\left(\frac{a}{c}\right)^2 - 0.4417\left(\frac{a}{c}\right)^3$$

$$B_1 = 2.415 - 6.901\left(\frac{a}{c}\right) + 5.928\left(\frac{a}{c}\right)^2 - 1.291\left(\frac{a}{c}\right)^3$$

$$B_2 = -6.251 + 13.282\left(\frac{a}{c}\right) - 8.097\left(\frac{a}{c}\right)^2 + \frac{1}{0.090 + \left(\frac{a}{c}\right)^{0.92}}$$

WEIGHT FUNCTION (Fig. 21) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1B} = \frac{3\pi}{\sqrt{Q}}\left(5F_1 - 3F_0\right) - 8$$

$$M_{2B} = \frac{15\pi}{\sqrt{Q}}\left(2F_0 - 3F_1\right) + 15$$

$$M_{3B} = \frac{3\pi}{\sqrt{Q}}\left(10F_1 - 7F_0\right) - 8$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t = R_o - R_i$$

$$F_0 = \left[C_0 + C_1\left(\frac{a}{t}\right)^2 + C_2\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{a}{c}}$$

$$C_0 = 1.2959 - 0.2935\left(\frac{a}{c}\right) + 0.1203\left(\frac{a}{c}\right)^2$$

$$C_1 = 0.1256 + 27.96\left(\frac{a}{c}\right) - 143.547\left(\frac{a}{c}\right)^2 + 293.879\left(\frac{a}{c}\right)^3 - 270.492\left(\frac{a}{c}\right)^4 + 92.502\left(\frac{a}{c}\right)^5$$

$$C_2 = -2.065 + 1.15\left(\frac{a}{c}\right) + \frac{1}{0.2 + \left(\frac{a}{c}\right)^{1.05}}$$

and

$$F_1 = \left[D_0 + D_1\left(\frac{a}{t}\right)^2 + D_2\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{a}{c}}$$

$$D_0 = 1.2959 - 0.8104\left(\frac{a}{c}\right) + 0.4901\left(\frac{a}{c}\right)^2$$

$$D_1 = 0.3311 + 15.433\left(\frac{a}{c}\right) - 81.361\left(\frac{a}{c}\right)^2 + 167.357\left(\frac{a}{c}\right)^3 - 153.789\left(\frac{a}{c}\right)^4 + 52.309\left(\frac{a}{c}\right)^5$$

$$D_2 = -1.879 + 1.087\left(\frac{a}{c}\right) + \frac{1}{0.299 + \left(\frac{a}{c}\right)^{1.05}}$$

RANGE OF APPLICATION: $R_0/R_i = 1.1$, $0 = a/t = 0.8$, and $0 = a/c = 1.0$

ACCURACY: Better than 5% when compared to the FEM data.

REFERENCES:

- weight function:

Wang X. J., Lambert S. B., 1996, "Stress Intensity Factors and Weight Functions for Longitudinal Semi-Elliptical Surface Cracks in Thin Pipes," *Int. J.Pres.Ves. & Piping*, Vol. 65, pp 75 -87.

Shen. G., Liebster T. D., Glinka G. ,1993, "Calculation of Stress Intensity Factors for Cracks in Pipes,"*Proc. of the 12th Int. Conf. on Offshore Mech. and Arctic Engng..,* *ASME*,M. Salama et al., ed., Vol. III-B, *Materials Engineering*, pp.847-854. (this paper contains the older version of the weight function, limited to 0 = a/t = 0.8 and 0.2 = a/c = 1.0)

- reference data:

Raju I. S., Newman J. C., 1982, “Stress Intensity Factors for Internal and External Surface Cracks in Cylindrical Vessels,“ *J. Press. Ves. Technol*., Vol. 104, pp. 293-298.

Andrasic C. P., Parker A. P., 1984, “Dimensionless Stress Intensity Factors for Cracked Thick Cylinder Under Polynomial Crack Face Loading,“ *Engng. Fract. Mech.*, Vol. 19, 1984, pp. 187-193.

9.9 External axial semi-elliptical surface crack in a thick cylinder (R_o/R_i=2.0)

WEIGHT FUNCTION (Fig. 22) - **Deepest Point A**

$$m_A(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1A} = \frac{2\pi}{\sqrt{2Q}}\left(-Y_0 + 3Y_1\right) - \frac{24}{5}$$

$$M_{2A} = 3$$

$$M_{3A} = \frac{6\pi}{\sqrt{2Q}}\left(Y_0 - 2Y_1\right) + \frac{8}{5}$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t = R_o - R_i$$

$$Y_0 = A_0 + A_1\left(\frac{a}{t}\right) + A_2\left(\frac{a}{t}\right)^2 + A_3\left(\frac{a}{t}\right)^4$$

$$A_0 = -0.98463 + 0.10409\exp\left[11.93568\left(\frac{a}{c}\right) - 44.74122\left(\frac{a}{c}\right)^2 + 53.45540\left(\frac{a}{c}\right)^3 - 21.48712\left(\frac{a}{c}\right)^4\right]$$

$$A_1 = 0.02208 - 0.04469\exp\left[35.69496\left(\frac{a}{c}\right) - 138.51103\left(\frac{a}{c}\right)^2 + 185.87345\left(\frac{a}{c}\right)^3 - 85.1922\left(\frac{a}{c}\right)^4\right]$$

$$A_2 = 0.41663 + 0.41917\exp\left[22.34798\left(\frac{a}{c}\right) - 87.48907\left(\frac{a}{c}\right)^2 + 118.30990\left(\frac{a}{c}\right)^3 - 55.88622\left(\frac{a}{c}\right)^4\right]$$

$$A_3 = -0.07482 + 17.78423\exp\left[-23.84462\left(\frac{a}{c}\right) + 71.77065\left(\frac{a}{c}\right)^2 - 118.12933\left(\frac{a}{c}\right)^3 + 63.64254\left(\frac{a}{c}\right)^4\right]$$

and

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right) + B_2\left(\frac{a}{t}\right)^2 + B_3\left(\frac{a}{t}\right)^4$$

$$B_0 = 0.71344 + 0.00525\exp\left[25.42681\left(\frac{a}{c}\right) - 82.30859\left(\frac{a}{c}\right)^2 + 39.63312\left(\frac{a}{c}\right)^3 + 16.09569\left(\frac{a}{c}\right)^4\right]$$

$$B_1 = 0.02433 - 0.08062\exp\left[24.45619\left(\frac{a}{c}\right) - 103.67649\left(\frac{a}{c}\right)^2 + 158.27997\left(\frac{a}{c}\right)^3 - 87.15246\left(\frac{a}{c}\right)^4\right]$$

$$B_2 = 0.12875 + 0.46420\exp\left[14.79057\left(\frac{a}{c}\right) - 66.68409\left(\frac{a}{c}\right)^2 + 103.18837\left(\frac{a}{c}\right)^3 - 58.96618\left(\frac{a}{c}\right)^4\right]$$

$$B_3 = 1.00553 - 8.68986\left(\frac{a}{c}\right) + 35.59049\left(\frac{a}{c}\right)^2 - 85.96909\left(\frac{a}{c}\right)^3 + 118.96622\left(\frac{a}{c}\right)^4 - 84.66299\left(\frac{a}{c}\right)^5 + 23.75335\left(\frac{a}{c}\right)^6$$

WEIGHT FUNCTION (Fig. 22) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1B} = \frac{3\pi}{\sqrt{Q}}\left(2F_0 - 5F_1\right) - 8$$

$$M_{2B} = \frac{15\pi}{\sqrt{Q}}\left(3F_1 - F_0\right) + 15$$

$$M_{3B} = \frac{3\pi}{\sqrt{Q}}\left(3F_0 - 10F_1\right) - 8$$

where:

$$F_0 = C_0 + C_1\left(\frac{a}{t}\right) + C_2\left(\frac{a}{t}\right)^2 + C_3\left(\frac{a}{t}\right)^4 + C_4\left(\frac{a}{t}\right)^5$$

$$C_0 = 1.40290\left\{1 - \exp\left[-6.93070\left(\frac{a}{c}\right) + 21.48633\left(\frac{a}{c}\right)^2 - 36.14294\left(\frac{a}{c}\right)^3 + 28.68609\left(\frac{a}{c}\right)^4 - 8.80402\left(\frac{a}{c}\right)^5\right]\right\}$$

$$C_1 = -15.12461\left(\frac{a}{c}\right)\left[1 - 1.17520\left(\frac{a}{c}\right)\right]\exp\left[-3.64659\left(\frac{a}{c}\right) - 5.78270\left(\frac{a}{c}\right)^2 + 13.55459\left(\frac{a}{c}\right)^3 - 6.78266\left(\frac{a}{c}\right)^4\right]$$

$$C_2 = 1.11589\left\{1 - \exp\left[-0.60488\left(\frac{a}{c}\right) - 9.42291\left(\frac{a}{c}\right)^2 + 17.10918\left(\frac{a}{c}\right)^3 - 7.51091\left(\frac{a}{c}\right)^4\right]\right\}$$

$$C_3 = 55.90204\left(\frac{a}{c}\right)\left[1 - 1.17520\left(\frac{a}{c}\right)\right]\exp\left[-10.21823\left(\frac{a}{c}\right) + 8.20568\left(\frac{a}{c}\right)^2 - 7.52675\left(\frac{a}{c}\right)^3 + 2.66400\left(\frac{a}{c}\right)^4\right]$$

$$C_4 = -16.95222\left(\frac{a}{c}\right)\left[1 - 0.88925\left(\frac{a}{c}\right)\right]\exp\left[-9.87203\left(\frac{a}{c}\right) + 19.30584\left(\frac{a}{c}\right)^2 - 26.66380\left(\frac{a}{c}\right)^3 + 14.35426\left(\frac{a}{c}\right)^4\right]$$

and

$$F_1 = D_0 + D_1\left(\frac{a}{t}\right) + D_2\left(\frac{a}{t}\right)^2$$

$$D_0 = 0.20478\left\{1 - \exp\left[-11.38873\left(\frac{a}{c}\right) + 28.07640\left(\frac{a}{c}\right)^2 - 30.62707\left(\frac{a}{c}\right)^3 + 9.86369\left(\frac{a}{c}\right)^4\right]\right\}$$

$$D_1 = -9.73349\left(\frac{a}{c}\right)\left[1 - 1.09108\left(\frac{a}{c}\right)\right]\exp\left[-4.22628\left(\frac{a}{c}\right) - 4.99914\left(\frac{a}{c}\right)^2 + 10.82767\left(\frac{a}{c}\right)^3 - 4.84826\left(\frac{a}{c}\right)^4\right]$$

$$D_2 = 14.67302\left(\frac{a}{c}\right)\left[1 - 0.90366\left(\frac{a}{c}\right)\right]\exp\left[-5.27930\left(\frac{a}{c}\right) + 1.71158\left(\frac{a}{c}\right)^2 + 1.33890\left(\frac{a}{c}\right)^3\right]$$

RANGE OF APPLICATION: R_0/R_i = 2.0, 0 = a/t = 1.0, a/c = 0 and 0.2 = a/c = 1.0

ACCURACY: Better than 3% when compared to the FEM data.

REFERENCES:

- weight function:

Kiciak A., Glinka G., Burns D. J., 1996, "Weight Functions and Stress Intensity Factors for Longitudinal Semi-Elliptical Cracks in Thick-wall Cylinders," *J. Press. Vess. Technol.*, ASME (to be published)

- reference data:

Mettu S. R., Raju I. S., Forman R. G., 1988, "Stress Intensity Factors for Part-Through Surface Cracks in Hollow Cylinders," NASA Technical Report, No. JSC 25685 LESC 30124.

Andrasic C. P., Parker A. P., 1984, "Dimensionless Stress Intensity Factors for Cracked Thick Cylinder Under Polynomial Crack Face Loading," *Engng. Fract. Mech.*, Vol. 19, 1984, pp. 187-193.

Murakami Y., et al., 1986, *Stress Intensity Factor Handbook*, Vol. 2, Pergamon Press, pp. 309-317.

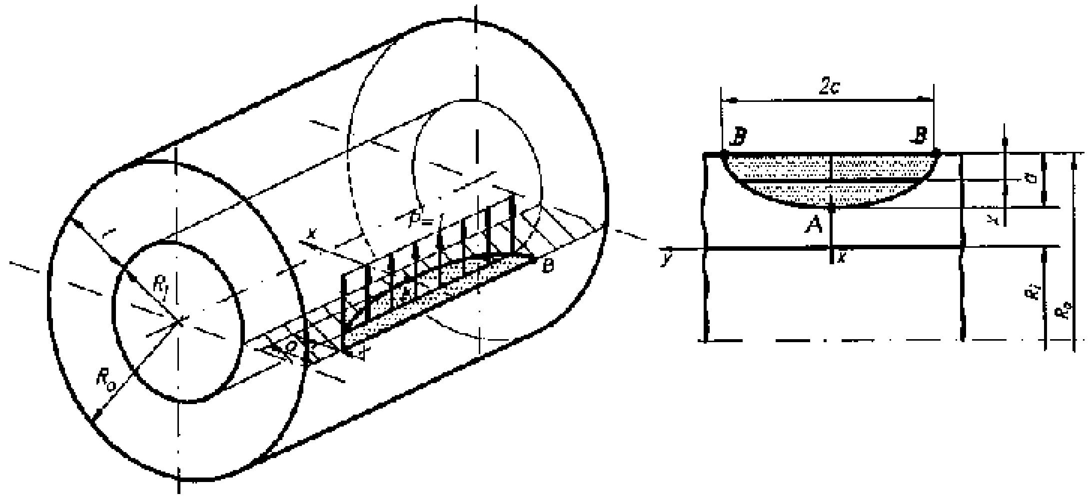

Figure 22. External axial semi-elliptical surface crack in a thick cylinder (R_o/R_i=2.0).

External axial semi-elliptical surface crack in a thick cylinder (R_o/R_i=1.5)

WEIGHT FUNCTION (Fig. 22) - **Deepest Point A**

$$m_A(x,a)=\frac{2}{\sqrt{2\pi(a-x)}}\left[1+M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+M_{2A}\left(1-\frac{x}{a}\right)^{1}+M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1A}=\frac{2\pi}{\sqrt{2Q}}\left(-Y_0+3Y_1\right)-\frac{24}{5}$$

$$M_{2A}=3$$

$$M_{3A}=\frac{6\pi}{\sqrt{2Q}}\left(Y_0-2Y_1\right)+\frac{8}{5}$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \qquad \text{and} \qquad t = R_o - R_i$$

$$Y_0 = A_0 + A_1\left(\frac{a}{t}\right) + A_2\left(\frac{a}{t}\right)^2 + A_3\left(\frac{a}{t}\right)^4$$

$$A_0 = 0.69716 + \exp\left[\begin{array}{l} -0.85208 + 0.14072\left(\frac{a}{c}\right) - 1.88533\left(\frac{a}{c}\right)^2 + 2.38695\left(\frac{a}{c}\right)^3 \\ -0.92014\left(\frac{a}{c}\right)^4 \end{array}\right]$$

$$A_1 = 0.83400 - \exp\left[\begin{array}{l} -1.89775 + 21.36041\left(\frac{a}{c}\right) - 84.13521\left(\frac{a}{c}\right)^2 + 142.22714\left(\frac{a}{c}\right)^3 \\ -109.84164\left(\frac{a}{c}\right)^4 + 32.02799\left(\frac{a}{c}\right)^5 \end{array}\right]$$

$$A_2 = 0.25833 + \exp\left[\begin{array}{l} 0.38467 + 3.46411\left(\frac{a}{c}\right) - 3.41241\left(\frac{a}{c}\right)^2 + 28.40776\left(\frac{a}{c}\right)^3 \\ -23.50662\left(\frac{a}{c}\right)^4 \end{array}\right]$$

$$A_3 = -0.024225 + \exp\left[\begin{array}{l} 0.18341 + 12.66325\left(\frac{a}{c}\right) - 148.59759\left(\frac{a}{c}\right)^2 + 400.31551\left(\frac{a}{c}\right)^3 \\ -419.88103\left(\frac{a}{c}\right)^4 + 150.43104\left(\frac{a}{c}\right)^5 \end{array}\right]$$

and

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right) + B_2\left(\frac{a}{t}\right)^2 + B_3\left(\frac{a}{t}\right)^4$$

$$B_0 = 0.22856 + \exp\left[\begin{array}{l} -0.78477 + 0.07367\left(\frac{a}{c}\right) - 0.19984\left(\frac{a}{c}\right)^2 - 0.04996\left(\frac{a}{c}\right)^3 \\ -0.25050\left(\frac{a}{c}\right)^4 \end{array}\right]$$

$$B_1 = 0.43259 - \exp\left[\begin{array}{l} -2.27787 + 20.51173\left(\frac{a}{c}\right) - 84.19018\left(\frac{a}{c}\right)^2 + 154.08510\left(\frac{a}{c}\right)^3 \\ -132.35299\left(\frac{a}{c}\right)^4 + 43.43167\left(\frac{a}{c}\right)^5 \end{array}\right]$$

$$B_2 = 0.14247 + \exp\left[\begin{array}{l} -1.11085 + 13.46249\left(\frac{a}{c}\right) - 55.49509\left(\frac{a}{c}\right)^2 + 74.92487\left(\frac{a}{c}\right)^3 \\ -37.080007\left(\frac{a}{c}\right)^4 \end{array}\right]$$

$$B_3 = -0.02704 + \exp\left[\begin{array}{l} -0.44818 - 0.17795\left(\frac{a}{c}\right) - 53.30479\left(\frac{a}{c}\right)^2 + 25.51119\left(\frac{a}{c}\right)^3 \\ +24.67589\left(\frac{a}{c}\right)^4 \end{array}\right]$$

WEIGHT FUNCTION (Fig. 22) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1B} = \frac{3\pi}{\sqrt{Q}}\left(2F_0 - 5F_1\right) - 8$$

$$M_{2B} = \frac{15\pi}{\sqrt{Q}}\left(3F_1 - F_0\right) + 15$$

$$M_{3B} = \frac{3\pi}{\sqrt{Q}}\left(3F_0 - 10F_1\right) - 8$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t = R_o - R_i$$

$$F_0 = C_0 + C_1\left(\frac{a}{t}\right) + C_2\left(\frac{a}{t}\right)^2 + C_3\left(\frac{a}{t}\right)^4$$

$$C_0 = 0.69519\left\{1 + 0.31674\left(\frac{a}{c}\right) + 0.34206\left(\frac{a}{c}\right)^2 - \exp\left[-29.87363\left(\frac{a}{c}\right) + 139.14830\left(\frac{a}{c}\right)^2 - 382.37581\left(\frac{a}{c}\right)^3\right]\right\}$$

$$C_1 = -\left(\frac{a}{c}\right)\left[55.208999 - 68.89759\left(\frac{a}{c}\right)\right]\exp\left[-16.47111\left(\frac{a}{c}\right) + 38.49788\left(\frac{a}{c}\right)^2 - 55.27126\left(\frac{a}{c}\right)^3 + 27.25396\left(\frac{a}{c}\right)^4\right]$$

$$C_2 = 2.70539\left\{1 - 1.50789\left(\frac{a}{c}\right) + 0.72855\left(\frac{a}{c}\right)^2 - \exp\left[-23.29526\left(\frac{a}{c}\right) + 84.84981\left(\frac{a}{c}\right)^2 - 144.62398\left(\frac{a}{c}\right)^3\right]\right\}$$

$$C_3 = \left(\frac{a}{c}\right)\left[27.52544 - 14.91620\left(\frac{a}{c}\right) - 12.89888\left(\frac{a}{c}\right)^2\right]\exp\left[-17.21310\left(\frac{a}{c}\right) + 21.22175\left(\frac{a}{c}\right)^2 - 7.53042\left(\frac{a}{c}\right)^3\right]$$

and

$$F_1 = D_0 + D_1\left(\frac{a}{t}\right) + D_2\left(\frac{a}{t}\right)^2 + D_3\left(\frac{a}{t}\right)^4$$

$$D_0 = 0.09646\left\{1 + 1.51333\left(\frac{a}{c}\right) - 1.27723\left(\frac{a}{c}\right)^2 + 0.79576\left(\frac{a}{c}\right)^3 - \exp\left[-27.55715\left(\frac{a}{c}\right)\right]\right\}$$

$$D_1 = -0.66304\left(\frac{a}{c}\right)\left\{1 - \exp\left[-11.75502\left(\frac{a}{c}\right)\right]\right\}\left[1 - 2.30088\left(\frac{a}{c}\right) + 0.66328\left(\frac{a}{c}\right)^2 - 0.58371\left(\frac{a}{c}\right)^3\right]$$

$$D_2 = 2.07021\left\{1 - \exp\left[-3.57311\left(\frac{a}{c}\right)\right]\right\}\left[1 - 2.31235\left(\frac{a}{c}\right) + 1.38520\left(\frac{a}{c}\right)^2\right]$$

$$D_3 = \left[0.23183 - 2.02421\left(\frac{a}{c}\right) + 4.64517\left(\frac{a}{c}\right)^2 - 2.861145\left(\frac{a}{c}\right)^3\right]\left\{1 - \exp\left[-52.19501\left(\frac{a}{c}\right)^{0.25}\right]\right\}$$

RANGE OF APPLICATION: $R_0/R_i = 1.5$, $0 = a/t = 1.0$, $a/c = 0$ and $0.2 = a/c = 1.0$

ACCURACY: Better than 3% when compared to the FEM data.

REFERENCES:

- weight function:

Kiciak A., Glinka G., Burns D. J., 1996, University of Waterloo, to be published.

- reference data:

Mettu S. R., Raju I. S., Forman R. G., 1988, "Stress Intensity Factors for Part-Through Surface Cracks in Hollow Cylinders," NASA Technical Report, No. JSC 25685 LESC 30124.

Andrasic C. P., Parker A. P., 1984, "Dimensionless Stress Intensity Factors for Cracked Thick Cylinder Under Polynomial Crack Face Loading," *Engng. Fract. Mech.*, Vol. 19, 1984, pp. 187-193.

Murakami Y., et al., 1986, *Stress Intensity Factor Handbook*, Vol. 2, Pergamon Press, pp. 309-317.

9.10 External axial semi-elliptical surface crack in a thick cylinder (R_o/R_i=1.25)

WEIGHT FUNCTION (Fig. 22) - **Deepest Point A**

$$m_A(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1A} = \frac{2\pi}{\sqrt{2Q}}\left(-Y_0 + 3Y_1\right) - \frac{24}{5}$$

$$M_{2A} = 3$$

$$M_{3A} = \frac{6\pi}{\sqrt{2Q}}\left(Y_0 - 2Y_1\right) + \frac{8}{5}$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t = R_o - R_i$$

$$Y_0 = A_0 + A_1\left(\frac{a}{t}\right) + A_2\left(\frac{a}{t}\right)^2 + A_3\left(\frac{a}{t}\right)^4$$

$$A_0 = 0.94235 + \exp\left[-1.68499 - 8.56013\left(\frac{a}{c}\right) + 23.08395\left(\frac{a}{c}\right)^2 - 26.59266\left(\frac{a}{c}\right)^3 + 10.62456\left(\frac{a}{c}\right)^4\right]$$

$$A_1 = -0.38656 + \exp\left[-0.82502 + 4.66494\left(\frac{a}{c}\right) - 12.06481\left(\frac{a}{c}\right)^2 + 11.96283\left(\frac{a}{c}\right)^3 - 4.23850\left(\frac{a}{c}\right)^4\right]$$

$$A_2 = -0.57088 + \exp\left[1.58221 - 3.16636\left(\frac{a}{c}\right) - 4.515307\left(\frac{a}{c}\right)^2 + 11.48057\left(\frac{a}{c}\right)^3 - 5.95069\left(\frac{a}{c}\right)^4\right]$$

$$A_3 = 0.198498 - \exp\left[\begin{array}{l} -15.94324 + 112.58315\left(\frac{a}{c}\right) - 340.20961\left(\frac{a}{c}\right)^2 + 497.99358\left(\frac{a}{c}\right)^3 \\ -354.74017\left(\frac{a}{c}\right)^4 + 98.71664\left(\frac{a}{c}\right)^5 \end{array}\right]$$

and

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right) + B_2\left(\frac{a}{t}\right)^2 + B_3\left(\frac{a}{t}\right)^3 + B_4\left(\frac{a}{t}\right)$$

$$B_0 = 050249 + \exp\left[\begin{array}{l} -1.71519 - 2.14909\left(\frac{a}{c}\right) + 7.58393\left(\frac{a}{c}\right)^2 - 10.00652\left(\frac{a}{c}\right)^3 \\ +4.71914\left(\frac{a}{c}\right)^4 \end{array}\right]$$

$$B_1 = 1.53122 - \exp\left[\begin{array}{l} 0.31448 + 0.59120\left(\frac{a}{c}\right) - 2.21570\left(\frac{a}{c}\right)^2 + 3.08396\left(\frac{a}{c}\right)^3 \\ -1.37007\left(\frac{a}{c}\right)^4 \end{array}\right]$$

$$B_2 = 0.04652 + \exp\left[0.04497 - 3.84029\left(\frac{a}{c}\right) - 1.32259\left(\frac{a}{c}\right)^2\right]$$

$$B_3 = 0.02569 + \exp\left[\begin{array}{l} -0.55548 + 0.88270\left(\frac{a}{c}\right) - 13.45912\left(\frac{a}{c}\right)^2 - 2.45236\left(\frac{a}{c}\right)^3 \\ +8.50684\left(\frac{a}{c}\right)^4 \end{array}\right]$$

$$B_4 = -0.01828 - \exp\left[\begin{array}{l} -2.10419 + 7.85207\left(\frac{a}{c}\right) - 33.70843\left(\frac{a}{c}\right)^2 + 25.85419\left(\frac{a}{c}\right)^3 \\ -4.64059\left(\frac{a}{c}\right)^4 \end{array}\right]$$

WEIGHT FUNCTION (Fig. 22) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^1 + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1B} = \frac{3\pi}{\sqrt{Q}}\left(2F_0 - 5F_1\right) - 8$$

$$M_{2B} = \frac{15\pi}{\sqrt{Q}}\left(3F_1 - F_0\right) + 15$$

$$M_{3B} = \frac{3\pi}{\sqrt{Q}}\left(3F_0 - 10F_1\right) - 8$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t = R_o - R_i$$

$$F_0 = C_0 + C_1\left(\frac{a}{t}\right) + C_2\left(\frac{a}{t}\right)^2 + C_3\left(\frac{a}{t}\right)^4$$

$$C_0 = 0.47251\left\{1 + 1.58454\left(\frac{a}{c}\right) - 0.19842\left(\frac{a}{c}\right)^2 - \exp\left[-14.02505\left(\frac{a}{c}\right)\right]\right\}$$

$$C_1 = -0.22958\left\{12.79131\left(\frac{a}{c}\right)1.47068\left(\frac{a}{c}\right)^2 - \exp\left[-14.11736\left(\frac{a}{c}\right)\right]\right\}$$

$$C_2 = 1.80687\left\{11.27898\left(\frac{a}{c}\right) + 0.55332\left(\frac{a}{c}\right)^2 - \exp\left[-11.39742\left(\frac{a}{c}\right) + 34.86421\left(\frac{a}{c}\right)^2 - 78.88988\left(\frac{a}{c}\right)^3\right]\right\}$$

$$C_3 = 0.24024\left\{1 - 2.08060\left(\frac{a}{c}\right) + 1.01701\left(\frac{a}{c}\right)^2 - \exp\left[-3.18934\left(\frac{a}{c}\right) - 3.79275\left(\frac{a}{c}\right)^2\right]\right\}$$

and

$$F_1 = D_0 + D_1\left(\frac{a}{t}\right) + D_2\left(\frac{a}{t}\right)^2 + D_3\left(\frac{a}{t}\right)^4$$

$$D_0 = 0.12603\left\{1 + 0.54116\left(\frac{a}{c}\right) - \exp\left[-3.19737\left(\frac{a}{c}\right)\right]\right\}$$

$$D_1 = -0.15562\left\{\left[1 - 2.30358\left(\frac{a}{c}\right) + 1.03870\left(\frac{a}{c}\right)^2 - \exp\left[-5.24795\left(\frac{a}{c}\right) + 2.35842\left(\frac{a}{c}\right)^2\right]\right]\right\}$$

$$D_2 = 2.17158\left\{1 - \exp\left[-1.21697\left(\frac{a}{c}\right)\right]\right\}\left[1 - 1.89058\left(\frac{a}{c}\right) + 0.96829\left(\frac{a}{c}\right)^2\right]$$

$$D_3 = \left(\frac{a}{c}\right)\left[\begin{array}{l} 1.08880 - 12.47981\left(\frac{a}{c}\right) + 46.80652\left(\frac{a}{c}\right)^2 - 77.95158\left(\frac{a}{c}\right)^3 \\ + 60.62440\left(\frac{a}{c}\right)^4 - 18.09600\left(\frac{a}{c}\right)^5 \end{array}\right]$$

RANGE OF APPLICATION: $R_0/R_i = 1.25$, $0 = a/t = 1.0$, $a/c = 0$ and $0.2 = a/c = 1.0$

ACCURACY: Better than 3% when compared to the FEM data.

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9.11 External axial semi-elliptical surface crack in a thick cylinder (R_o/R_i=1.1)

WEIGHT FUNCTION (Fig. 22) - **Deepest Point A**

$$m_A(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1A} = \frac{2\pi}{\sqrt{2Q}}(2Y_0 - 3Y_1) - \frac{24}{5}$$

$$M_{2A} = 3$$

$$M_{3A} = \frac{6\pi}{\sqrt{2Q}}(2Y_1 - Y_0) + \frac{8}{5}$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t = R_o - R_i$$

$$Y_0 = A_0 + A_1\left(\frac{a}{t}\right)^2 + A_2\left(\frac{a}{t}\right)^4$$

$$A_0 = 1.1492 - 0.4322\left(\frac{a}{c}\right) + 0.2984\left(\frac{a}{c}\right)^2$$

$$A_1 = 4.00 - 8.98\left(\frac{a}{c}\right) + 5.29\left(\frac{a}{c}\right)^2$$

$$A_2 = -7.44 + 14.559\left(\frac{a}{c}\right) - 8.305\left(\frac{a}{c}\right)^2 + \frac{1}{0.066 + \left(\frac{a}{c}\right)^{1.094}}$$

and

$$Y_1 = B_0 + B_1\left(\frac{a}{t}\right)^2 + B_2\left(\frac{a}{t}\right)^4$$

$$B_0 = 0.4840 - 0.5211\left(\frac{a}{c}\right) + 0.788\left(\frac{a}{c}\right)^2 - 0.453\left(\frac{a}{c}\right)^3$$

$$B_1 = 2.4478 - 5.0937\left(\frac{a}{c}\right) + 2.850\left(\frac{a}{c}\right)^2$$

$$B_2 = -5.69 + 9.653\left(\frac{a}{c}\right) - 5.062\left(\frac{a}{c}\right)^2 + \frac{1}{0.097 + \left(\frac{a}{c}\right)^{1.006}}$$

WEIGHT FUNCTION (Fig. 22) - **Surface Point B**

$$m_B(x,a) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^1 + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right]$$

PARAMETERS

$$M_{1B} = \frac{3\pi}{\sqrt{Q}}(5F_1 - 3F_0) - 8$$

$$M_{2B} = \frac{15\pi}{\sqrt{Q}}(2F_0 - 3F_1) + 15$$

$$M_{3B} = \frac{3\pi}{\sqrt{Q}}(10F_1 - 7F_0) - 8$$

where:

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{and} \quad t = R_o - R_i$$

$$F_0 = \left[C_0 + C_1\left(\frac{a}{t}\right)^2 + C_2\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{a}{c}}$$

$$C_0 = 1.2964 - 0.2532\left(\frac{a}{c}\right) + 0.0895\left(\frac{a}{c}\right)^2$$

$$C_1 = -0.1110 + 25.953\left(\frac{a}{c}\right) - 115.107\left(\frac{a}{c}\right)^2 + 210.832\left(\frac{a}{c}\right)^3 - 177.848\left(\frac{a}{c}\right)^4 + 56.8339\left(\frac{a}{c}\right)^5$$

$$C_2 = -1.405 - 3.746\left(\frac{a}{c}\right) + 2.262\left(\frac{a}{c}\right)^2$$

and

$$F_1 = \left[D_0 + D_1\left(\frac{a}{t}\right)^2 + D_2\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{a}{c}}$$

$$D_0 = 1.2531 - 0.7814\left(\frac{a}{c}\right) + 0.4668\left(\frac{a}{c}\right)^2$$

$$D_1 = 0.2128 + 14.4065\left(\frac{a}{c}\right) - 67.574\left(\frac{a}{c}\right)^2 + 129.93\left(\frac{a}{c}\right)^3 - 115.252\left(\frac{a}{c}\right)^4 + 38.6732\left(\frac{a}{c}\right)^5$$

$$D_2 = 0.7704 - 1.3817\left(\frac{a}{c}\right) + 0.58529\left(\frac{a}{c}\right)^2$$

RANGE OF APPLICATION: $R_0/R_i = 1.1$, $0 = a/t = 1.0$, and $0 = a/c = 1.0$

ACCURACY: Better than 5% when compared to the FEM data.

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10. NOTATION

a - crack length for one-dimensional cracks or crack depth (minor semi-axis) for semi-elliptical cracks

A_i, B_i - coefficients of the linearized stress function in the sub-interval “i”

c - crack length (major semi-axis) for semi-elliptical crack

C_{ij} - coefficients of the integrated weight function $m_A(x,a)$ in association with the linearised stress field s(x)

D_{ij} - coefficients of the integrated weight function $m_B(x,a)$in association with the linearised stress field s(x)

K - stress intensity factor

K_I - mode I stress intensity factor (general)

K_0^A - mode I reference stress intensity factor for the deepest point A of a crack under the uniform stress field

K_0^B - mode I reference stress intensity factor for the surface point B of a crack under the uniform stress field

K^A - mode I stress intensity factor for the deepest point A on the crack front

K^B - mode I reference stress intensity factor for the surface point B on the crack front

M_i - parameters of the weight function (i = 1, 2, 3)

M_{iA} - coefficients of the weight functions for the deepest point A (i= 1,2,3)

M_{iB} - coefficients of the weight functions for the surface point B (i= 1,2,3)

m(x,a) - weight function

$m_A(x,a)$ - weight function for the deepest point A of a crack

$m_B(x,a)$ - weight function for the surface point B of a crack

S - area under a monotonic curve m(x,a)

S_i^* - area under the linearized stress function s(x) corresponding to the sub-interval “i”

S_i - area under the weight function curve m(x,a) corresponding to the sub-interval "i"

Q - the semi-elliptical crack shape factor: $Q = 1 + 1.464(a/c)^{1.65}$ for $a/c < 1$

t - wall thickness of a cylinder or plate thickness

x - the local, through the thickness co-ordinate along the crack depth

X - co-ordinate x of the centroid of the area S

X_i - co-ordinate x of the centroid of the area S_i corresponding the sub-interval "i"

X_i^* - co-ordinate x of the centroid of the area S_i^* corresponding to the sub-interval "i"

$\sigma(x)$ - stress distribution

$\sigma(x_i)$ -value of stress function at $x = x_i$

$\sigma(X)$ - value of stress function at $x = X$

$\sigma_\rho(x)$ - a reference stress distribution

Y_{nA} - the geometric stress intensity correction factor for the deepest point A

Y_{nB} - the geometric stress intensity correction factor for the surface point B

11. LITERATURE

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