Hsu Model

AFRL-RB-WP-TR-2008-3

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Title	**Hsu Model**
Doc. Ref.	**TWD03ER009-1**
Revision	1
Date	01-29-2004

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Checked by	**Thomas W. Deiters**	**01-29-2004**

ABSTRACT

This report documents the Hsu Load Interaction (Retardation) Model implementation in AFGROW. The Hsu model originally developed at Lockheed-Georgia uses an effective stress and closure concept. It assumes that the stress singularity does not exist if the crack surface is closed, and that the crack propagates only during that portion of the load cycle in which the crack surface is fully open. A complete review of background and overview of the process is presented to establish the Hsu implementation model basis.

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1 INTRODUCTION

This report documents the Hsu Load Interaction (Retardation) Model implementation in AFGROW. The Hsu model originally developed at Lockheed-Georgia uses an effective stress and closure concept. It assumes that the stress singularity does not exist if the crack surface is closed, and that the crack propagates only during that portion of the load cycle in which the crack surface is fully open. A complete review of background and overview of the process is presented to establish the Hsu implementation model basis. This is offered to give the entire foundation of the Hsu model process in one place because there are certain aspects that are not apparent on the surface. The overview and background is followed by details and flow of the Hsu model process.

1.1 Background and Overview

The retardation premise has been well established experimentally, that normal crack growth rate under constant amplitude loading changes if the load application is preceded by different amplitude. The tensile overload causes permanent plastic deformation at the crack tip and along the crack flanks which, in turn, delays the crack growth at subsequent load cycles, while a tension-compressive load reduce the beneficial retardation effect.

Hsu was associated with the Advanced Structures Department of Lockheed Georgia. At the time Hsu developed his model, several researchers in academia and government had published their ideas, reference 1, 2, and 3. These authors are familiar names to practicing fracture analysts today. Ideas were also presented by reference 4, 5, and 6. Different aspects of the problem were being addressed in papers by names such as, Hardrath, H.F., Hertzberg, R.W., Wei, R.P., Shih, T.T., Von Euw, E.F.J., Rice, R.C., Paris, P., Forman, R.G., and Schijve, J., many of these may also be remembered and recognized today.

When Hsu was tasked to come up with a predictive technology for Lockheed, it should be remembered that many of these concepts were new and none had been completely verified, the only thing that could be said was that all investigators understood how complex predicting

variable loading crack growth was. Several models had been suggested to account for the effects of delay on the prediction of fatigue crack growth. Two models, Wheeler [1] and Willenborg et al [2], used the plastic zone size (for either plane stress or plane strain) associated with the applied load levels directly and one model, Elber [3], used plasticity indirectly to characterize the load interaction effect. The two models, [1] [2], assume that if the size of the plastic zone size, r_p, developed due to the application of the current load cycle extends to or past the extremities of a previously developed overload interaction zone, r_{pOL} that is if $(\Delta a+r_p) \geq r_{pOL}$), there will be no load interaction and the growth increment is the same as the one generated under constant amplitude loading. Conversely, the crack growth rate will be reduced (retarded) if $(\Delta a+r_p) \leq r_{pOL}$). The load interaction model developed by Hsu borrowed and built on the approaches offered by Wheeler, Willenborg et al, and Elber. These 3 methods are widely discussed in textbooks today but they were leading edge technology when Hsu put forth his model. For completeness a brief presentation of each will be given to provide a reference to understand the Hsu model process.

1.1.1 The Wheeler Model, reference 1, 7, and 8

Wheeler modifies baseline constant amplitude da/dN according to the following formula

$$\left(\frac{da}{dN}\right)_{ret} = \phi\left(\frac{da}{dN}\right)_{CA} = \phi\, f(\Delta K);\ \text{where}\ \phi = f(r_p, \text{crack length})^m \qquad (1$$

The ϕ function is the ratio of the current plastic zone size and the remaining size of the plastic enclave formed at an overload raised to 'm'. The exponent 'm' shapes the retardation parameter ϕ to correlate with test data.

$$\phi = \left(\frac{r_{pi}}{a_0 + r_{pOL} - a_i}\right)^m \qquad (2$$

In his experiments Wheeler found values for 'm' of 1.43 for D6AC steel and 3.4 for Ti-6AL-4V titanium. If m=0 then no retardation influence exists. Although the Wheeler model is a substantial improvement over the linear cumulative damage rule, it is more of a data fitting technique than it is a predictive technique. The exponent 'm' has been found to be dependent not only on the material, but also upon the manner in which spectrum loads were applied. The

bottom line is that this model simply shifts the constant amplitude baseline da/dN curve downward. Thus slowing down the crack growth rate. Figure 1 shows the affect of equation 1.

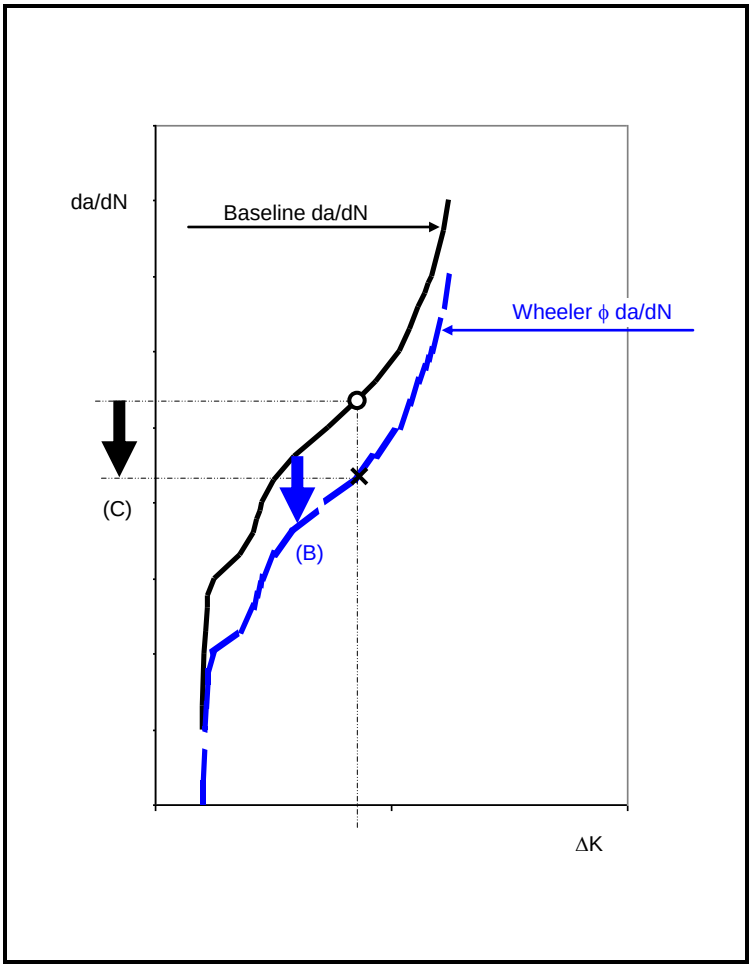

Figure 1

Although Wheeler never went beyond this formulation, it is possible to continue by simple convention. In those days the Paris equation was available and so equation can be expressed as,

$$\left(\frac{da}{dN}\right)_{ret} = \phi\, c_P\, (\Delta K)^{n_P} = c_P\, (\Delta K_{eff})^{n_P}$$

Where

$$\Delta K_{eff} = \phi^{\frac{1}{n_P}}\, \Delta K$$

Further from convention

$$\Delta K_{eff} = K_{maxeff} - K_{mineff}$$

Where

$$K_{maxeff} = \phi^{\frac{1}{n_P}} K_{max}$$

$$K_{mineff} = \phi^{\frac{1}{n_P}} K_{min}$$

The effective stress ratio becomes

$$R_{eff} = \frac{\phi^{\frac{1}{n_P}} K_{min}}{\phi^{\frac{1}{n_P}} K_{max}} = \frac{K_{min}}{K_{max}}$$

Therefore, in terms of stress intensity, Wheeler's relationship can also be understood as shown in Figure 2. It is an equivalent picture to Figure 1. Whether the baseline da/dN curve is factored down as shown by arrow (B) or the ΔK is reduced shown by arrow (A), the same thing is accomplished – a reduction in the crack growth rate arrow (C). The danger in interpretation is that due to the shape of the baseline da/dN curve. In both Figures, the baseline da/dN has been realistically depicted and therefore the direct equivalence of Figure 1 and 2 can be disputed depending where one is located on the da/dN data. However, if strict Region II or Paris straight line was presented then the equivalence would be apparent. The idea to keep in mind is that Wheeler's model holds R-ratio constant and varies ΔK.

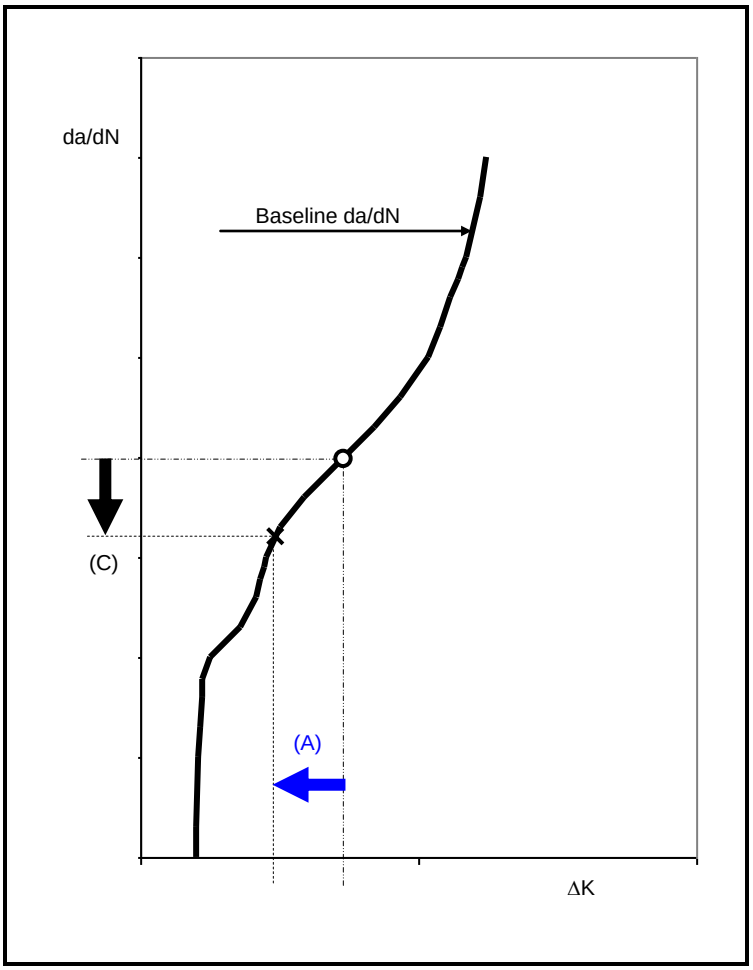

Figure 2

1.1.2 The Willenborg, Engle, and Wood Model, reference 2, 7, and 8

The Willenborg model uses an effective stress concept to reduce the applied stresses and hence the crack tip stress intensity factor. The Willenborg et al model like Wheeler compares plasticity at the crack tip but expresses the plastic zone size and remaining overload plastic enclave in terms of stress or stress intensity. This is an improvement over the Wheeler formulation because both $\Delta K_{effective}$ and $R_{effective}$ can be calculated. The Willenborg et al model reduces the maximum

stress intensity and minimum stress intensity by a reduction stress intensity factor. Willenborg solves for a stress intensity factor that would be required to produce a plastic zone size from the current crack size to the boundary of the overload plastic enclave. The reduction stress intensity factor is set equal to the difference between the required overload stress intensity and the current maximum stress intensity. It is a curious by-product of the formulation, that the current maximum stress intensity reduces itself. This is a consequence of the math not physics but the intent of the model is to reduce the amount of crack growth due to an overload. The Willenborg et al model accomplishes this by reducing the stress ratio, R. The Willenborg et al model reduces both the maximum and minimum stress intensity by the same amount to an effective state and since both are reduced by the same amount, ΔK is unaffected and in fact, $\Delta K_{effective}=\Delta K$. The stress ratio, R, is reduced because $R_{eff}=K_{mineff}/K_{maxeff}$. This is true provided the effective minimum stress is greater than zero; else K_{mineff} is set to zero and then $\Delta K_{effective}$ is less than ΔK. The bottom line is that the Willenborg et al model shifts the constant amplitude baseline da/dN curve to a new lower R-ratio. Thus slowing down the crack. In Figure 3, arrow (B) represents the Willenborg R_{eff} shift and arrow (C) shows the reduction in crack growth rate. The major issue is that Willenborg et al can be calculated whereas Wheeler is empirical. The idea to keep in mind is that Willenborg et al model holds ΔK constant and varies R-ratio.

The basics of the Willenborg et al relationships are as follows.

$$K_{reduction} = K_{max\ req} - K_{max}$$

And

$$K_{max\ req} = [\ (\ a_0 + r_{pOL} - a_i\ )\ F_{TY}^{\ 2}/C\ ]^{0.5}$$

Where

a_0 = Crack length at overload

r_{pOL} = Plastic radius at overload

a_i = Current crack length

$C = \pi$ times constraint

And

$$K_{maxeff} = K_{max} - K_{reduction}$$

$$K_{mineff} = K_{min} - K_{reduction}$$

Which gives

$$\Delta K_{eff} = K_{maxeff} - K_{mineff} = K_{max} - K_{min}$$

And

$$R_{eff} = K_{mineff} / K_{maxeff}$$

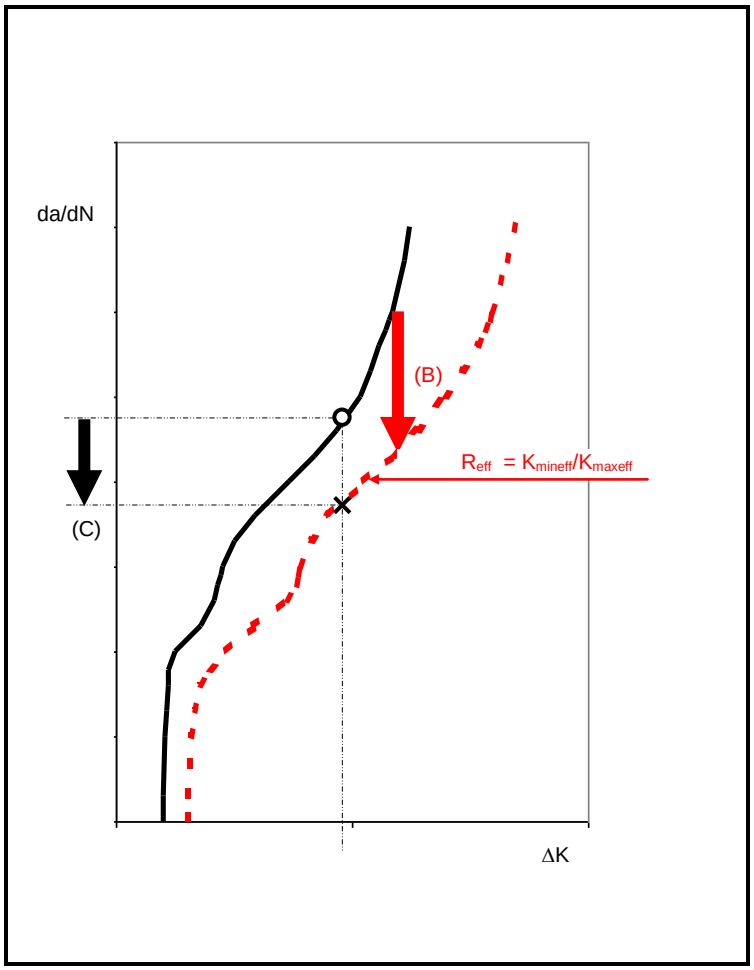

Figure 3

1.1.3 The Elber Model, reference 3 and 7

The Elber crack closure model is an empirically based model that uses an effective stress range concept to incorporate interaction effects in variable amplitude fatigue crack growth life predictions. Elber found that fatigue cracks subjected to plane stress tension-tension loading

close before the remotely applied stress becomes zero. This closing effect is the result of plasticity along the crack flanks. Significant compressive stresses are transmitted across the crack face when the remote load is zero. Elber assumed that crack extension occurs only when the applied stress is greater than the crack opening stress. The stress range that contributes to crack extension is called the effective stress range,

$\Delta\sigma_{eff} = \sigma_{max}-\sigma_{op}$

Where

σ_{op} is the crack tip opening stress determined experimentally.

Defining a closure factor γ as

$\gamma = \sigma_{op}/\sigma_{max}$

Results in

$\Delta\sigma_{eff} = \sigma_{max}(1-\gamma)$

γ is dependent on many factors, including material, thickness, temperature, corrosive environment, and stress peaks.

The effective stress intensity factor can be calculated from the equation,

$\Delta K_{eff} = \Delta\sigma_{eff}$ sqrt(π a) β_T.

The crack growth rate is computed from the growth rate equation by replacing ΔK with ΔK_{eff} in the familiar Paris equation,

$$\frac{da}{dN} = c_P\left(\Delta K_{eff}\right)^{n_P}$$

Where baseline constant amplitude rate data in terms of ΔK and R-ratio must be expressed or transformed into terms of ΔK effective; Elber introduced a closure function U to accomplish this,

$\Delta K_{eff} = U\ \Delta K$

Where

$U = f(R, \ldots)$,

The closure function U and the normalized opening stress function, γ, (closure factor) are related as follows,

$\gamma = 1 - (1-R)\ U$

Elber's model requires performing baseline da/dN tests for a range of R-ratios in order to develop the closure function U, but once found, any ΔK and R can be transformed to ΔK_{eff} using γ or U. Elber's formulation generally works fairly well in Region II (the Paris region) but breaks down in Region I and III where closure is dependent crack size and geometrical constraint, not just R-ratio. But in the late 1960's these were subtle nuances and remember the only available crack growth law was Paris. Figure 4 presents an example of transforming ΔK into ΔK_{eff} for 7075-T6511 extrusion AFGROW data at several R-ratios by use of de Koning's, reference 9, 7075-T6 U-function.

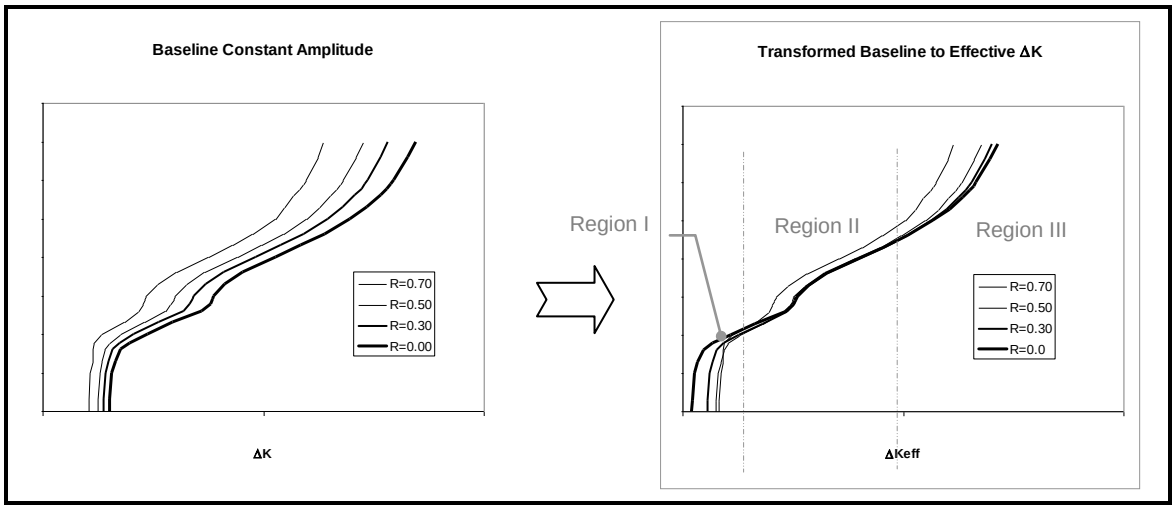

Figure 4

It can be seen that R-ratios of 0.0, 0.30, and 0.50 are fairly consolidated in Region II but are not in Region I and III. R-ratio of 0.70 does not consolidate nearly as well. This figure was presented to give an idea of the downside involved in closure models in spite of the effort that has to be taken to obtain the data. As was stated 7075-T6511 extrusion baseline data was obtained from AFGROW (Harter T-Method). The U-function was obtained for 7075-T6. Therefore, a reason that the ΔK does not completely consolidate in Region II is because the U-function was not derived on the baseline reference 7075-T6511 extrusion data, if it were then ΔK_{eff} would consolidate in Region II for the R-ratios considered. This points out another aspect, that the closure function U is very sensitive to material form.

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In summary, these 3 retardation models may be somewhat artificial; they do contain the relevant parameters. The Wheeler model reduces ΔK, the Willenborg et al reduces R-ratio, and the Elber model transforms both ΔK and R-ratio. All these models are based on plasticity considerations and it should be emphasized for clarity that, with increasing crack size (plastic zone size), it becomes increasingly difficult for the elastic material to restore the zero strain field after unloading. Therefore, in addition to the parameters already mentioned, it must be remembered that retardation is crack size and panel size dependent and therefore requires more than equal stress intensity, reference 8.

1.1.4 Hsu Model

The Hsu model originally developed at Lockheed Georgia uses an effective stress and closure concept. The model is not only capable of accounting for the retardation effect due to tensile overload, but also accounts for the effect of the compression portions of tension-compression load cycles on the fatigue crack growth rate during subsequent load cycles. The current Hsu model is unable to account for compression-compression cycles, although it is known that a compressive-compressive load cycle will negate the retardation of crack growth due to tensile overload, reference 8. In general, overloads decelerate or retard while underloads accelerate crack growth.

The Hsu model assumes that the stress singularity does not exist if the crack surface is closed, and that the crack propagates only during that portion of the load cycle in which the crack surface is fully open. The effective stress range of the load cycle during crack propagation is defined as,

$$\Delta\sigma_{eff} = \sigma_{max}-\sigma_0,$$

Where

σ_0 is the far-field stress corresponding to the onset of crack opening.

When the opening stress is less than the minimum stress of the applied load cycle, the effective stress range is given by,

$$\Delta\sigma_{eff} = \sigma_{max}-\sigma_{min}.$$

If the maximum stress of the applied load cycle is less than the crack opening stress due to prior loads, the crack surface will be fully closed and the fatigue crack will not propagate. However, experimental evidence indicates that, below the closure K, strain concentration in the vicinity of the crack tip still exists. Since fatigue damage is normally related to the cyclic strain range, the effective K_{min} is likely to be somewhat lower than the level at the onset of closure and higher than K_{min} under steady state (constant amplitude) condition. The effective stress range and effective load R-ratio can than be rewritten as,

$\Delta\sigma_{eff} = \sigma_{max}-(\sigma_{min})_{eff}$, and

$R_{eff} = (\sigma_{min})_{eff}/\sigma_{max}$ respectively,

Where $\sigma_{min} < (\sigma_{min})_{eff} < \sigma_0$.

The effective stress intensity factor can be calculated from the equation,

$\Delta K_{eff} = \Delta\sigma_{eff}\ \mathrm{sqrt}(\pi\ a)\ \beta_T$.

The crack growth rate is computed from the growth rate equation,

$da/dN = f(\Delta K_{eff}, R_{eff},\ldots)$

As can be seen, Hsu's equations are similar to the form of equations presented for Elber's crack closure model. Analytical experimental correlations of the Hsu model were historically made using the Forman's crack growth rate equation rather than Paris. The form of Forman's equation used was,

$$\frac{da}{dN} = \frac{c_f(\Delta K)^{n_f}}{(1-R)FK_c - \Delta K}$$

Where

F_{Kc} = fitting parameter based partly on test and experience (not the fracture toughness of the material, as a rule around 4 times the average plane strain fracture toughness).

It is important to remember that the Forman equation merely shifts constant amplitude baseline da/dN data up and down within the range of ΔK values used to define it. This is shown in Figure 5. The item to make note of is that the resultant sideways shift is constant. For most materials, the shape of da/dN changes as R-ratio changes from zero. For example, usually Region II exhibits a rotation and Region III a shift along with a change in slope, whilst Region I is relatively unaffected with slight shift as shown in left hand side Figure 4. Furthermore, the

rotation and shift are not constant with R-ratio. Using the above form of Forman relationship, there can be no rotation in Region II or shifting in Region III as R-ratio is increased or decreased from the baseline reference as shown in Figure 5. The da/dN keeps the same shape. In other words, as discussed and presented in the Elber presentation above, most da/dN data shift sideways non-linearly with R-ratio, i.e., as the ΔK increases the sideways shift also increases. Thus the above form of Forman's equation does not model da/dN realistically. In addition, the use of the above form of Forman's equation also indirectly places a limit on the positive R-ratio than can be used within the range of baseline ΔK values. This R-ratio limit is intrinsic in the value chosen for F_{Kc} in the denominator. As the baseline da/dN curve is shifted upward to model positive R-ratios, ΔK values starting on the extreme right and progressing to the left as R-ratio is shifted upward (increased), go out of range and asymptotic assumptions must be made. Also the value, F_{Kc} determines magnitude of the shift that is to be made from one R-ratio to another. The larger the value of F_{Kc} the smaller the shift will be. While the smaller the value of F_{Kc} the smaller the maximum positive R-ratio will be to cover the baseline range of ΔK. It is almost as if one must know the answer before one can get the answer. The selection of F_{Kc} is important, since F_{Kc} tunes how the baseline da/dN data will be transformed. F_{Kc} values ranging from 90 to 150 for aluminum alloys have been used. Unfortunately, there is no documentation of the value of F_{Kc} that was used in the historical Hsu model verification nor is there documentation of how best to determine F_{Kc}. Therefore, it can be seen that not only does the Hsu model account for the effects of variable amplitude loading and compression in tension-compression cycles but also accounts for the deficiencies of Forman equation. Therefore, because R-ratio cutoffs, material exponents, and spectra treatment were based on the Hsu/Forman combination, the effect of using the Hsu model with other expressions for crack growth rate or even parametric data should be verified.

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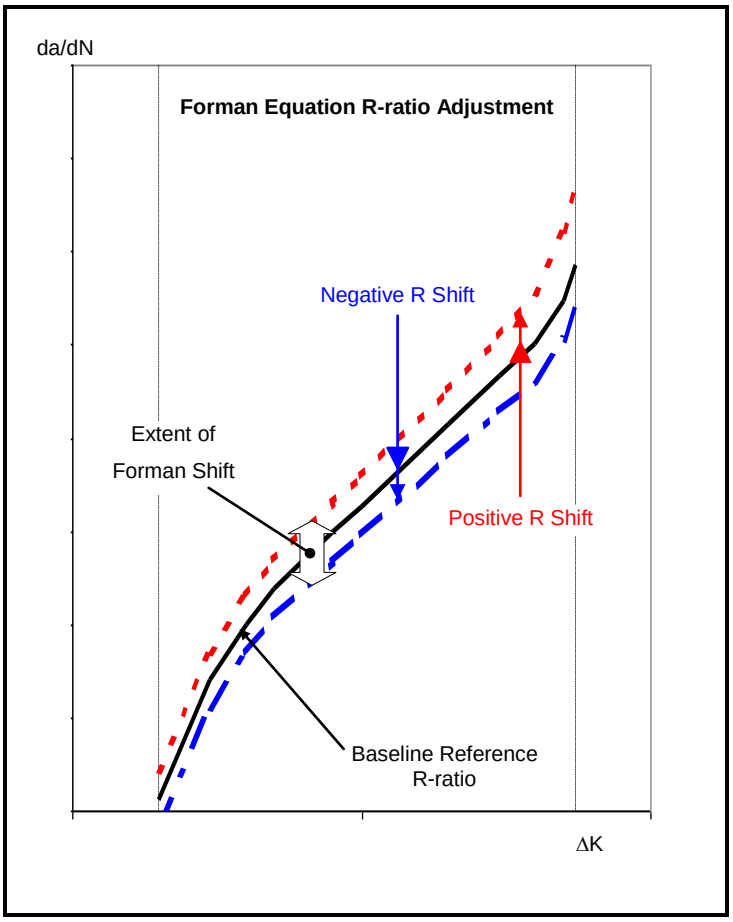

Figure 5

The mechanics of using Forman's equation with Hsu's ΔK_{eff} and R_{eff} are illustrated in Figure 6. This illustration is greatly exaggerated in order to clearly show how the Hsu process works. Hsu translates the circle at coordinate (ΔK, R) to the triangle at coordinate (ΔK_{eff}, R_{eff}). The Hsu reduction from ΔK to ΔK_{eff} is shown by arrow (A). The increase from R to R_{eff} is shown by arrow (B). Note that Hsu's ΔK_{eff} enters the graph as "ΔK", there is no need to transform the baseline constant amplitude da/dN to an effective basis when using Hsu which is necessary for the Elber model. The net effect is arrow (C) showing a reduction in crack growth rate.

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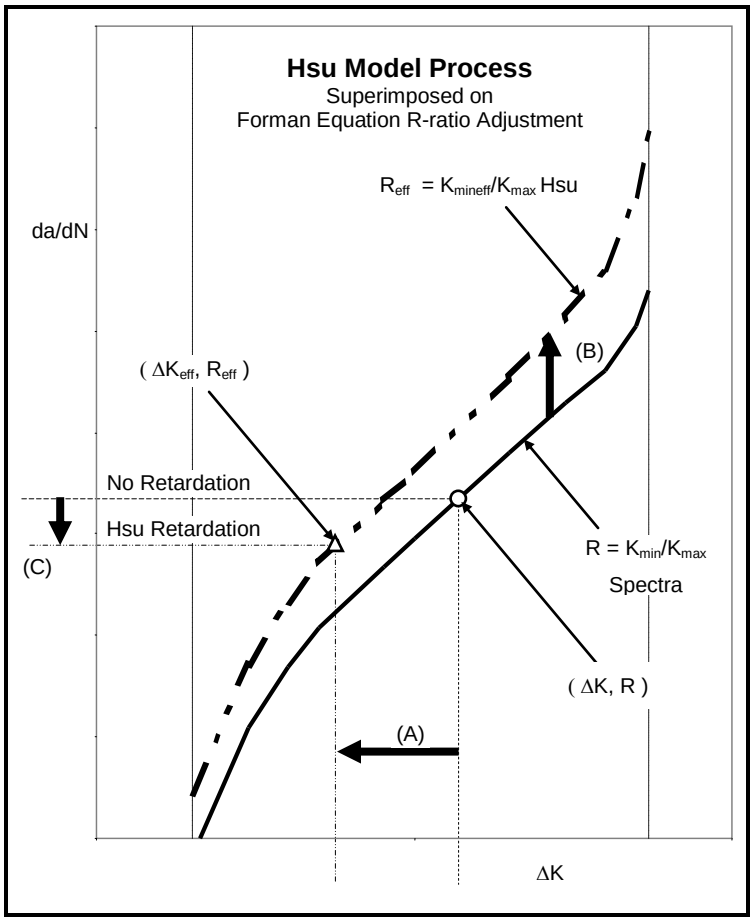

Figure 6

Notice THAT Hsu increases R-ratio and reduces ΔK to R_{eff} and ΔK_{eff} respectively, but keep in mind these are different than Willenborg et al R_{eff} and Elber's ΔK_{eff}, this is why Hsu said his model is based on closure and effective stress 'concepts'. The Hsu model can be seen to contain elements of Wheeler (change ΔK), Willenborg (change R-ratio), and Elber (change only the minimum stress).

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2 IMPLEMENTATION FLOW DIAGRAMS

This section presents FLOW chart implementations. Figure 7 starts out the FLOW chart presentations with a Non-Retarded AFGROW Flow Chart. Figure 8 presents a Generic Hsu Model Crack Growth Flow Chart. Figure 8 Flow Chart was implemented in AFGROW. Figures 9 and 10, present the equivalent C-5 FLOW charts. Figure 9 presents the initializations and spectra checks and stress intensity truncation used before entering into the actual Hsu model process. Figure 10 presents the actual Hsu model process implementation. Figures 9 and 10, are presented in order to show that correlation to C-5 crack growth prediction will require more than Hsu, establish the foundation and to see the variables that are involved. In other words on the surface the implementation of Hsu should be straightforward but there are subtle aspects which have to be considered especially if correlation or compatibility to LMAS is desired, specifically in this case cutoff of Kmax greater than Kc and reordering of the spectra to start on the maximum stress.

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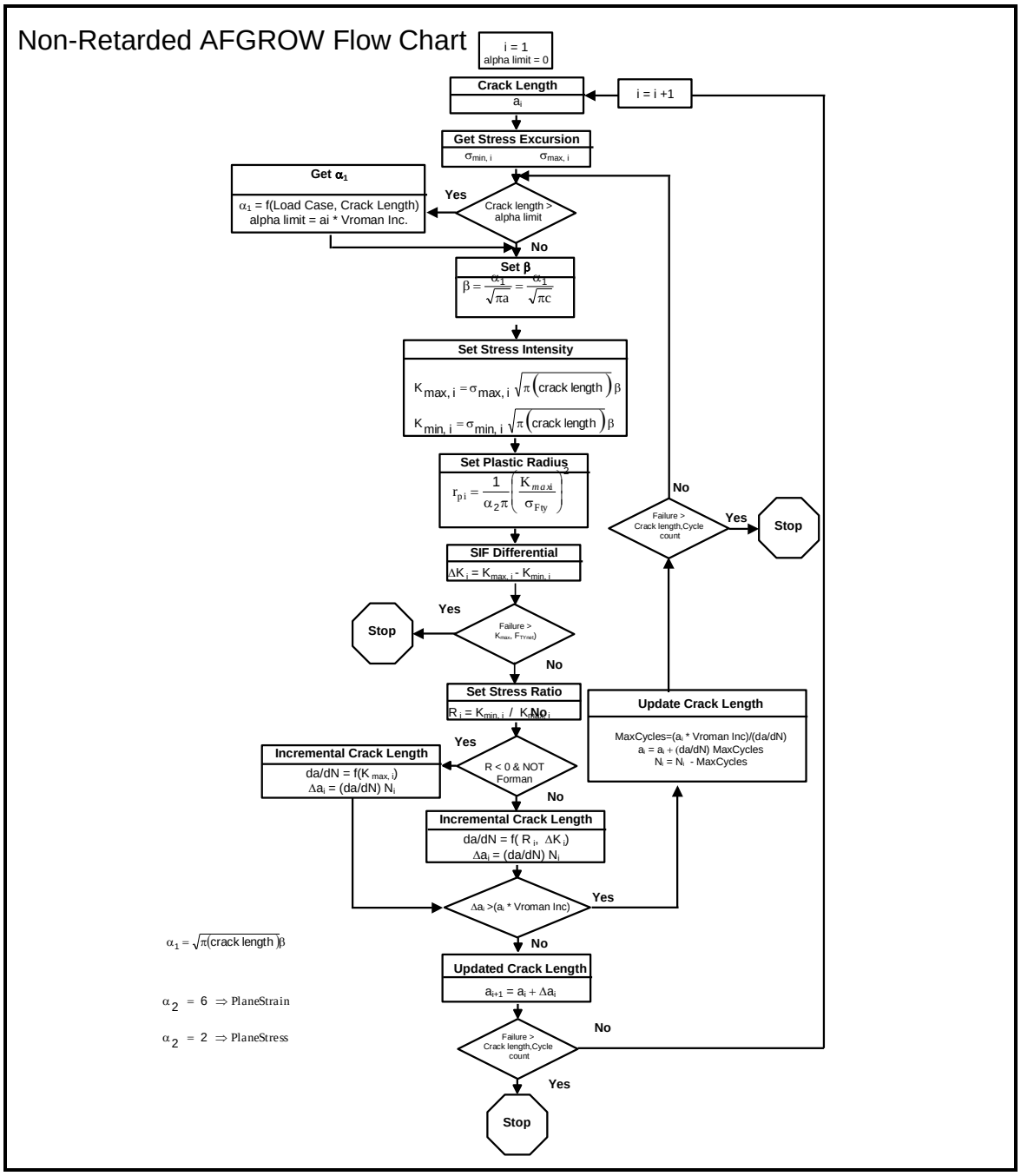
Non-Retarded AFGROW Flow Chart
i = 1
alpha limit = 0
Crack Length
a_i
i = i +1
Get Stress Excursion
$\sigma_{min,\,i}$ $\sigma_{max,\,i}$
Get α_1
α_1 = f(Load Case, Crack Length)
alpha limit = ai * Vroman Inc.
Yes
Crack length > alpha limit
No
Set β
$\beta = \frac{\alpha_1}{\sqrt{\pi a}} = \frac{\alpha_1}{\sqrt{\pi c}}$
Set Stress Intensity
$K_{max,\,i} = \sigma_{max,\,i}\sqrt{\pi(\text{crack length})}\beta$
$K_{min,\,i} = \sigma_{min,\,i}\sqrt{\pi(\text{crack length})}\beta$
Set Plastic Radius
$r_{p\,i} = \frac{1}{\alpha_2\pi}\left(\frac{K_{max\,i}}{\sigma_{Fty}}\right)^2$
No
Failure > Crack length,Cycle count
Yes
Stop
SIF Differential
$\Delta K_i = K_{max,\,i} - K_{min,\,i}$
Yes
Stop
Failure > K_{max}, $F_{Ty/ult}$
No
Set Stress Ratio
$R_i = K_{min,\,i} / K_{max\,i}$
No
Update Crack Length
MaxCycles=(a_i * Vroman Inc)/(da/dN)
$a_i = a_i$ + (da/dN) MaxCycles
$N_i = N_i$ - MaxCycles
Yes
Incremental Crack Length
da/dN = f($K_{max,\,i}$)
Δa_i = (da/dN) N_i
R < 0 & NOT Forman
No
Incremental Crack Length
da/dN = f(R_i, ΔK_i)
Δa_i = (da/dN) N_i
Yes
Δa_i >(a_i * Vroman Inc)
No
$\alpha_1 = \sqrt{\pi(\text{crack length})}\beta$
$\alpha_2 = 6 \Rightarrow$ PlaneStrain
$\alpha_2 = 2 \Rightarrow$ PlaneStress
Updated Crack Length
$a_{i+1} = a_i + \Delta a_i$
No
Failure > Crack length,Cycle count
Yes
Stop

Figure 7

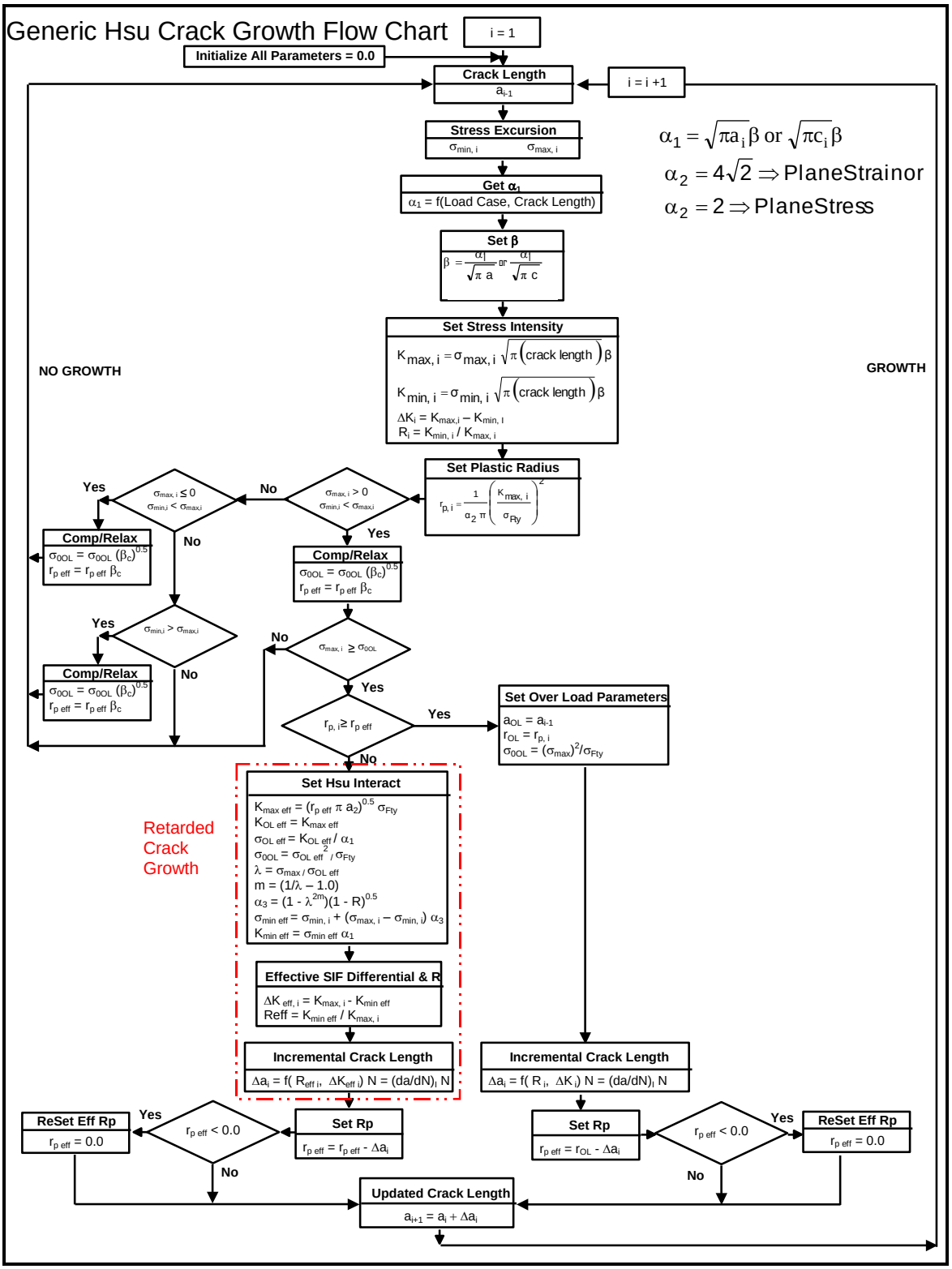
Generic Hsu Crack Growth Flow Chart
i = 1
Initialize All Parameters = 0.0
Crack Length
a_{i-1}
i = i +1
Stress Excursion
$\sigma_{min,\,i}$ $\sigma_{max,\,i}$
Get α_1
α_1 = f(Load Case, Crack Length)
Set β
$\beta = \frac{\alpha_1}{\sqrt{\pi\, a}}$ or $\frac{\alpha_1}{\sqrt{\pi\, c}}$
$\alpha_1 = \sqrt{\pi a_i}\beta$ or $\sqrt{\pi c_i}\beta$
$\alpha_2 = 4\sqrt{2} \Rightarrow$ PlaneStrainor
$\alpha_2 = 2 \Rightarrow$ PlaneStress
Set Stress Intensity
$K_{max,\,i} = \sigma_{max,\,i}\sqrt{\pi(\text{crack length})}\beta$
$K_{min,\,i} = \sigma_{min,\,i}\sqrt{\pi(\text{crack length})}\beta$
$\Delta K_i = K_{max,i} - K_{min,\,i}$
$R_i = K_{min,\,i} / K_{max,\,i}$
NO GROWTH
GROWTH
Set Plastic Radius
$r_{p,\,i} = \frac{1}{\alpha_2 \pi}\left(\frac{K_{max,\,i}}{\sigma_{Fty}}\right)^2$
$\sigma_{max,\,i} \le 0$ $\sigma_{min,i} < \sigma_{max,i}$
Yes
No
$\sigma_{max,\,i} > 0$ $\sigma_{min,i} < \sigma_{max,i}$
Comp/Relax
$\sigma_{0OL} = \sigma_{0OL}\,(\beta_c)^{0.5}$
$r_{p\,eff} = r_{p\,eff}\,\beta_c$
$\sigma_{min,i} > \sigma_{max,i}$
$\sigma_{max,\,i} \ge \sigma_{0OL}$
$r_{p,\,i} \ge r_{p\,eff}$
Set Over Load Parameters
$a_{OL} = a_{i-1}$
$r_{OL} = r_{p,\,i}$
$\sigma_{0OL} = (\sigma_{max})^2/\sigma_{Fty}$
Retarded Crack Growth
Set Hsu Interact
$K_{max\,eff} = (r_{p\,eff}\,\pi\,a_2)^{0.5}\,\sigma_{Fty}$
$K_{OL\,eff} = K_{max\,eff}$
$\sigma_{OL\,eff} = K_{OL\,eff} / \alpha_1$
$\sigma_{0OL} = \sigma_{OL\,eff}^2 / \sigma_{Fty}$
$\lambda = \sigma_{max} / \sigma_{OL\,eff}$
$m = (1/\lambda - 1.0)$
$\alpha_3 = (1 - \lambda^{2m})(1 - R)^{0.5}$
$\sigma_{min\,eff} = \sigma_{min,\,i} + (\sigma_{max,\,i} - \sigma_{min,\,i})\,\alpha_3$
$K_{min\,eff} = \sigma_{min\,eff}\,\alpha_1$
Effective SIF Differential & R
$\Delta K_{eff,\,i} = K_{max,\,i} - K_{min\,eff}$
Reff = $K_{min\,eff} / K_{max,\,i}$
Incremental Crack Length
$\Delta a_i = f(R_{eff\,i},\ \Delta K_{eff\,i})\ N = (da/dN)_i\ N$
Incremental Crack Length
$\Delta a_i = f(R_i,\ \Delta K_i)\ N = (da/dN)_i\ N$
ReSet Eff Rp
$r_{p\,eff} = 0.0$
$r_{p\,eff} < 0.0$
Set Rp
$r_{p\,eff} = r_{p\,eff} - \Delta a_i$
Set Rp
$r_{p\,eff} = r_{OL} - \Delta a_i$
$r_{p\,eff} < 0.0$
ReSet Eff Rp
$r_{p\,eff} = 0.0$
Updated Crack Length
$a_{i+1} = a_i + \Delta a_i$

Figure 8

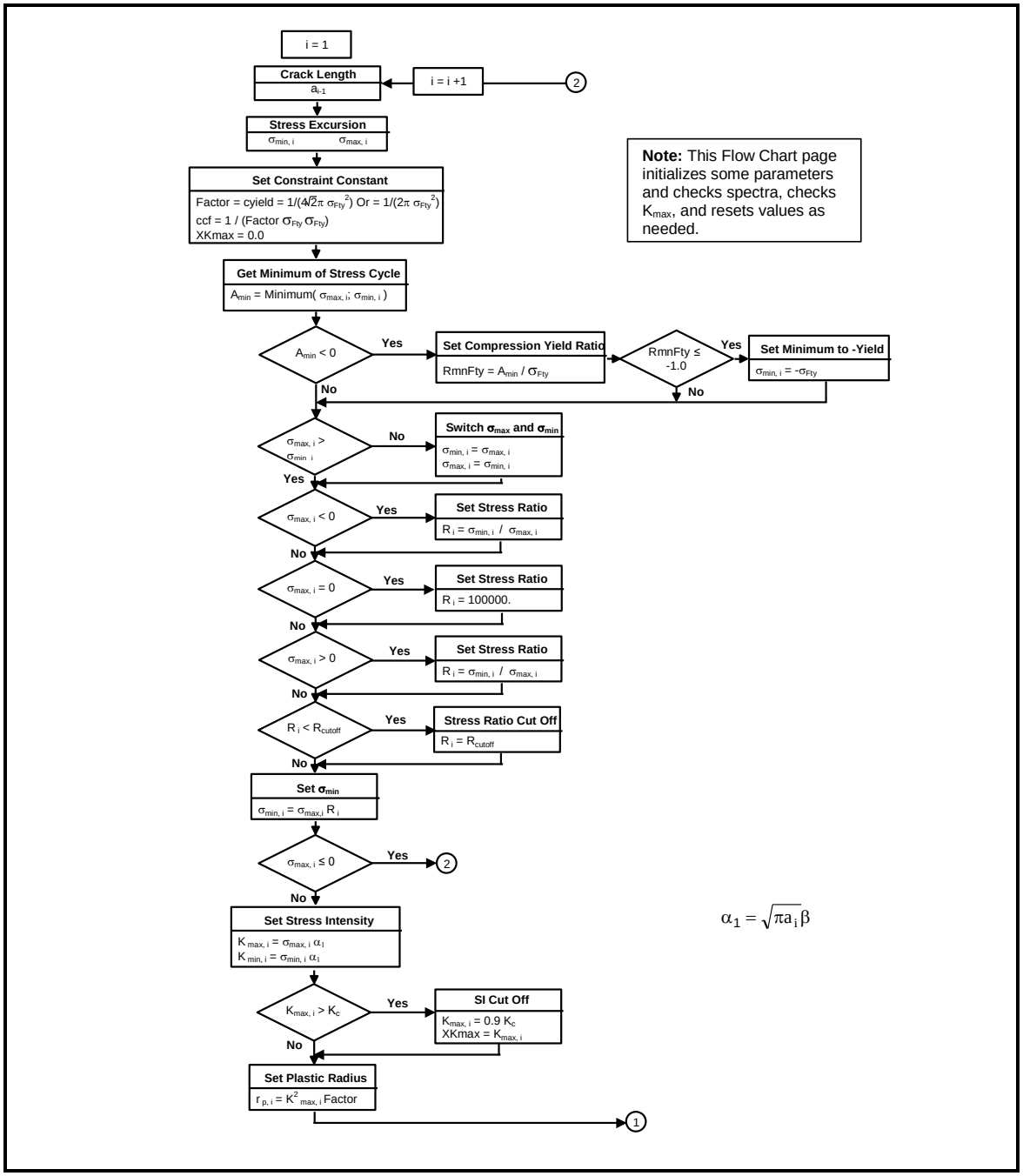
i = 1
Crack Length
a_{i-1}
i = i +1
2
Stress Excursion
$\sigma_{min,\,i}$ $\sigma_{max,\,i}$
Set Constraint Constant
Factor = cyield = 1/(4/2π σ_{Fty}^2) Or = 1/(2π σ_{Fty}^2)
ccf = 1 / (Factor σ_{Fty} σ_{Fty})
XKmax = 0.0
Get Minimum of Stress Cycle
A_{min} = Minimum($\sigma_{max,\,i}$, $\sigma_{min,\,i}$)
A_{min} < 0
Yes
Set Compression Yield Ratio
RmnFty = A_{min} / σ_{Fty}
RmnFty ≤ -1.0
Yes
Set Minimum to -Yield
$\sigma_{min,\,i}$ = -σ_{Fty}
No
No
$\sigma_{max,\,i}$ > $\sigma_{min,\,i}$
No
Switch σ_{max} and σ_{min}
$\sigma_{min,\,i}$ = $\sigma_{max,\,i}$
$\sigma_{max,\,i}$ = $\sigma_{min,\,i}$
Yes
$\sigma_{max,\,i}$ < 0
Yes
Set Stress Ratio
R_i = $\sigma_{min,\,i}$ / $\sigma_{max,\,i}$
No
$\sigma_{max,\,i}$ = 0
Yes
Set Stress Ratio
R_i = 100000.
No
$\sigma_{max,\,i}$ > 0
Yes
Set Stress Ratio
R_i = $\sigma_{min,\,i}$ / $\sigma_{max,\,i}$
No
R_i < R_{cutoff}
Yes
Stress Ratio Cut Off
R_i = R_{cutoff}
No
Set σ_{min}
$\sigma_{min,\,i}$ = $\sigma_{max,i}$ R_i
$\sigma_{max,\,i}$ ≤ 0
Yes
2
No
Set Stress Intensity
$K_{max,\,i}$ = $\sigma_{max,\,i}$ α_1
$K_{min,\,i}$ = $\sigma_{min,\,i}$ α_1
$K_{max,\,i}$ > K_c
Yes
SI Cut Off
$K_{max,\,i}$ = 0.9 K_c
XKmax = $K_{max,\,i}$
No
Set Plastic Radius
$r_{p,\,i}$ = $K^2_{max,\,i}$ Factor
1
Note: This Flow Chart page initializes some parameters and checks spectra, checks K_{max}, and resets values as needed.
$\alpha_1 = \sqrt{\pi a_i}\beta$

Figure 9

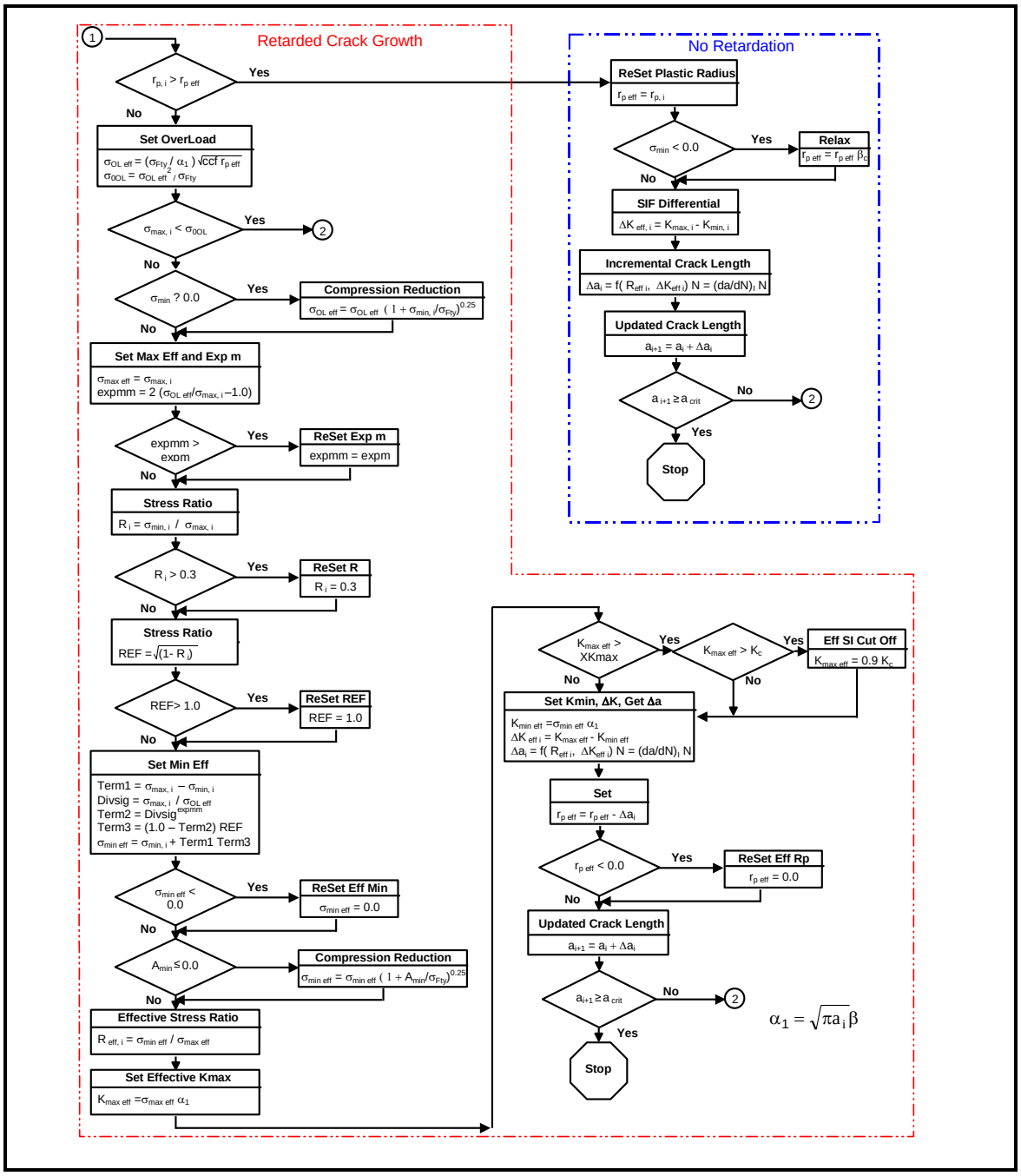
Retarded Crack Growth
No Retardation
Yes
No
Set OverLoad
Compression Reduction
Set Max Eff and Exp m
ReSet Exp m
Stress Ratio
ReSet R
ReSet REF
Set Min Eff
ReSet Eff Min
Compression Reduction
Effective Stress Ratio
Set Effective Kmax
ReSet Plastic Radius
Relax
SIF Differential
Incremental Crack Length
Updated Crack Length
Stop
Eff SI Cut Off
Set Kmin, ΔK, Get Δa
Set
ReSet Eff Rp
Updated Crack Length
Stop

Figure 10

3 HSU MODEL METHOD

This section presents the main derivation of the Hsu formulation and process. A quick time history of the Hsu process is given first. The Hsu process starts by making an innovative assumption by checking both opening stress level and plastic zone size. Therefore, sections are provided to present the definition of the opening stress and effective load interaction zone and plastic zone size. Next, the modification for retardation is presented. This section is exhaustive. Separate issues addressing R-ratio cutoff and closure effects and compression effects are then presented and discussed completely.

3.1 A Time History Account of the Hsu Process

The spectrum is assumed to start on σ_{min} and growth is calculated for the stress (load) going from σ_{min} to σ_{max}. Crack growth occurs for the first half of the load cycle - on up ticks. At the instance of the first half cycle, an opening stress and effective load interaction zone is calculated. The subsequent half cycle – down tick, does not contribute to crack growth. The ensuing up tick half cycles, are processed starting with a check on σ_{max} versus opening stress, σ_{oOL}. If opening stress check is passed, the plastic zone is checked at the end of the half cycle using σ_{max}. Should the current plastic zone be less than the residual effective load interaction zone size, crack growth will be retarded by modifying the minimum stress of the cycle, if not then the residual effective load interaction zone size and opening stress are reset. The minimum stress of the cycle is checked for compression. If it is compressive then corrections are made to both the residual effective load interaction zone size and the required overload stress (i.e., Willenborg et al required stress) and minimum effective stress (if retarded).

3.2 Opening stress

At the instance of the first half load cycle and every overload half cycle thereafter, Hsu calculates an opening stress of overload cycle as follows.

$$\sigma_{oOL} = \frac{\sigma_{max}^2}{\sigma_{Fty}}$$

For subsequent non-overload half cycles or in between half cycles, σ_{oOL} is set to the following.

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$$\sigma_{oOL} = \frac{\sigma_{OL\ eff}^2}{\sigma_{Fty}}$$

Where σ_{OLeff} is the Willenborg et al stress that is required to produce the effective interaction zone, r_{peff}, at the current crack length. It is derived in the next section.

If $\sigma_{max} >= \sigma_{oOL}$ then the stress cycle is considered for crack growth and the process continues to the check on plastic zone size. If $\sigma_{max} < \sigma_{oOL}$ then this cycle is assumed to produce no crack growth and the process continues to the next half cycle. In both instances, the minimum stress is checked for compression and appropriate corrections are made as covered in the compression effect section. Thus this check is a screening or threshold check. The σ_{max} must be greater than σ_{oOL} or there can be no growth. The initial setting can be explored to gain insight into this check by simple factoring.

$$\sigma_{oOL} = \frac{\sigma_{max}^2}{\sigma_{Fty}} \Rightarrow \frac{\sigma_{oOL}}{\sigma_{max}} = \frac{\sigma_{max}}{\sigma_{Fty}}$$

This equation states that the ratio of opening stress to maximum stress is the same as the maximum stress to yield strength. It can be recognized that the maximum spectra stress for transport aircraft could be around 20 KSI and yield strength could be around 60 KSI, so that the ratio of opening stress to maximum stress could be around 0.333. Therefore, the Hsu process only turns away applied load half cycles whose maximum stress is less than 0.333 times 20 or 6.7 KSI but even this number is reduced during intermediate cycles and so even less cycles are turned away. At the time of its creation, computer time was outlandish costing $800 per crack run; therefore Hsu implemented this check in an effort to keep processing costs down. If no similar constraint exists today this check step could be eliminated.

3.3 Effective Load Interaction Zone and Plastic Zone Size

The Hsu model uses a load interaction zone concept based on the Irwin plastic zone model as a criterion to determine whether the crack growth of the current applied half cycle will be altered from that of constant amplitude. Figure 11 shows the basic dimensions while Figures 12 and 13

develop the load interaction zone in terms of Hsu model parameters that are derived on the basis Wheeler.

To start assume that an over load stress occurred. By definition this will have occurred in Figure 11 at a_0 and produced K_{maxOL} which produced an over load plastic zone equal to the following.

$$r_{pOL} = \frac{1}{\alpha \pi}\left[\frac{(K_{max})_{OL}}{\sigma_{Fty}}\right]^2$$

Next assume that the application of a subsequent half cycle produced growth equal to Δa. Then by definition the effective load interaction zone is determined as follows.

$$r_{peff} = r_{pOL} - \Delta a$$

As the crack grows further away from a_0, the load interaction zone, r_{peff} decreases. The plastic zone of the current crack, a_i, is.

$$r_p = \frac{1}{\alpha \pi}\left[\frac{K_{max}}{\sigma_{Fty}}\right]^2$$

If $r_p >= r_{peff}$, there will be no load interaction and the crack growth rate associated with the cycle will be generated as under constant amplitude loading. Conversely, if $r_p < r_{peff}$, then the crack growth rate will be reduced by modifying the minimum stress of the cycle.

At crack length a_i we can associate a stress intensity factor, $K_{max\ eff}$ with the effective interaction zone by solving the following equation.

$$r_{peff} = \frac{1}{\alpha \pi}\left[\frac{K_{max\,eff}}{\sigma_{Fty}}\right]^2 = \frac{1}{\alpha \pi}\left[\frac{K_{OL\,eff}}{\sigma_{Fty}}\right]^2$$

And this stress intensity factor, $K_{max\ eff}$ can be converted into an effective load interaction zone stress, σ_{OLeff} easily as follows.

$$\sigma_{OL\ eff} = \frac{K_{max\,eff}}{\sqrt{\pi a}\,\beta} = \frac{K_{OL\,eff}}{\sqrt{\pi a}\,\beta}$$

This is exactly the same as the required Willenborg et al stress. This is used in the calculation of σ_{oOL} above in the opening stress section.

3.3.1 Load Interaction Zone

Figure 11 shows the relationship of crack sizes and plastic zones, i.e., the load interaction zone.

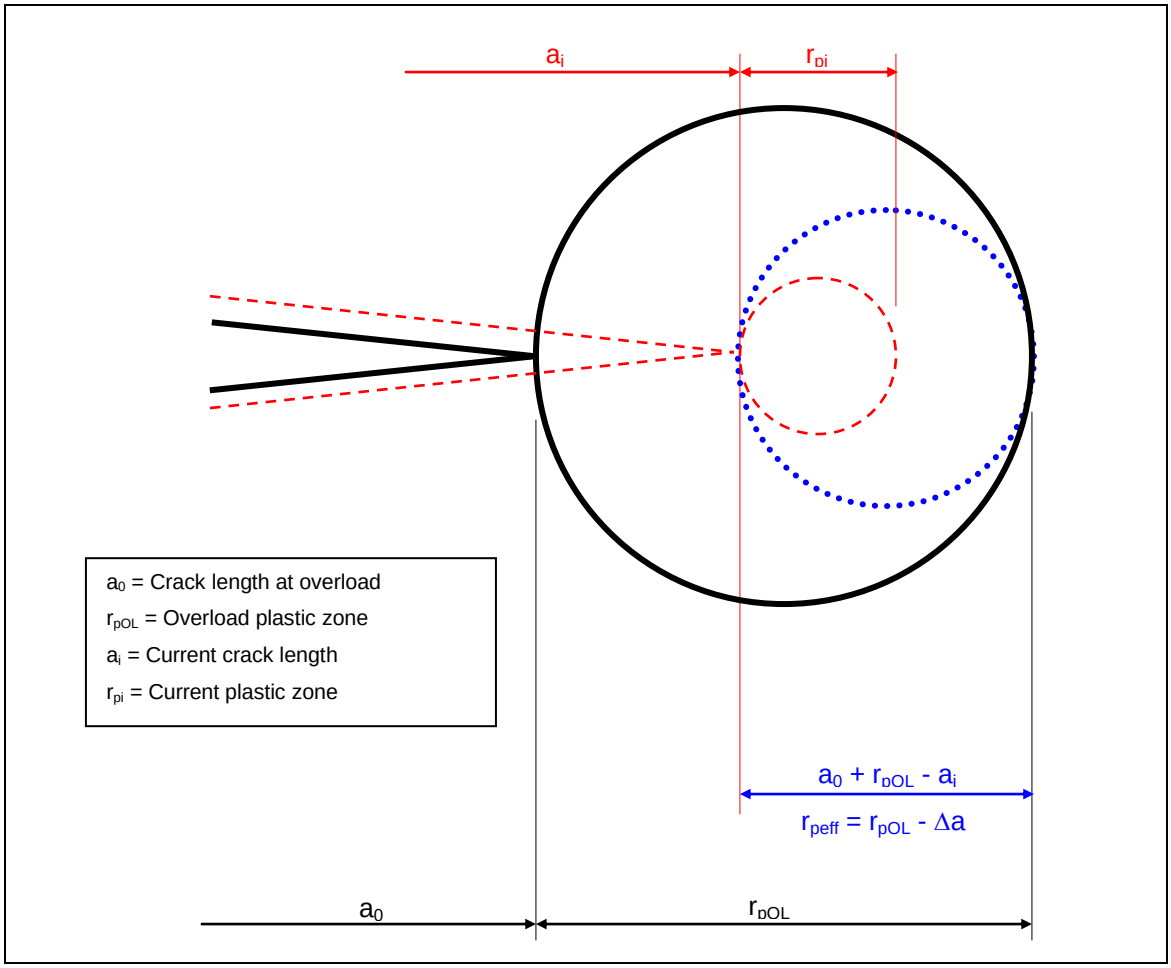

Figure 11

3.4 Modification for Retardation

As stated above, the Hsu formulation modifies the minimum stress of the applied half cycle to take into account variable amplitude load interaction. Therefore, if the plastic zone size of the current half cycle is less than the effective load interaction zone Hsu redefines the minimum stress to be an effective minimum stress as follows.

$$\sigma_{min\ eff} = \sigma_{min\ i} + \alpha\ \Delta\sigma$$

Where

$$\Delta\sigma = \sigma_{max\ i} - \sigma_{min\ i}$$

$$\alpha = \left(1 - \lambda_H^{2m}\right)\sqrt{(1-R)};\ 0 < \alpha < 1.0$$

R must be positive in order to limit α to 1.0

The explanation of α, λ_H and m, requires their own subsection as follows.

3.4.1 α, λ_H, and m

To start, a detailed review of the Wheeler model is necessary and helpful. Remembering that the denominator in Wheeler's model is also equal to the required plastic zone in the Willenborg model as follows.

$$r_{p\ req} = a_0 + r_{pOL} - a_i$$

This is also the same as the effective interaction zone, r_{peff} per Hsu as shown in Figure 11.

Then Wheeler's equation becomes in terms of plastic zone.

$$\phi = \left[\frac{r_{pi}}{r_{p\,req}}\right]^m$$

This can be expressed in terms of stress intensity, K. Since

$$r_{pi} = C\left(\frac{K_{max\,i}}{\sigma_{FTY}}\right)^2$$

$$r_{p\,req} = C\left(\frac{K_{max\,req}}{\sigma_{FTY}}\right)^2$$

Where

$$C = \frac{1}{\alpha\,\pi} \text{ and } \alpha = \text{constraint factor}$$

Then

$$\phi = \left[\frac{r_{pi}}{r_{p\,req}}\right]^m = \left[\left(\frac{K_{max\,i}}{K_{max\,req}}\right)^2\right]^m = \left[\frac{K_{max\,i}}{K_{max\,req}}\right]^{2m}$$

This in turn can be expressed in terms of stress, since

$$K_{max\,i} = \sigma_{max\,i}\sqrt{\pi\,a_i}\ \beta_{(a_i)}$$

And

$$K_{max\,req} = \sigma_{max\,req}\sqrt{\pi a_i}\;\beta_{(a_i)}$$

Finally

$$\phi = \left[\frac{\sigma_{max\,i}}{\sigma_{max\,req}}\right]^{2m} = \left[\left(\frac{\sigma_{max\,i}}{\sigma_{max\,req}}\right)^2\right]^m$$

Hsu defines,

$$\lambda_H = \frac{\sigma_{max\,i}}{\sigma_{max\,req}}$$

Then

$$\phi = \left[\lambda_H{}^2\right]^m$$

In summary, Wheeler's equation may be expressed in the following forms.

$$\phi = \left[\frac{r_{pi}}{r_{p\,req}}\right]^m = \left[\left(\frac{K_{max\,i}}{K_{max\,req}}\right)^2\right]^m = \left[\left(\frac{\sigma_{max\,i}}{\sigma_{max\,req}}\right)^2\right]^m = \left[\lambda_H^2\right]^m$$

The exponent, 'm', in Wheeler's equation is empirically derived to give the best fit to test data. In Wheeler's expression it can be seen that 'm' acts as an effectivity constant on the ratio's of; plasticity, Ks, or stresses, that is 'm' determines how effective the ratio's are. If 'm' equals 1.0, the ratios are unaffected. Hsu formulated an expression in terms of 'm' that does not rely on empirically derived parameters -- except as a limiting case. Remembering that Hsu defines the minimum effective stress as follows.

$$\sigma_{min\,eff} = \sigma_{min\,i} + \alpha\,\Delta\sigma$$

Where

$$\Delta\sigma = \sigma_{max\,i} - \sigma_{min\,i}$$

$$\alpha = \left(1 - \lambda_H^{2m}\right)\sqrt{(1-R)};\; 0 < \alpha < 1.0$$

R must be positive in order to limit α to 1.0

To understand the physical significance of this formulation, Figure 12 presents the plastic zone illustration of Figure 11, with the required plastic zone normalized to 1.0 by $a_0 + r_{pOL} - a_i$ or $r_{p\,required}$. And Figure 13 presents Figure 11 in terms of effectivity, i.e., including 'm'. Load retardation requires that the current plastic zone be less than the required plastic zone or that, r_{pi}

$< a_0 + r_{pOL} - a_i$ or r_{peff} which is the same thing as saying that λ^2_H will always be less than one, this is easily seen in the normalized figures. The normalized current plastic zone is equal to λ^2_H in Figure 12 and the effective normalized current plastic zone equal to λ^{2m}_H in Figure 13. These figures dimension the distance between the current plastic radius and the overload plastic boundary which can be seen to be equal to Hsu's α except for the square root of (1-R) term. The inclusion of the square root term is evidently a correction refinement that provided better correlation to test and suggests that Hsu found that the effect of closure decreased with increasing R-ratio. As R-ratios increase the effect is to reduce α, and reducing α, reduces the effective cyclical stress, which in turn reduces the effective minimum stress, so as R-ratios increase the difference between the effective minimum stress and the minimum stress decreases and in the limit the effective minimum stress equals the minimum stress and there is no load interaction effect. In summary, Hsu bases his formulation on the available plasticity ahead of the current plastic zone to the overload plastic boundary and modifies its effectivity by 'm' and square root of (1-R). Hsu formulates 'm' as follows.

$$m = \frac{1}{\lambda_H} - 1 \leq m_0$$

Where

m_0 = The limiting value where the delay in crack growth starts to decrease or where the effect of retardation starts being reduced.

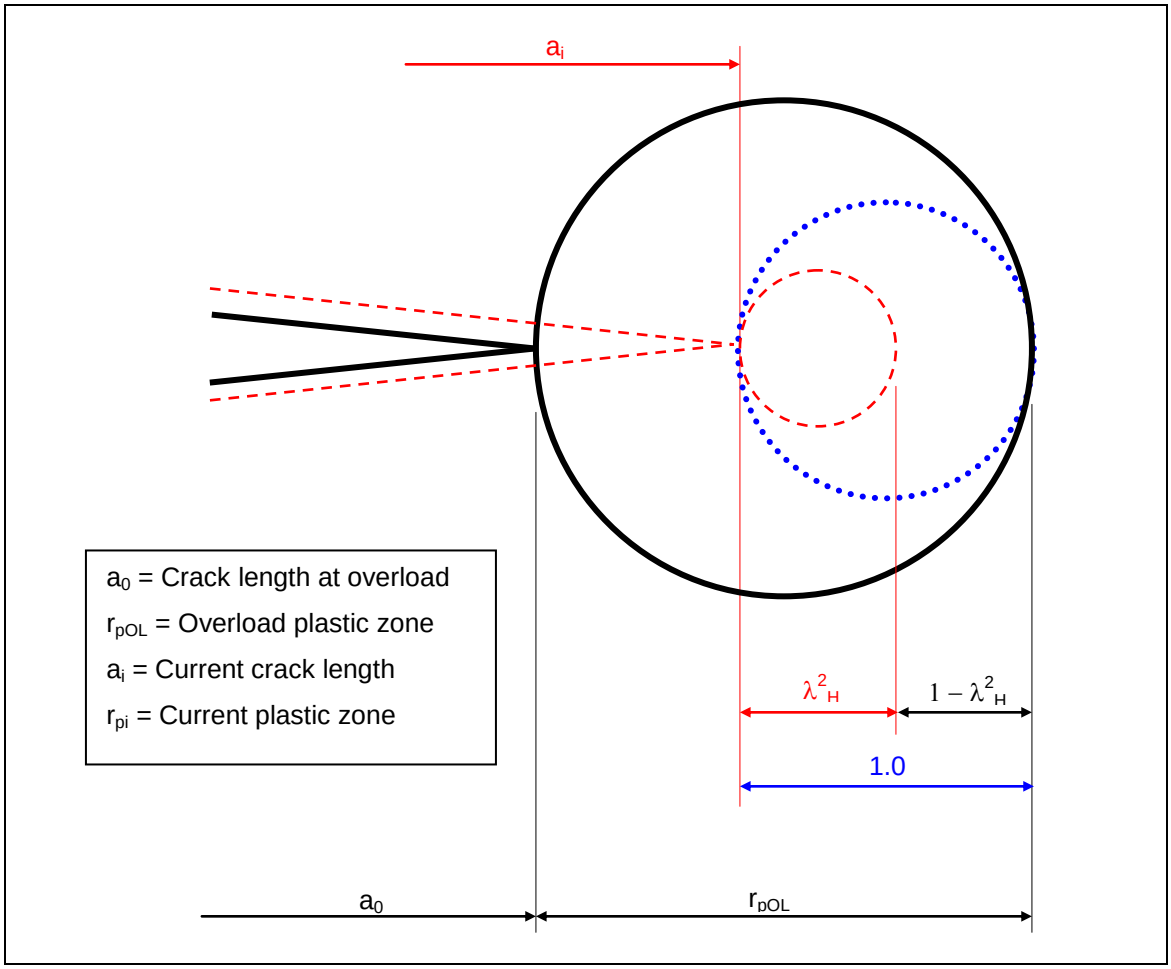
a_i
a_0 = Crack length at overload
r_{pOL} = Overload plastic zone
a_i = Current crack length
r_{pi} = Current plastic zone
λ^2_H
$1 - \lambda^2_H$
1.0
a_0
r_{pOL}

Figure 12

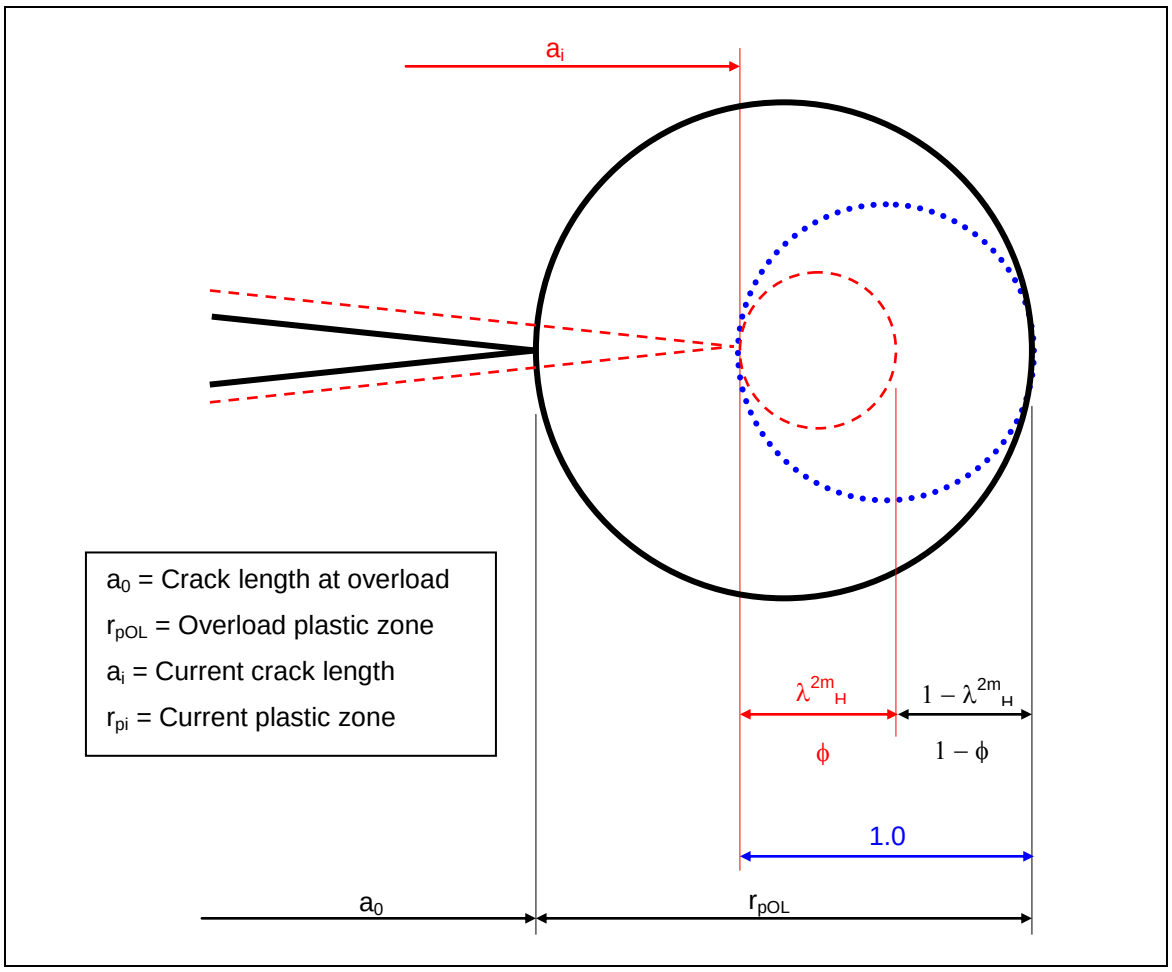

Figure 13

Figures 13 shows that the Wheeler's ϕ is an expression of the proportion of the current plastic zone to the remaining overload plastic zone while Hsu's α is an expression of the proportion of the plasticity between the current plastic zone and the overload plastic boundary times the square root of (1-R). Therefore, Hsu can be expressed in terms of Wheeler as follows.

$$\alpha = (1-\phi)\sqrt{(1-R)}$$

Provided the appropriate expression for 'm' is used.

It should be obvious that the Hsu 'm' is not equal to the Wheeler 'm'. The Hsu 'm' is an expression, which has a shut off value of m_0. Hsu modifies each $\lambda = (\sigma_{max\ i}/\sigma_{max\ req})$ ratio in the spectrum *differently* provided 'm < m_0'. The Wheeler 'm' modifies every ratio *equally*. The determination of 'm_0' from test data is dependent on the 'm' expression as well as the square root of (1-R). 'm_0' is essentially a tuning factor to adjust the acceleration or deceleration of

retardation of the overall spectrum and material. So while Hsu is an improvement over Wheeler and Willenborg et al in that the parameters can be calculated it is still to a degree empirically based due to the dependency on 'm_0' in the limit.

It is insightful to plot m, ϕ, 1-ϕ, as functions of λ_H as shown in Figure 14 and to plot α in Figure 15. Notice that in these presentations 'm' is not restricted to a limiting 'm_0'.

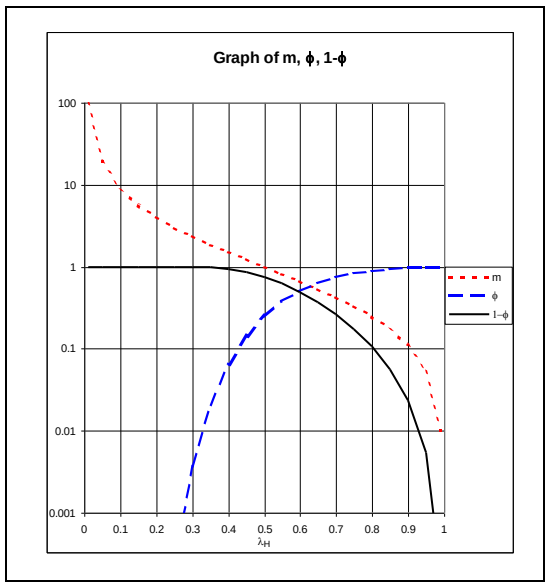

Figure 14

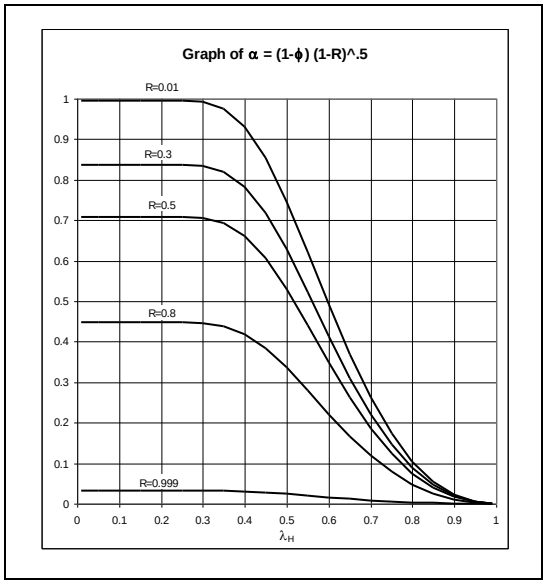

Figure 15

Figure 15 indicates that α varies between 0.0 and 1.0 by ensuring that the R-ratio varies between 0.999 and 0.01, or essentially 1.0 and 0.0. In other words, the R-ratio is not allowed to go negative else the square root (1-R) term would be greater than 1.0 and when multiplied by (1-ϕ) would produce an α greater than 1.0 which when factored by $\Delta\sigma$ would produce a $\sigma_{min\ eff}$ greater than $\sigma_{max\ i}$, by definition this can not be allowed to happen. Notice also that as R-ratio increases, α decreases and that as λ_H increases α decreases. This means that as the maximum stress, $\sigma_{max\ i}$, approaches the maximum required stress, $\sigma_{max\ req}$ (the stress that is required to produce the plastic zone at the current crack length), α tends to zero. This is the same thing as saying the retardation goes to zero and the minimum effective stress is equal to the minimum stress, $\sigma_{min\ eff} = \sigma_{min\ i}$. On the other hand when λ_H tends to zero, α tends to its maximum for the given R-ratio. This means that when the maximum stress, $\sigma_{max\ i}$, is a small fraction of the maximum required stress, $\sigma_{max\ req}$,

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retardation is maximized and the minimum effective stress approaches the maximum stress minimizing the effective cyclic stress, that is $\Delta\sigma_{eff}$ is at a minimum. It is a little more complicated than this because at the same time this is happening the effective R-ratio is increasing but the net effect is a decrease in da/dN, which is the desired result.

Figures 14 and 15 will be affected by the 'm_0' limit on 'm', remembering that Hsu does not allow 'm' to increase indefinitely, he found the rate of retardation would start to decrease. In general, the higher the overload the greater the retardation effect will be. It is conceivable that if the spectrum contains an absurdly high overload, crack growth could be all but stopped. However, Hsu observed that there appears to be a limit to the amount of retardation an overload could produce. Hsu accounted for this limit in shunting 'm' by 'm_0'. Hsu acknowledged 'm_0' to be a function of material and spectra. For 7075-T6, Ti-6Al-4V, and 2219-T851 materials, 'm_0' is approximately equal to 1.0, while for 7050-T73, 'm_0' is about 0.6, and for D6AC 'm_0' is about 1.5. These 'm_0's were determined as follows; 7050-T73 aluminum was run with a C-141 flight-by-flight spectra (with tension-compression and compression-compression cycles and a spectra without tension-compression and compression-compression cycles), 2219-T851 aluminum was run with a fighter (containing tension-compression cycles) and a bomber (containing compression-compression cycles) spectra, D6AC steel was run for an F-111 spectrum (no compression). No warrant can be made today if these 'm_0' values are reflective of current air force usage or materials. The following figures present the effects of 'm_0' on Figures 14 and 15. Figures 16 and 17, present m_0=1.5, Figures 18 and 19, present m_0=1.0, and Figures 20 and 21, present m_0=0.6.

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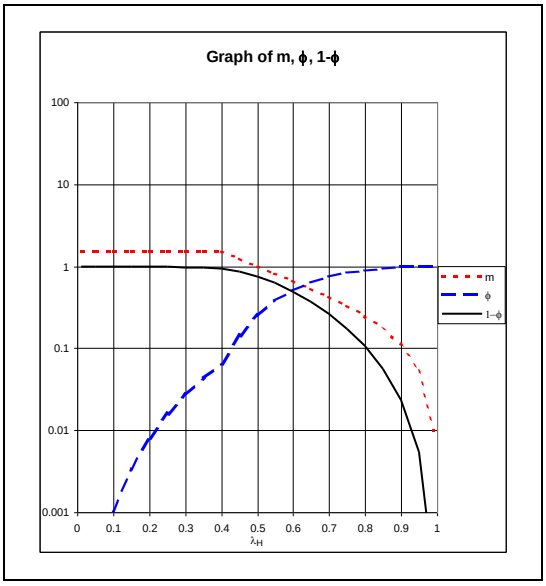

Figure 16, m_0=1.5

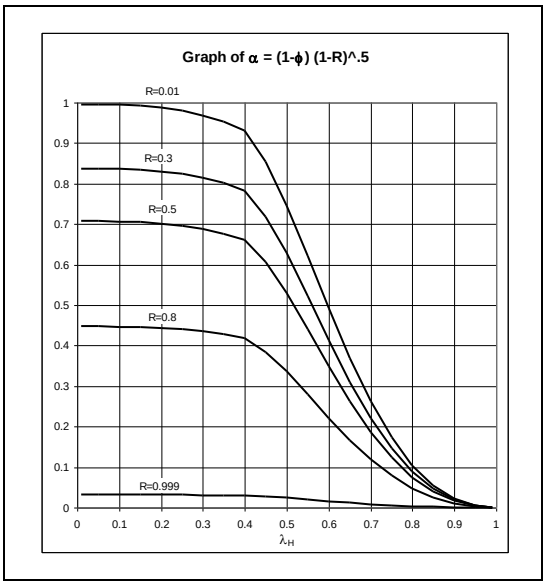

Figure 17, m_0=1.5

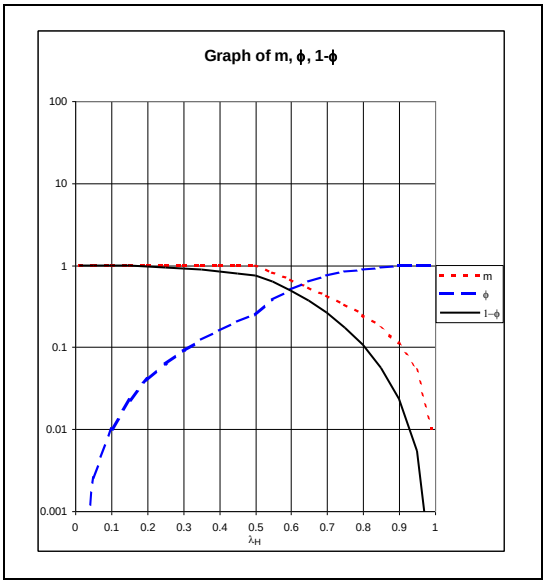

Figure 18, m_0=1.0

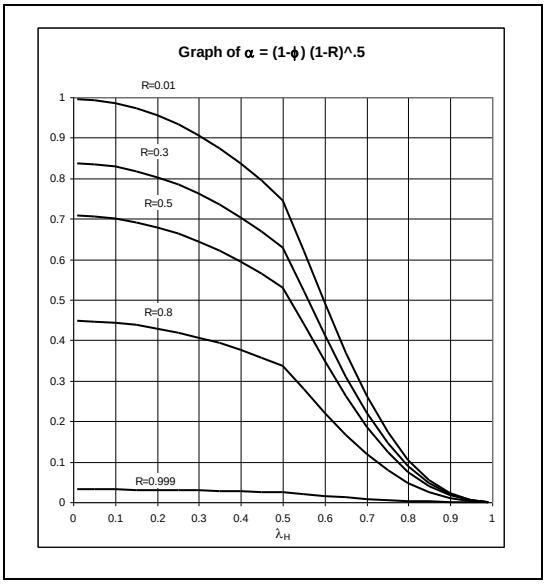

Figure 19, m_0=1.0

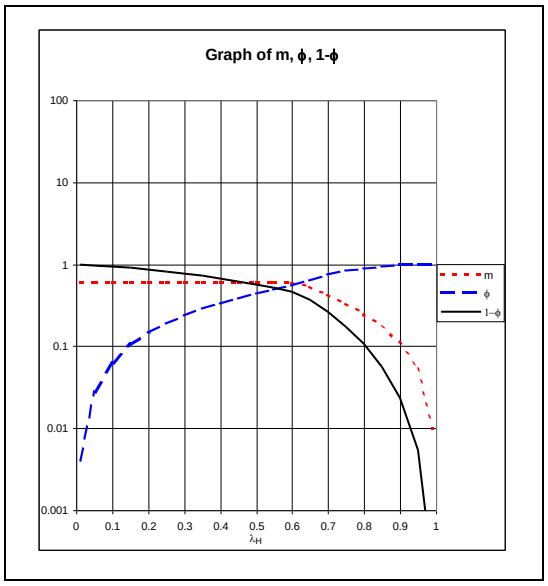

Figure 20, m_0=0.6

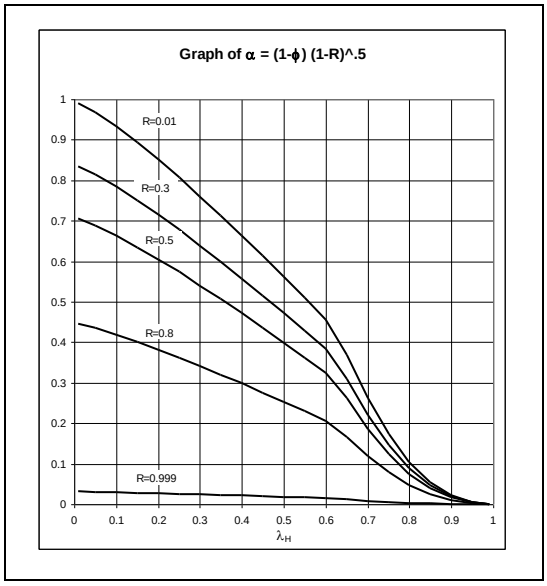

Figure 21, m_0=0.6

Notice by including the 'm_0' limit on 'm', a knee results in the α versus λ_H plot, Figures 17, 19, and 21. Notice also that the slope in the α plot is changed below the knee resulting in a reduced value for α, and for λ_H values above the knee, α is unaffected.

In summary, the Hsu 'm' and Wheeler 'm' have the same basic formulation however, Hsu modifies 'm' for each load cycle provided 'm' is less than m_0. Because m_0 is test based, the Hsu model like all the previous load interaction models is still empirical.

3.5 R-ratio Cutoff and Closure Effects

Historically Hsu found that Shih and Wei reference 10, conducted a study on crack closure in fatigue for Ti-6Al-4V titanium and observed no crack closure for R-ratio greater than 0.3. The statement that no closure exists above a certain R-ratio can be interpreted today in terms of the γ function versus R-ratio graph where,

$$\gamma = \sigma_{opening}/\sigma_{max}$$

With the almost universal understanding that $\sigma_{opening}$ is approximately equal to $\sigma_{closure}$. Therefore the statement that no closure to exist means that $\sigma_{opening}$ is equal to $\sigma_{min\ i}$. In Figure 22, the results of reference 10, 11, and 9 are shown in terms of γ. Reference 10, Shih and Wei, results do not reflect the normal γ behavior. Reference 11, Katcher and Kaplan, show that closure is

limited in their study to about R = 0.35 that is at R greater than 0.35 set σ_{open} equal to $\sigma_{min\ i}$. Reference 9, de Koning, exhibits the expected closure behavior. Notice that Shih and Wei results fall between deKoning and Katcher and Kaplan. In the Shih and Katcher study, spectra stresses were very small considering the material is titanium, 12 KSI and less. Also Katcher's geometry was a compact tension specimen 0.719 inches thick. Shih and Wei [10] results are given in Appendix A.

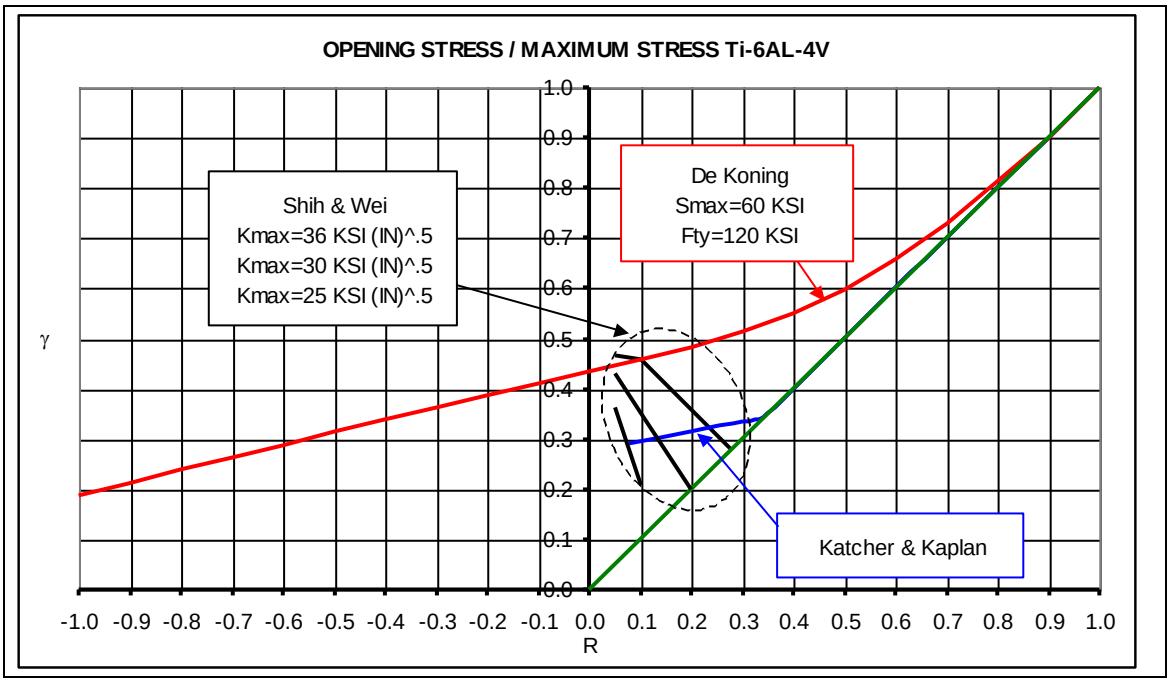

Figure 22

Based on the Shih Wei [10] study, Hsu stated to set R = 0.3 if R is greater than 0.3 in the α equation. Remembering that Hsu's minimum effective stress is written as.

$$\sigma_{min\ eff} = \sigma_{min\ i} + \alpha\ \Delta\sigma$$

and that

$$\alpha = (1 - \phi)\sqrt{(1 - R)}$$

What does limiting the R-ratio in the α equation do and how does this compare to the γ functions for titanium from reference 10 (11 and 9)? In order to answer these questions the Hsu Forman relations have to be put in terms of closure functions themselves. To do this requires that Forman equation be set equal to Elber's equation and solved for an equivalent U function. See Appendix B for this development.

Finally, the effect of limiting R to 0.3 in the α equation is shown in Figure 23. Figure 23 is busy; the solid lines present γ_{HF} for R = 1.0 in the α equation and for 11 λ_H values. The dashed lines present the γ_{HF} for R cutoff of 0.3 in the α equation and for 11 λ_H values. The first thing to note is that as λ_H goes to 1.0 the γ_{HF} collapses to γ_F as shown in red. This makes sense, because as the affects of retardation or α go to 0.0 the γ function ought to tend to the basic constant amplitude γ value. Secondly, note that by using the Forman equation either with or with out Hsu there is acceleration above R=0.6 for da/dN Segment 5 (see Appendix B for all Segments, Segment 6 shows acceleration above R=0.4). That is the function plots to the right of the no closure line. The third and main aspect to notice is that by limiting R to 0.3 -- the dashed lines plot above the solid lines indicating an increase in opening stress in Figure 23. (Hind sight is always 20:20 but it appears to use Shih and Wei's results the R-ratio should be limited to 0.3 outside the load interaction model by setting the minimum spectra stress of the half cycle so that for spectra R's greater than 0.3, the σ_{min} = σ_{max} 0.3, and then use the basic constant amplitude crack growth equation.) The red line represents baseline Segment 5 closure level without retardation (all 7 segments are presented in Appendix B for reference), as retardation increases, λ_H values decrease and closure values increase and by restricting R to 0.3, closure values increase even more. de Koning does not collapse to no closure. However, de Koning[9] is generic in terms of Fty and σ_{max} and the paper compares 7075-T6 not Ti. Therefore, de Koning's equation may not be applicable even though it tracks fairly well with equivalent Forman γ_F for Segment 5, see Appendix A Figure A2.

Figure 24 presents the same information as Figure 23 but with reference 9, 10, and 11 data added.

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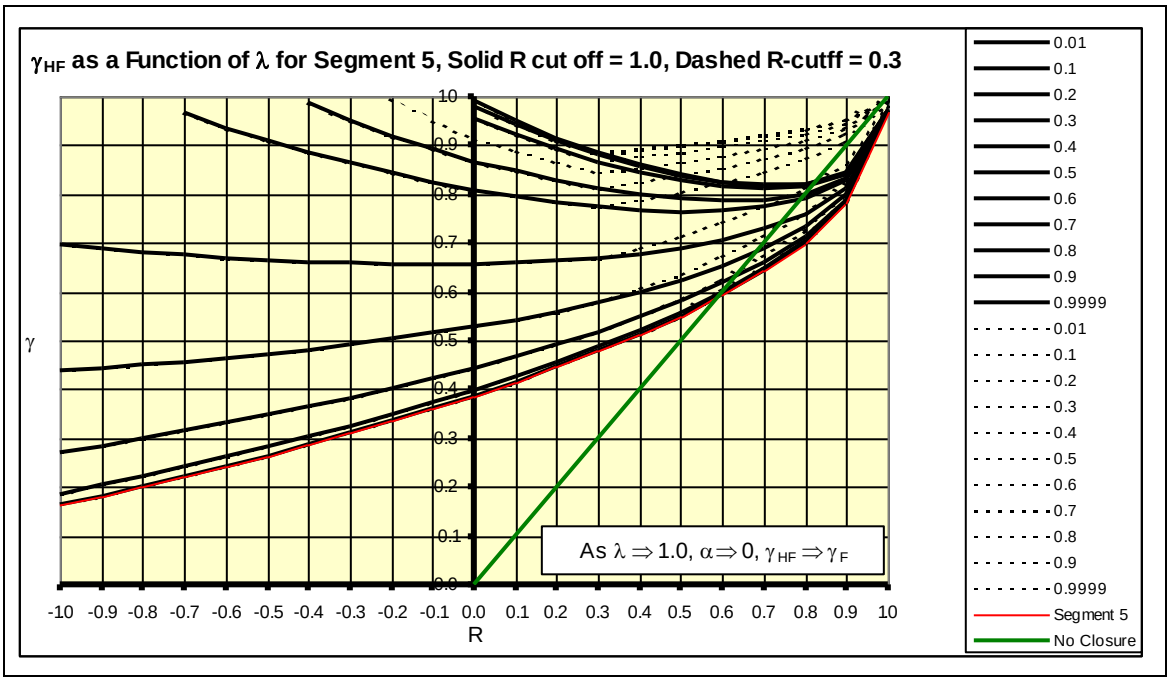

Figure 23

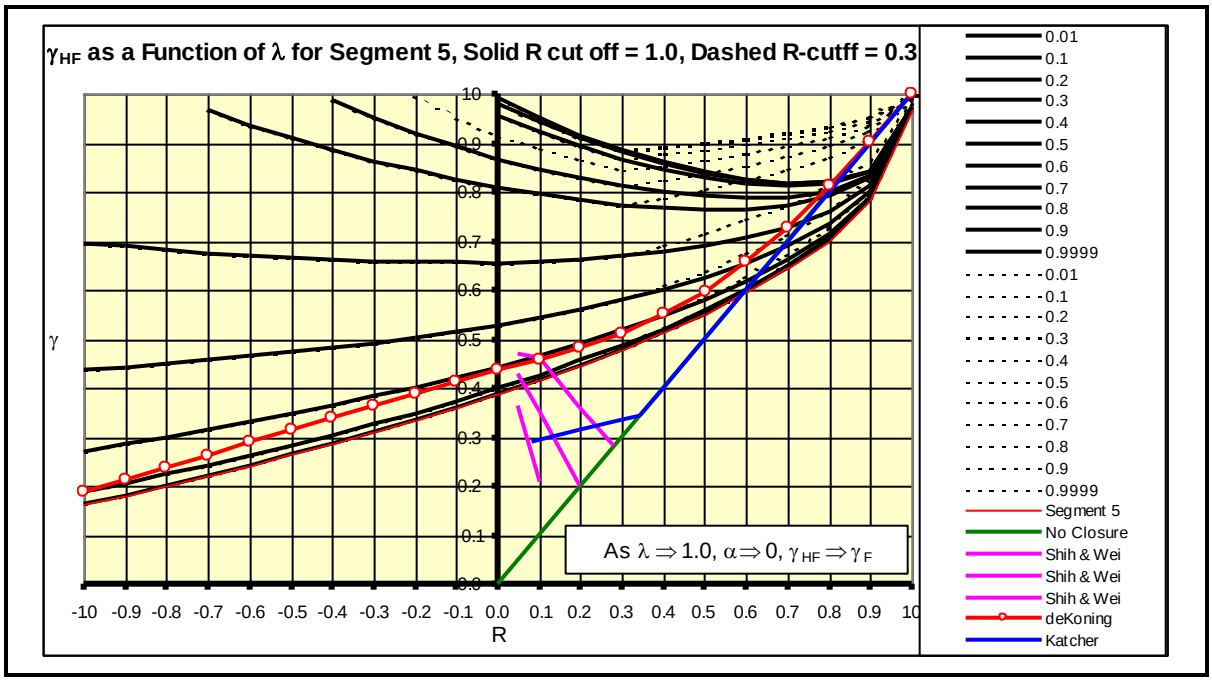

Figure 24

In summary, Figures 23 and 24 present the affects of limiting R to 0.3 in the α equation.

3.6 Compression Effects

The section presents Hsu model aspects affected by compression. A compression load will accelerate the fatigue crack growth and shorten the life. If the compression load is neglected, the fatigue crack growth life prediction will be un-conservative. Therefore, for the case where the minimum load is compressive, modification of the effective plastic zone and its corresponding effective tensile overload is necessary [The clarity in time history of when and where these modifications are to made indicates some hurried last minute thinking]. During unloading of an overload cycle, the change of stress field and the plastic zone will behave linearly. However, Hsu has stated, should the minimum stress of the subsequent applied load cycle continuously decrease from tension into compression, reverse (or compressive) yielding will start to occur and the benefit of residual strain created by the tensile overload will begin to decrease. Therefore, one may assume that the effect of compressive load on cyclic fatigue growth depends upon the magnitude of the compressive load and compressive yield strength. The compressive correction factor follows.

$$\beta_c = \left(\frac{\sigma_{Fty} - \sigma_c}{\sigma_{Fty}} \right)^{\frac{1}{2}} = \left(1 - \frac{\sigma_c}{\sigma_{Fty}} \right)^{\frac{1}{2}}$$

The form of β_c is based on the following reasoning.

1. If there is no compressive load then $\beta_c = 1.0$, i.e., no effect,
2. If the compressive load reaches the compressive yield strength, $\beta_c = 0$, completely nullifies tension overload,
3. The choice of the exponent ½ is based on the argument that the compressive load effect is proportional to the square root of the plastic zone size, since the plastic zone is proportional to the square of the applied stress. The basis for this was by considering the relations of the terms in the plastic zone equation.

The compressive correction factor is applied to the effective overload plastic zone at the encounter of a compressive minimum stress as follows.

$$(r_{eff})_c = \beta_c \, (r_{eff})_t$$

Where

Subscripts ‘c’ and ‘t’ are compression and tension,

$(r_{eff})_t$ is the size of the effective tensile plastic zone prior to the encounter with compressive load

$(r_{eff})_c$ is the size of the effective tensile plastic zone after the encounter with the compressive load

$$r_{eff} = \frac{1}{\alpha \pi}\left[\frac{(K_{max})_{eff}}{\sigma_{Fty}}\right]^2$$

The effective over load stress following the encounter with a compressive stress will become.

$$(\sigma_{OL})_{effc} = \beta_c^{1/2} (\sigma_{OL})_{eff}$$

The effective minimum stress of the half cycle that contains the compressive minimum is to be set as follows

$$(\sigma_{min})_{effc} = \beta_c^{1/2} (\sigma_{min})_{eff}$$

This essentially drives the effective opening (minimum effective) stress to a lower value, which reduces the retardation effect – this is the desired compression effect.

In summary, the compressive load effect is developed and applied consistently by modifying the effective residual plastic zone, the minimum effective stress value of the minimum stress half cycle, and the required stress at the current crack length to give the residual plastic zone.

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5 APPENDIX A – TITANIUM CLOSURE COMPARISONS

This appendix presents the results of Shih and Wei[10]. Wherein it is stated that their results are in substantial disagreement with Elber and further they state that Elber's data cannot be considered valid. Reference 10 is a paper submitted in partial fulfillment for a Master's Degree from Lehigh University. Reference 10 additionally states that their results are in agreement with previous findings from T.T. Shih, Masters Thesis, also Lehigh University. The main purpose of the paper appears to have been to discredit Elber's results. It is hard to determine the maximum spectrum stress used in reference 10 but it is believed to vary from 10 KSI, 16.66 KSI and 20 KSI. Figure A1 presents the closure function, U, results data from reference 10.

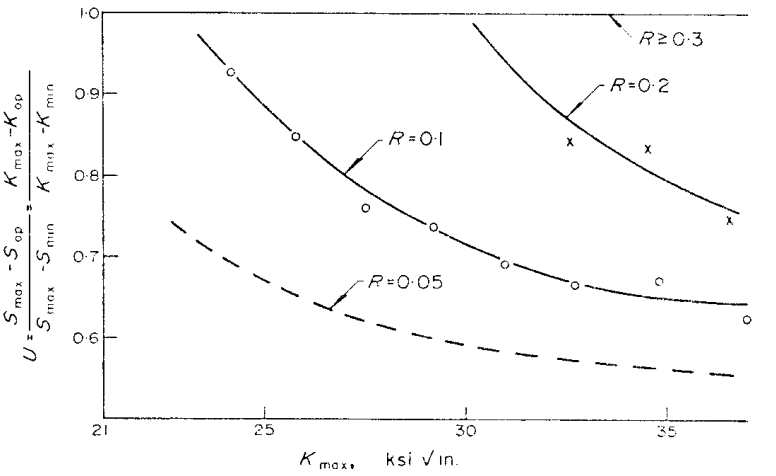

Fig. 4. Relationship between U and K_{max} for different stress ratios.

(From reference 10.)

Figure A1

For comparison purposes convert the U data in Figure A1 to γ for 3 K_{max} values, namely 25 KSI(IN)$^{0.5}$, 30 KSI(IN)$^{0.5}$, and 35 KSI(IN)$^{0.5}$. The γ results are plotted in Figure A2 along with Katcher and Kaplan [11], and de Koning [9] and equivalent Forman Segment 5.

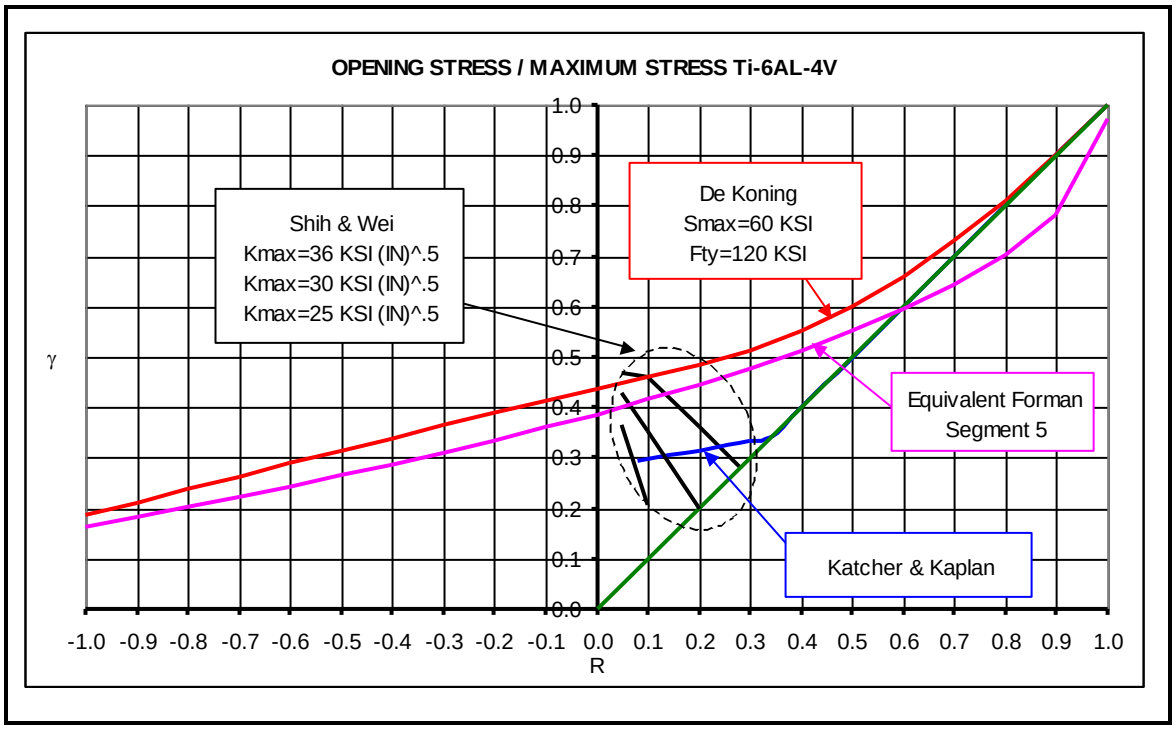

Figure A2

The results of Shih and Wei [10] are disconcerting, as R-ratio increases γ decreases; this trend is not consistent with closure but spans deKoning to Katcher and Kaplan results.

6 APPENDIX B – Equivalent Forman Closure Function Development

The Hsu Model was developed in conjunction with the Forman equation. The closure model functions, U, have been developed for use in Elber's equation. Reference 10, 11, and 9 are based on Elber's formulation. Therefore to be able to compare apples to apples, the Forman, and the Hsu Forman combination must be rewritten in the form of the Elber's equation or in terms of equivalent closure functions U_F and U_{HF} and finally in terms of γ_F and γ_{HF}. This appendix develops these equivalent forms.

Remembering Elber's equation follows.

$$\frac{da}{dN} = C\left(\Delta K_{Eff}\right)^n = C\left(U\,\Delta K\right)^n$$

Where,

U = Crack closure function

The γ function is derived from the crack closure function as follows.

$\gamma = 1 - (1\text{-}R)\ U = = \sigma_{opening}/\sigma_{max}$

To keep from getting distracted, subscripts on the coefficients will be introduced as E for Elber's and F for Forman's equations and HF for Hsu Forman.

Elber's equation then becomes

$$\frac{da}{dN} = EC\left(\Delta K_{Eff}\right)^{EN} = EC\ \left(U\,\Delta K_E\right)^{EN}\ \text{(Elber's equation)}$$

And Forman's equation becomes

$$\frac{da}{dN} = \frac{FC\ \left(\Delta K_F\right)^{FN}}{(1-R)F_{KC} - \Delta K_F}\ \text{(Forman's equation)}$$

We can write the Forman equation in terms of Elber's as follows.

$$\frac{da}{dN} = EC\left(\Delta K_{Eff}\right)^{EN} = EC\left(U_F\ \Delta K_F\right)^{EN}$$

The Forman's equation recast in terms of Elber's equation becomes.

$$\frac{da}{dN} = EC\left(\left(\frac{FC}{EC}\right)^{\frac{1}{EN}} \frac{(\Delta K_F)^{\left(\frac{FN}{EN}-1\right)}}{((1-R)F_{KC} - \Delta K_F)^{\frac{1}{EN}}} \Delta K_F\right)^{EN}$$

So then by grouping terms, an equivalent Forman crack closure function U_F, can be obtained as follows.

$$U_F = \left(\frac{FC}{EC}\right)^{\frac{1}{EN}} \frac{(\Delta K_F)^{\left(\frac{FN}{EN}-1\right)}}{((1-R)F_{KC} - \Delta K_F)^{\frac{1}{EN}}}$$

As mentioned above, the Hsu model develops an effective stress intensity factor, $\Delta K_{Eff\ Hsu}$ that is substituted in the Forman crack growth equation as ΔK_F. Remembering that,

$$\Delta K_F = \Delta K_{Eff\ Hsu} = K_{max\ i} - K_{min\ Eff}$$

$$\Delta K_{Eff\ Hsu} = K_{max\ i} - (K_{min\ i} + \Delta K\ \alpha)$$

The last equation can be rewritten in terms of a closure function as,

$$\Delta K_{Eff\ Hsu} = U_H\ \Delta K$$

Then equating in terms of $\Delta K_{Eff\ Hsu}$.

$$\Delta K_{Eff\ Hsu} = U_H\ \Delta K = K_{max\ i} - (K_{min\ i} + \Delta K\ \alpha)$$

And finally.

$$U_H = \frac{K_{max\ i} - (K_{min\ i} + \Delta K\ \alpha)}{\Delta K}$$

Or in terms of stress as follows.

$$U_H = \frac{\sigma_{max\ i} - (\sigma_{min\ i} + \Delta\sigma\ \alpha)}{\Delta\sigma}$$

Or simply.

$$U_H = 1 - \alpha$$

And then,

$$\Delta K = \Delta K_{Eff\ Hsu} = U_{HF}\ \Delta K_F = (1-\alpha)\ \Delta K_F$$

Performing this substitution in the above equations as follows.

The Hsu Forman equation in terms of Elber's becomes.

$$\frac{da}{dN} = EC\left(\Delta K_{Eff}\right)^{EN} = EC\left(U_{HF}\,\Delta K_F\right)^{EN}$$

The Hsu Forman's equation recast in terms of Elber's equation becomes.

$$\frac{da}{dN} = EC\left(\left(\frac{FC}{EC}\right)^{\frac{1}{EN}} \frac{\left((1-\alpha)\,\Delta K_F\right)^{\frac{FN}{EN}}}{\left((1-R)F_{KC} - (1-\alpha)\,\Delta K_F\right)^{\frac{1}{EN}}\,\Delta K_F}\,\Delta K_F\right)^{EN}$$

So then by grouping terms, an equivalent Hsu Forman crack closure function U_{HF}, can be obtained as follows.

$$U_{HF} = \left(\frac{FC}{EC}\right)^{\frac{1}{EN}} \frac{\left((1-\alpha)\,\Delta K_F\right)^{\frac{FN}{EN}}}{\left((1-R)F_{KC} - (1-\alpha)\,\Delta K_F\right)^{\frac{1}{EN}}\,\Delta K_F}$$

Finally with the two closure functions, U_F and U_{HF}, two gamma functions, γ_F and γ_{HF} can be developed using the standard equation above.

$\gamma = 1 - (1\text{-}R)\,U$

$\gamma_F = 1 - (1\text{-}R)\,U_F$

$\gamma_{HF} = 1 - (1\text{-}R)\,U_{HF}$

Unfortunately none of the foregoing can be obtained closed form and must be developed numerically. This was done for Titanium Ti-6Al-4V.

In order to accomplish this study:

1. A da/dN table was obtained for Ti-6AL-4V,

2. A closure function was obtained by curve fit to the formulation based on de Koning's work in reference 9 using σ_{max}= 60 KSI, σ_{Fty}= 120 KSI as follows,

$U_{Ti} = 0.0839R^6 - 0.0544R^5 - 0.2729R^4 + 0.0031R^3 + 0.3273R^2 + 0.3476R + 0.5653$

3. Elber's coefficients EC and EN were solved for using U_{Ti} to convert ΔK to ΔK_{eff}, for each straight line da/dN segment

4. Forman's coefficients FC and FN were solved for each straight line da/dN segment,

5. Equivalent Forman closure function, U_F and γ_F was solved for each segment

6. Hsu α values were solved for $-1.0 < R < 1.0$ at 0.1 increments at 11 λ values from $0.01 < \lambda < 0.9999$ at 0.1 increments with using an R-cutoff of 1.0 in the α equation.

7. Hsu α values were solved for $-1.0 < R < 1.0$ at 0.1 increments at 11 λ values from $0.01 < \lambda < 0.9999$ at 0.1 increments with using an R-cutoff of 0.3 in the α equation.

8. Equivalent Hsu Forman closure function, U_{HF} and γ_{HF} was solved for using $\alpha(R, \lambda)$ values for da/dN Segment 5, these are shown in Figures 23 and 24 in main body of report.

1. da/dN Ti-6AL-4V

Ti-6AL-4V Beta Annealed Titanium (LMAS SMN 366, Fig. 8.13.0)

Kpoint = 8 Segments = 7

K_{FC} = 200

ΔK	da/dN	ΔK	da/dN (μ inch/cycle)
3.5	1.00E-09	3.5	1.00E-03
4	4.50E-08	4	4.50E-02
5	1.40E-07	5	1.40E-01
10	1.50E-06	10	1.50E+00
40	8.00E-05	40	8.00E+01
80	5.20E-04	80	5.20E+02
100	1.10E-03	100	1.10E+03
125	1.00E-02	125	1.00E+04

F_{TU} = 130 F_{TY}=120 KIC =50

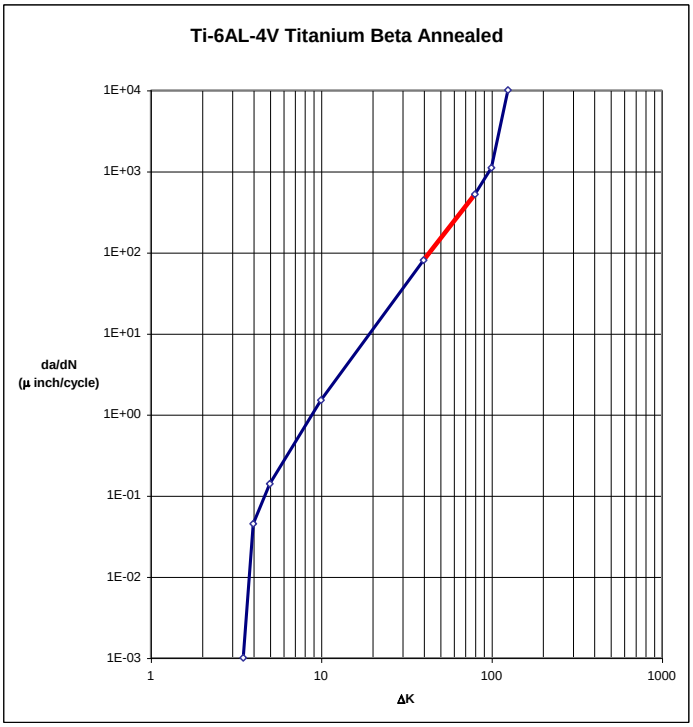

Figure B1

3. and 4. Crack growth coefficients

STRESS INTENSITY (DK) KSI X SQ. ROOT OF INCHES	CRACK GROWTH RATE (DA/DN) (MICROINCHES /CYCLE)	FORMAN'S CONSTANTS FC	FN
3.5	1.00E-03	5.60E-23	28.48637
4	4.50E-02	7.11E-09	5.06079
5	1.40E-01	1.06E-07	3.37964
10	1.50E+00	4.77E-07	2.72843
40	8.00E+01	3.17E-06	2.21501
80	5.20E+02	1.70E-06	2.35764
100	1.10E+03	3.31E-18	8.21257
125	1.00E+04		

KC VALUE USED IN FORMAN EQUATION IS 200.000 KSI*SQRT(IN.)
CONSTANT AMPLITUDE RATE IS 0.1000
MINIMUM ALLOWABLE STRESS RATIO IS -0.1000

STRESS INTENSITY (DK) KSI X SQ. ROOT OF INCHES	CRACK GROWTH RATE (DA/DN) (MICROINCHES /CYCLE)	ELBER'S CONSTANTS EC	EN
3.5	1.00E-03	5.58E-19	28.50763
4	4.50E-02	5.10E-10	5.08632
5	1.40E-01	3.20E-09	3.42146
10	1.50E+00	8.65E-09	2.86848
40	8.00E+01	1.48E-08	2.70044
80	5.20E+02	1.16E-09	3.35764
100	1.10E+03	2.68E-21	9.89173
125	1.00E+04		

Figure B2

5. Baseline Forman γ function for 7 da/dN Segments

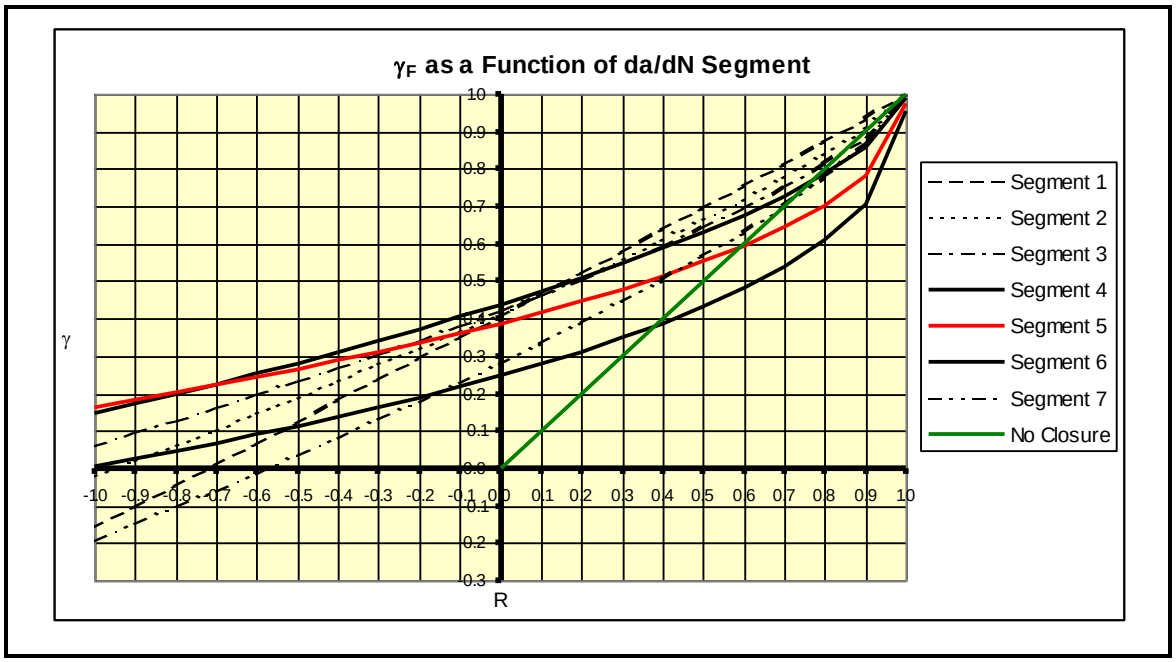

Figure B3

Forman γ function for 3 Region II da/dN Segments

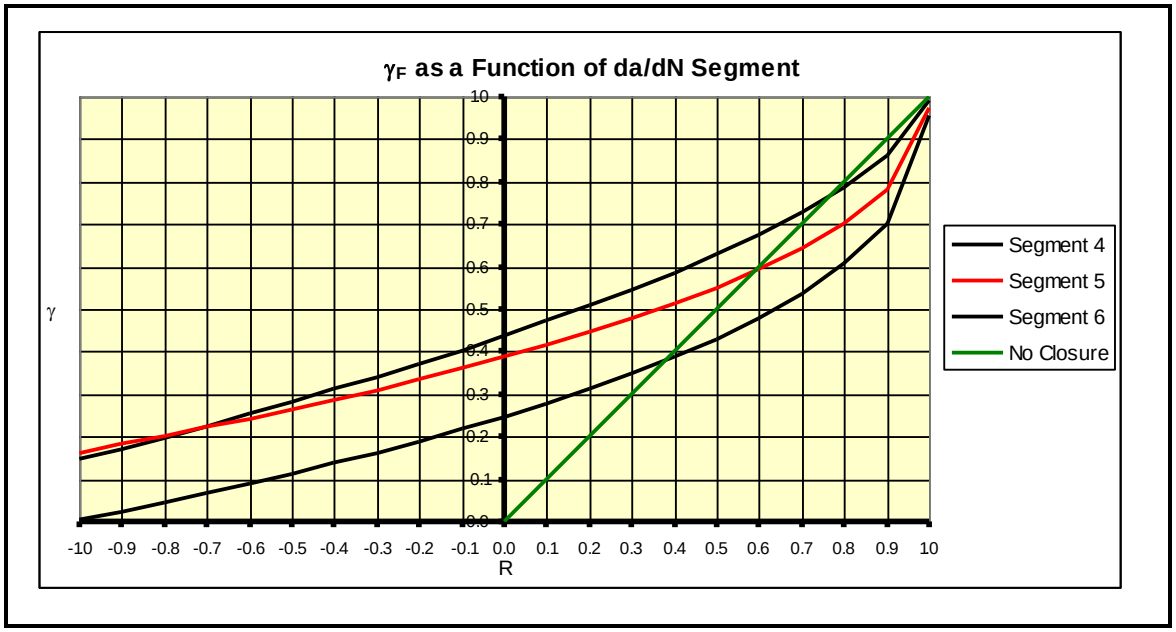

Figure B4

6. and 7. α

α at R=1.0

	λ	0.01	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	0.99
	α	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
a(R=-1)	Z	1.4141	1.4107	1.4001	1.3824	1.3576	1.3258	1.2869	1.2410	1.1879	1.1278	1.0607	0.8825	0.6985	0.5249	0.3725	0.2468	0.1493	0.0788	0.0327	0.0076	0.0000
a(R=-0.9)	AA	1.3783	1.3750	1.3646	1.3474	1.3233	1.2923	1.2543	1.2096	1.1579	1.0993	1.0338	0.8602	0.6808	0.5116	0.3631	0.2406	0.1455	0.0768	0.0319	0.0074	0.0000
a(R=-0.8)	AB	1.3415	1.3383	1.3282	1.3115	1.2880	1.2578	1.2209	1.1773	1.1270	1.0700	1.0062	0.8372	0.6627	0.4980	0.3534	0.2341	0.1416	0.0748	0.0310	0.0072	0.0000
a(R=-0.7)	AC	1.3037	1.3006	1.2908	1.2745	1.2517	1.2224	1.1865	1.1441	1.0952	1.0398	0.9779	0.8137	0.6440	0.4840	0.3434	0.2275	0.1377	0.0727	0.0302	0.0070	0.0000
a(R=-0.6)	AD	1.2648	1.2617	1.2523	1.2365	1.2143	1.1859	1.1511	1.1100	1.0625	1.0088	0.9487	0.7894	0.6248	0.4695	0.3332	0.2207	0.1335	0.0705	0.0293	0.0068	0.0000
a(R=-0.5)	AE	1.2246	1.2217	1.2125	1.1972	1.1758	1.1482	1.1145	1.0747	1.0288	0.9767	0.9186	0.7643	0.6050	0.4546	0.3226	0.2137	0.1293	0.0683	0.0283	0.0066	0.0000
a(R=-0.4)	AF	1.1831	1.1803	1.1714	1.1566	1.1359	1.1093	1.0767	1.0383	0.9939	0.9436	0.8874	0.7384	0.5844	0.4392	0.3117	0.2065	0.1249	0.0660	0.0274	0.0064	0.0000
a(R=-0.3)	AG	1.1401	1.1373	1.1288	1.1145	1.0946	1.0689	1.0376	1.0005	0.9577	0.9093	0.8551	0.7115	0.5632	0.4232	0.3003	0.1990	0.1204	0.0636	0.0264	0.0061	0.0000
a(R=-0.2)	AH	1.0953	1.0927	1.0845	1.0708	1.0516	1.0270	0.9969	0.9613	0.9202	0.8736	0.8216	0.6836	0.5411	0.4066	0.2885	0.1912	0.1156	0.0611	0.0254	0.0059	0.0000
a(R=-0.1)	AI	1.0487	1.0462	1.0383	1.0252	1.0069	0.9833	0.9544	0.9203	0.8810	0.8364	0.7866	0.6545	0.5180	0.3893	0.2763	0.1830	0.1107	0.0585	0.0243	0.0056	0.0000
a(R=0.01)	AJ	0.9949	0.9925	0.9850	0.9726	0.9552	0.9328	0.9054	0.8731	0.8358	0.7935	0.7462	0.6209	0.4915	0.3693	0.2621	0.1736	0.1050	0.0555	0.0230	0.0054	0.0000
a(R=0.1)	AK	0.9486	0.9463	0.9392	0.9273	0.9107	0.8894	0.8633	0.8325	0.7969	0.7566	0.7115	0.5920	0.4686	0.3521	0.2499	0.1656	0.1002	0.0529	0.0220	0.0051	0.0000
a(R=0.2)	AL	0.8943	0.8922	0.8855	0.8743	0.8587	0.8385	0.8139	0.7849	0.7513	0.7133	0.6708	0.5582	0.4418	0.3320	0.2356	0.1561	0.0944	0.0499	0.0207	0.0048	0.0000
a(R=0.3)	AM	0.8366	0.8346	0.8283	0.8178	0.8032	0.7844	0.7614	0.7342	0.7028	0.6672	0.6275	0.5221	0.4133	0.3106	0.2204	0.1460	0.0883	0.0466	0.0194	0.0045	0.0000
a(R=0.4)	AN	0.7745	0.7727	0.7669	0.7572	0.7436	0.7262	0.7049	0.6797	0.6507	0.6177	0.5809	0.4834	0.3826	0.2875	0.2040	0.1352	0.0818	0.0432	0.0179	0.0042	0.0000
a(R=0.5)	AO	0.7070	0.7053	0.7000	0.6912	0.6788	0.6629	0.6435	0.6205	0.5940	0.5639	0.5303	0.4413	0.3493	0.2625	0.1863	0.1234	0.0747	0.0394	0.0164	0.0038	0.0000
a(R=0.6)	AP	0.6324	0.6309	0.6261	0.6182	0.6072	0.5929	0.5755	0.5550	0.5313	0.5044	0.4743	0.3947	0.3124	0.2348	0.1666	0.1104	0.0668	0.0353	0.0146	0.0034	0.0000
a(R=0.7)	AQ	0.5477	0.5464	0.5422	0.5354	0.5258	0.5135	0.4984	0.4806	0.4601	0.4368	0.4108	0.3418	0.2705	0.2033	0.1443	0.0956	0.0578	0.0305	0.0127	0.0029	0.0000
a(R=0.8)	AR	0.4472	0.4461	0.4427	0.4372	0.4293	0.4193	0.4070	0.3924	0.3757	0.3567	0.3354	0.2791	0.2209	0.1660	0.1178	0.0780	0.0472	0.0249	0.0103	0.0024	0.0000
a(R=0.9)	AS	0.3162	0.3154	0.3131	0.3091	0.3036	0.2965	0.2878	0.2775	0.2656	0.2522	0.2372	0.1973	0.1562	0.1174	0.0833	0.0552	0.0334	0.0176	0.0073	0.0017	0.0000
a(R=0.9999	AT	0.0100	0.0100	0.0099	0.0098	0.0096	0.0094	0.0091	0.0088	0.0084	0.0080	0.0075	0.0062	0.0049	0.0037	0.0026	0.0017	0.0011	0.0006	0.0002	0.0001	0.0000

α at R = 0.3

	λ	0.01	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	0.99
	α	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
a(R=-1)	Z	1.4141	1.4107	1.4001	1.3824	1.3576	1.3258	1.2869	1.2410	1.1879	1.1278	1.0607	0.8825	0.6985	0.5249	0.3725	0.2468	0.1493	0.0788	0.0327	0.0076	0.0000
a(R=-0.9)	AA	1.3783	1.3750	1.3646	1.3474	1.3233	1.2923	1.2543	1.2096	1.1579	1.0993	1.0338	0.8602	0.6808	0.5116	0.3631	0.2406	0.1455	0.0768	0.0319	0.0074	0.0000
a(R=-0.8)	AB	1.3415	1.3383	1.3282	1.3115	1.2880	1.2578	1.2209	1.1773	1.1270	1.0700	1.0062	0.8372	0.6627	0.4980	0.3534	0.2341	0.1416	0.0748	0.0310	0.0072	0.0000
a(R=-0.7)	AC	1.3037	1.3006	1.2908	1.2745	1.2517	1.2224	1.1865	1.1441	1.0952	1.0398	0.9779	0.8137	0.6440	0.4840	0.3434	0.2275	0.1377	0.0727	0.0302	0.0070	0.0000
a(R=-0.6)	AD	1.2648	1.2617	1.2523	1.2365	1.2143	1.1859	1.1511	1.1100	1.0625	1.0088	0.9487	0.7894	0.6248	0.4695	0.3332	0.2207	0.1335	0.0705	0.0293	0.0068	0.0000
a(R=-0.5)	AE	1.2246	1.2217	1.2125	1.1972	1.1758	1.1482	1.1145	1.0747	1.0288	0.9767	0.9186	0.7643	0.6050	0.4546	0.3226	0.2137	0.1293	0.0683	0.0283	0.0066	0.0000
a(R=-0.4)	AF	1.1831	1.1803	1.1714	1.1566	1.1359	1.1093	1.0767	1.0383	0.9939	0.9436	0.8874	0.7384	0.5844	0.4392	0.3117	0.2065	0.1249	0.0660	0.0274	0.0064	0.0000
a(R=-0.3)	AG	1.1401	1.1373	1.1288	1.1145	1.0946	1.0689	1.0376	1.0005	0.9577	0.9093	0.8551	0.7115	0.5632	0.4232	0.3003	0.1990	0.1204	0.0636	0.0264	0.0061	0.0000
a(R=-0.2)	AH	1.0953	1.0927	1.0845	1.0708	1.0516	1.0270	0.9969	0.9613	0.9202	0.8736	0.8216	0.6836	0.5411	0.4066	0.2885	0.1912	0.1156	0.0611	0.0254	0.0059	0.0000
a(R=-0.1)	AI	1.0487	1.0462	1.0383	1.0252	1.0069	0.9833	0.9544	0.9203	0.8810	0.8364	0.7866	0.6545	0.5180	0.3893	0.2763	0.1830	0.1107	0.0585	0.0243	0.0056	0.0000
a(R=0.01)	AJ	0.9949	0.9925	0.9850	0.9726	0.9552	0.9328	0.9054	0.8731	0.8358	0.7935	0.7462	0.6209	0.4915	0.3693	0.2621	0.1736	0.1050	0.0555	0.0230	0.0054	0.0000
a(R=0.1)	AK	0.9486	0.9463	0.9392	0.9273	0.9107	0.8894	0.8633	0.8325	0.7969	0.7566	0.7115	0.5920	0.4686	0.3521	0.2499	0.1656	0.1002	0.0529	0.0220	0.0051	0.0000
a(R=0.2)	AL	0.8943	0.8922	0.8855	0.8743	0.8587	0.8385	0.8139	0.7849	0.7513	0.7133	0.6708	0.5582	0.4418	0.3320	0.2356	0.1561	0.0944	0.0499	0.0207	0.0048	0.0000
a(R=0.3)	AM	0.8366	0.8346	0.8283	0.8178	0.8032	0.7844	0.7614	0.7342	0.7028	0.6672	0.6275	0.5221	0.4133	0.3106	0.2204	0.1460	0.0883	0.0466	0.0194	0.0045	0.0000
a(R=0.4)	AN	0.8366	0.8346	0.8283	0.8178	0.8032	0.7844	0.7614	0.7342	0.7028	0.6672	0.6275	0.5221	0.4133	0.3106	0.2204	0.1460	0.0883	0.0466	0.0194	0.0045	0.0000
a(R=0.5)	AO	0.8366	0.8346	0.8283	0.8178	0.8032	0.7844	0.7614	0.7342	0.7028	0.6672	0.6275	0.5221	0.4133	0.3106	0.2204	0.1460	0.0883	0.0466	0.0194	0.0045	0.0000
a(R=0.6)	AP	0.8366	0.8346	0.8283	0.8178	0.8032	0.7844	0.7614	0.7342	0.7028	0.6672	0.6275	0.5221	0.4133	0.3106	0.2204	0.1460	0.0883	0.0466	0.0194	0.0045	0.0000
a(R=0.7)	AQ	0.8366	0.8346	0.8283	0.8178	0.8032	0.7844	0.7614	0.7342	0.7028	0.6672	0.6275	0.5221	0.4133	0.3106	0.2204	0.1460	0.0883	0.0466	0.0194	0.0045	0.0000
a(R=0.8)	AR	0.8366	0.8346	0.8283	0.8178	0.8032	0.7844	0.7614	0.7342	0.7028	0.6672	0.6275	0.5221	0.4133	0.3106	0.2204	0.1460	0.0883	0.0466	0.0194	0.0045	0.0000
a(R=0.9)	AS	0.8366	0.8346	0.8283	0.8178	0.8032	0.7844	0.7614	0.7342	0.7028	0.6672	0.6275	0.5221	0.4133	0.3106	0.2204	0.1460	0.0883	0.0466	0.0194	0.0045	0.0000
a(R=0.9999	AT	0.8366	0.8346	0.8283	0.8178	0.8032	0.7844	0.7614	0.7342	0.7028	0.6672	0.6275	0.5221	0.4133	0.3106	0.2204	0.1460	0.0883	0.0466	0.0194	0.0045	0.0000

Figure B5

7 APPENDIX C – Hsu Model Implementation

The Hsu Model implementation for this report was accomplished in Microsoft Visual Studio C++ 6.0. A Closure shell provided by Jim Harter was modified for these purposes. Essentially a stand-alone crack growth program incorporating the Hsu Model was implemented for check out before full incorporation into AFGROW. The implementation uses the Walker relation and Kmax when R is less than zero. Hsu will be available in AFGROW with Forman's relation; it was just easier with a one-segment da/dN Walker since the Closure shell included it. The implementation is included in a folder called app_C.

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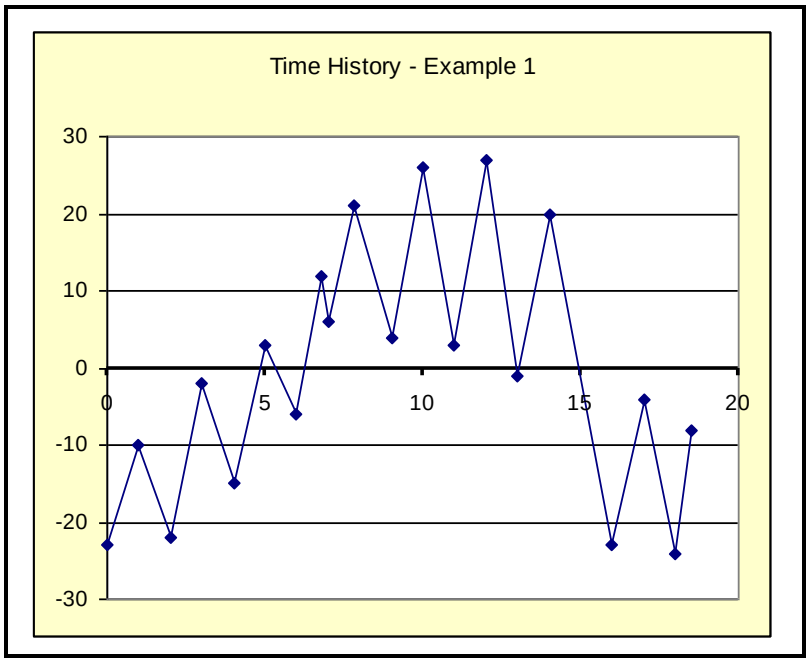
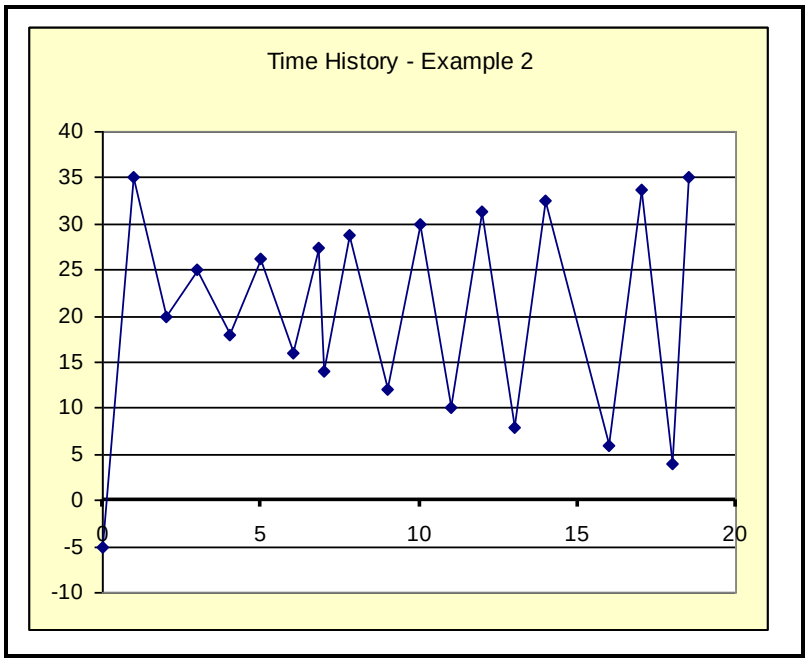
8 APPENDIX D – Examples

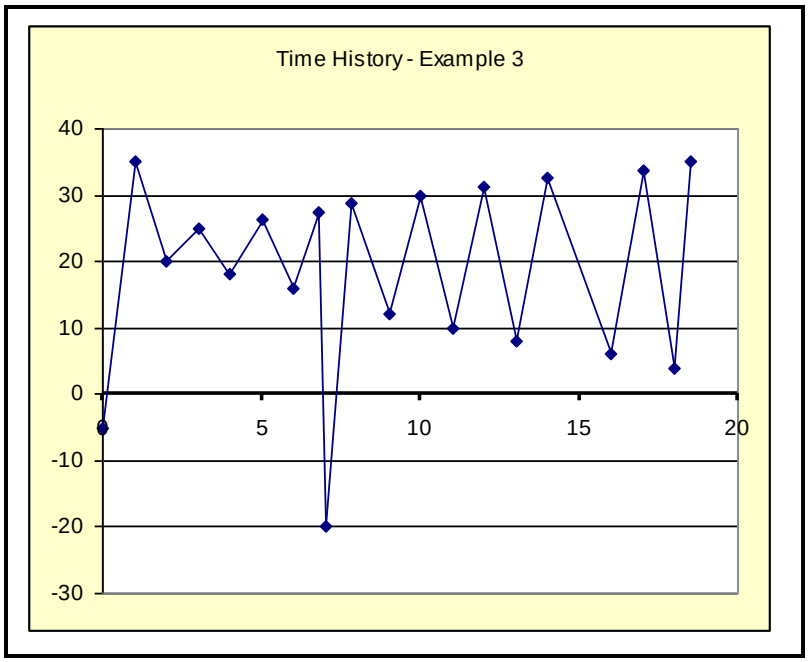
This appendix presents 2 problem cases, 1) m0 = 1.0 and 2) m0 = 1.3, for the 4 stress profile examples given in the following table. Then time history and screen captures of output results are presented.

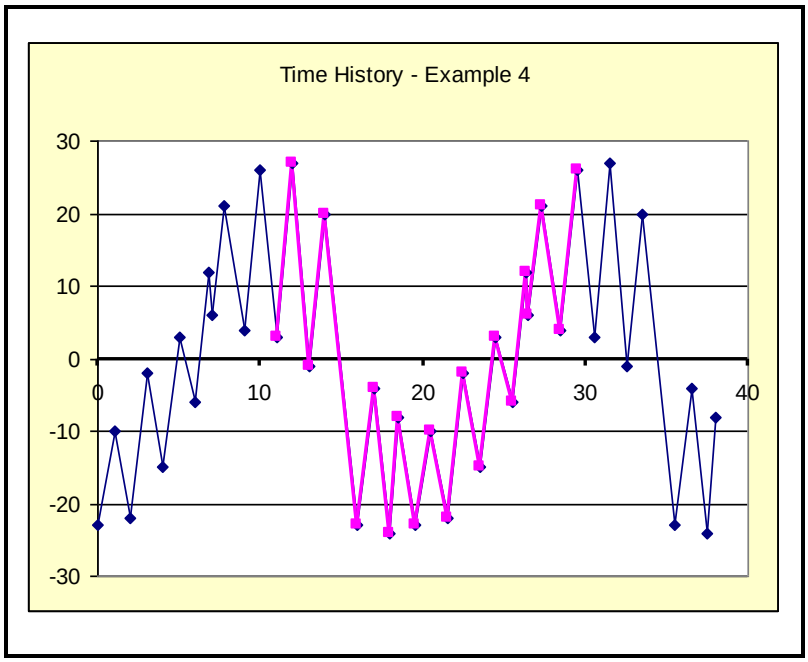
EXAMPLE 1		EXAMPLE 2		EXAMPLE 3		EXAMPLE 4	
Max Stress	Min Stress	Max Stress	Min Stress	Max Stress	Min Stress	Max Stress	Min Stress
-10.0	-23.0	35.0	-5.0	35.0	-5.0	27.0	3.0
-2.0	-22.0	25.0	20.0	25.0	20.0	20.0	-1.0
3.0	-15.0	26.3	18.0	26.3	18.0	-4.0	-23.0
12.0	-6.0	27.5	16.0	27.5	16.0	-8.0	-24.0
21.0	6.0	28.8	14.0	28.8	-20.0	-10.0	-23.0
26.0	4.0	30.0	12.0	30.0	12.0	-2.0	-22.0
27.0	3.0	31.3	10.0	31.3	10.0	3.0	-15.0
20.0	-1.0	32.5	8.0	32.5	8.0	12.0	-6.0
-4.0	-23.0	33.8	6.0	33.8	6.0	21.0	6.0
-8.0	-24.0	35.0	4.0	35.0	4.0	26.0	4.0

For reference these stress profiles are also presented in time history plots.

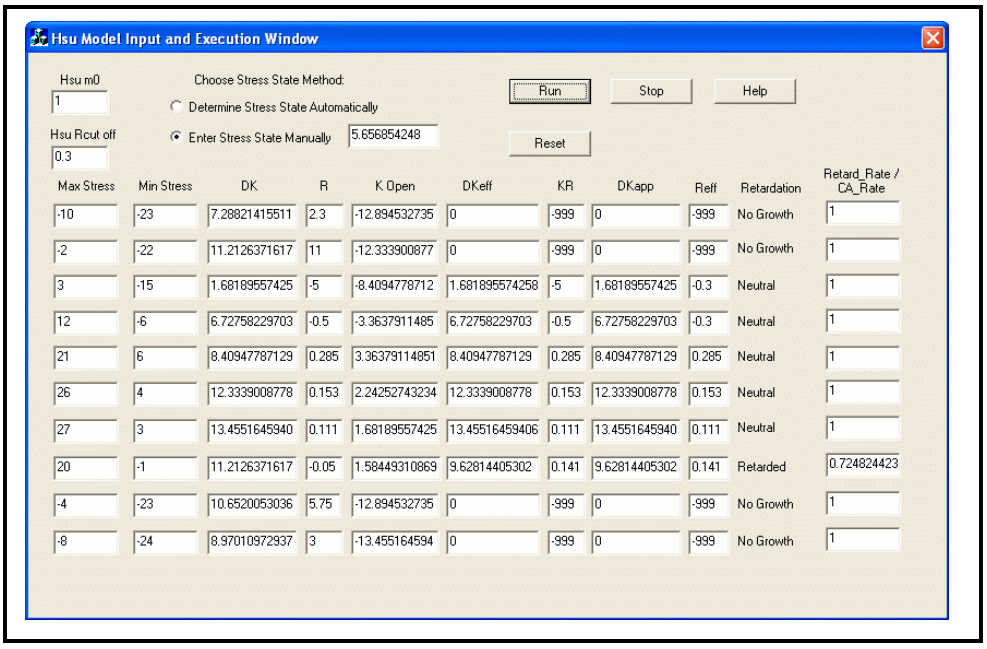



Time History - Example 2
40
35
30
25
20
15
10
5
0
-5
-10
0
5
10
15
20


Time History - Example 3
40
30
20
10
0
-10
-20
-30
0
5
10
15
20

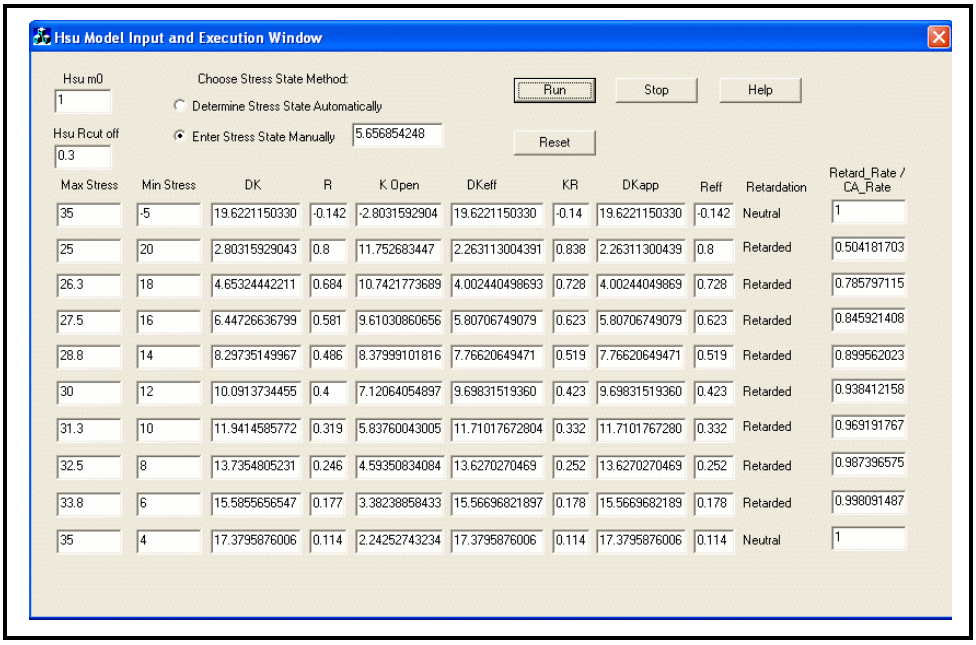


Example 1 Screen Capture – $m_0 = 1.0$



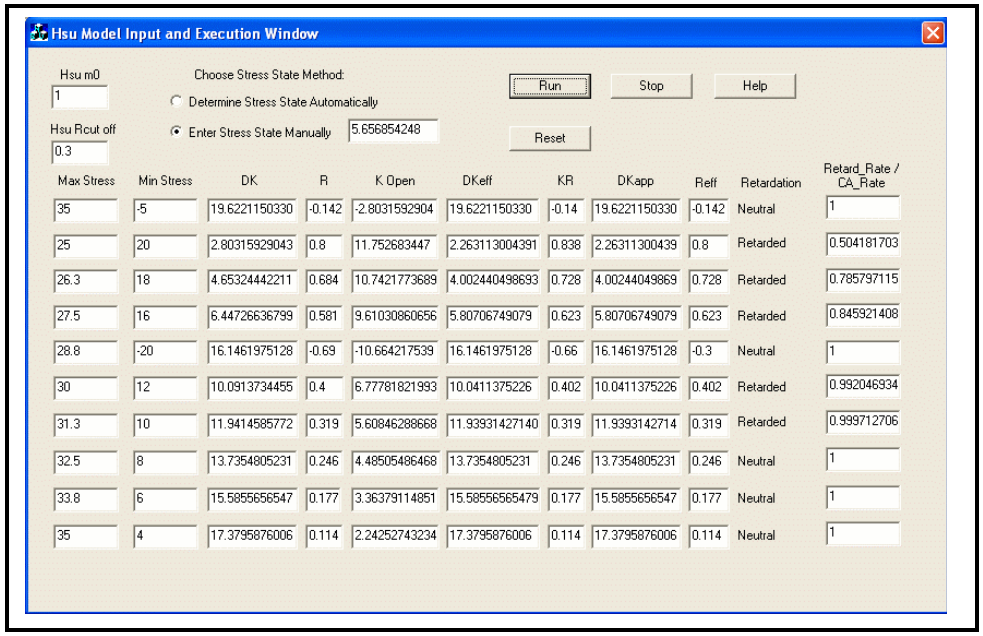
Max Stress	Min Stress	DK	R	K Open	DKeff	KR	DKapp	Reff	Retardation	Retard_Rate / CA_Rate
-10	-23	7.28821415511	2.3	-12.894532735	0	-999	0	-999	No Growth	1
-2	-22	11.2126371617	11	-12.333900877	0	-999	0	-999	No Growth	1
3	-15	1.68189557425	-5	-8.4094778712	1.681895574258	-5	1.68189557425	-0.3	Neutral	1
12	-6	6.72758229703	-0.5	-3.3637911485	6.72758229703	-0.5	6.72758229703	-0.3	Neutral	1
21	6	8.40947787129	0.285	3.36379114851	8.40947787129	0.285	8.40947787129	0.285	Neutral	1
26	4	12.3339008778	0.153	2.24252743234	12.3339008778	0.153	12.3339008778	0.153	Neutral	1
27	3	13.4551645940	0.111	1.68189557425	13.45516459406	0.111	13.4551645940	0.111	Neutral	1
20	-1	11.2126371617	-0.05	1.58449310869	9.62814405302	0.141	9.62814405302	0.141	Retarded	0.724824423
-4	-23	10.6520053036	5.75	-12.894532735	0	-999	0	-999	No Growth	1
-8	-24	8.97010972937	3	-13.455164594	0	-999	0	-999	No Growth	1

Example 2 Screen Capture – m0 = 1.0



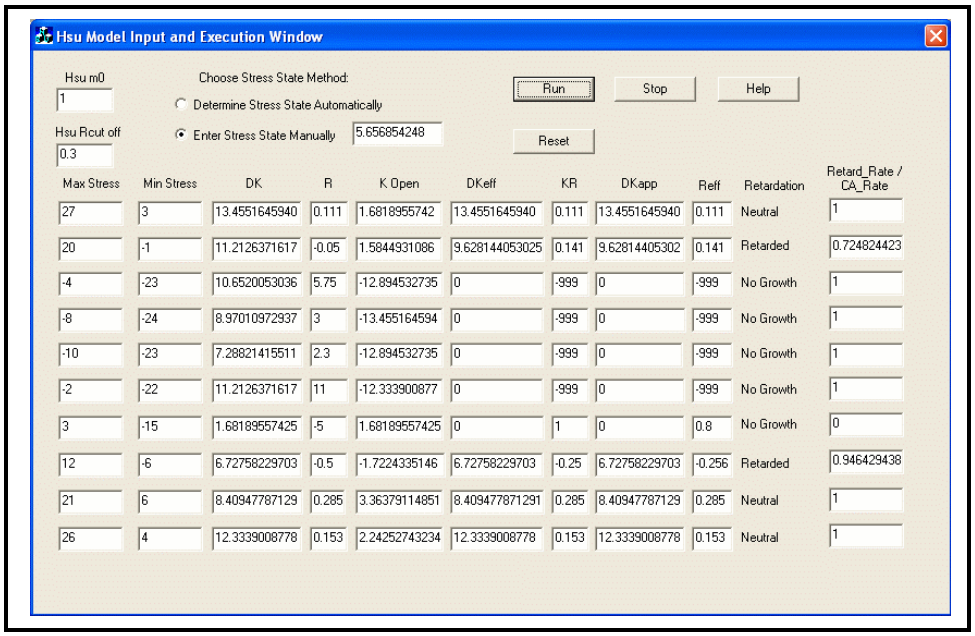
Max Stress	Min Stress	DK	R	K Open	DKeff	KR	DKapp	Reff	Retardation	Retard_Rate / CA_Rate
35	-5	19.6221150330	-0.142	-2.8031592904	19.6221150330	-0.14	19.6221150330	-0.142	Neutral	1
25	20	2.80315929043	0.8	11.752683447	2.263113004391	0.838	2.26311300439	0.8	Retarded	0.504181703
26.3	18	4.65324442211	0.684	10.7421773689	4.002440498693	0.728	4.00244049869	0.728	Retarded	0.785797115
27.5	16	6.44726636799	0.581	9.61030860656	5.80706749079	0.623	5.80706749079	0.623	Retarded	0.845921408
28.8	14	8.29735149967	0.486	8.37999101816	7.76620649471	0.519	7.76620649471	0.519	Retarded	0.899562023
30	12	10.0913734455	0.4	7.12064054897	9.69831519360	0.423	9.69831519360	0.423	Retarded	0.938412158
31.3	10	11.9414585772	0.319	5.83760043005	11.71017672804	0.332	11.7101767280	0.332	Retarded	0.969191767
32.5	8	13.7354805231	0.246	4.59350834084	13.6270270469	0.252	13.6270270469	0.252	Retarded	0.987396575
33.8	6	15.5855656547	0.177	3.38238858433	15.56696821897	0.178	15.5669682189	0.178	Retarded	0.998091487
35	4	17.3795876006	0.114	2.24252743234	17.3795876006	0.114	17.3795876006	0.114	Neutral	1

Example 3 Screen Capture – m0 = 1.0



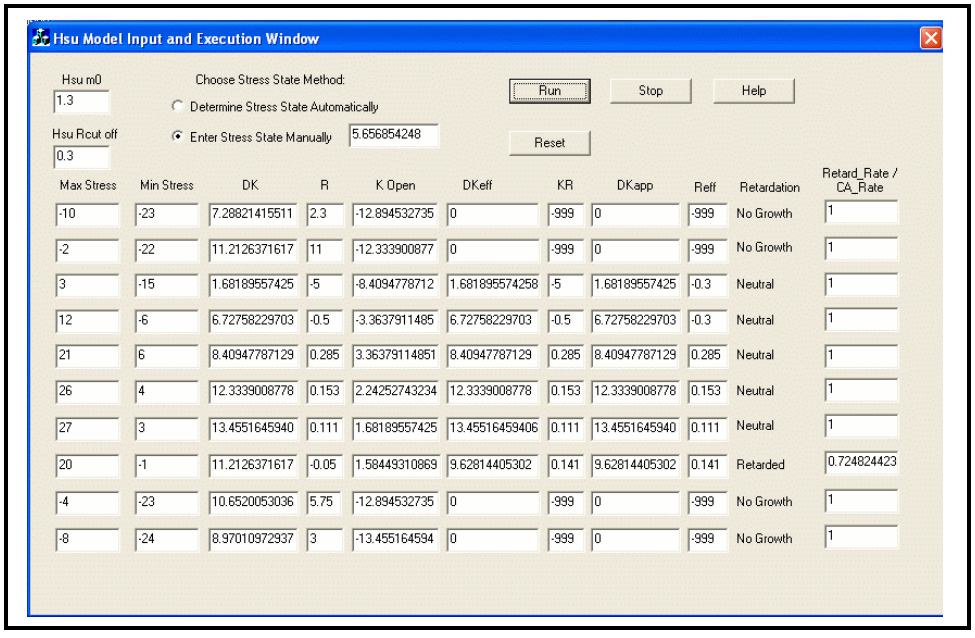
Max Stress	Min Stress	DK	R	K Open	DKeff	KR	DKapp	Reff	Retardation	Retard_Rate / CA_Rate
35	-5	19.6221150330	-0.142	-2.8031592904	19.6221150330	-0.14	19.6221150330	-0.142	Neutral	1
25	20	2.80315929043	0.8	11.752683447	2.263113004391	0.838	2.26311300439	0.8	Retarded	0.504181703
26.3	18	4.65324442211	0.684	10.7421773689	4.002440498693	0.728	4.00244049869	0.728	Retarded	0.785797115
27.5	16	6.44726636799	0.581	9.61030860656	5.80706749079	0.623	5.80706749079	0.623	Retarded	0.845921408
28.8	-20	16.1461975128	-0.69	-10.664217539	16.1461975128	-0.66	16.1461975128	-0.3	Neutral	1
30	12	10.0913734455	0.4	6.77781821993	10.0411375226	0.402	10.0411375226	0.402	Retarded	0.992046934
31.3	10	11.9414585772	0.319	5.60846288668	11.93931427140	0.319	11.9393142714	0.319	Retarded	0.999712706
32.5	8	13.7354805231	0.246	4.48505486468	13.7354805231	0.246	13.7354805231	0.246	Neutral	1
33.8	6	15.5855656547	0.177	3.36379114851	15.58556565479	0.177	15.5855656547	0.177	Neutral	1
35	4	17.3795876006	0.114	2.24252743234	17.3795876006	0.114	17.3795876006	0.114	Neutral	1

Example 4 Screen Capture – m0 = 1.0



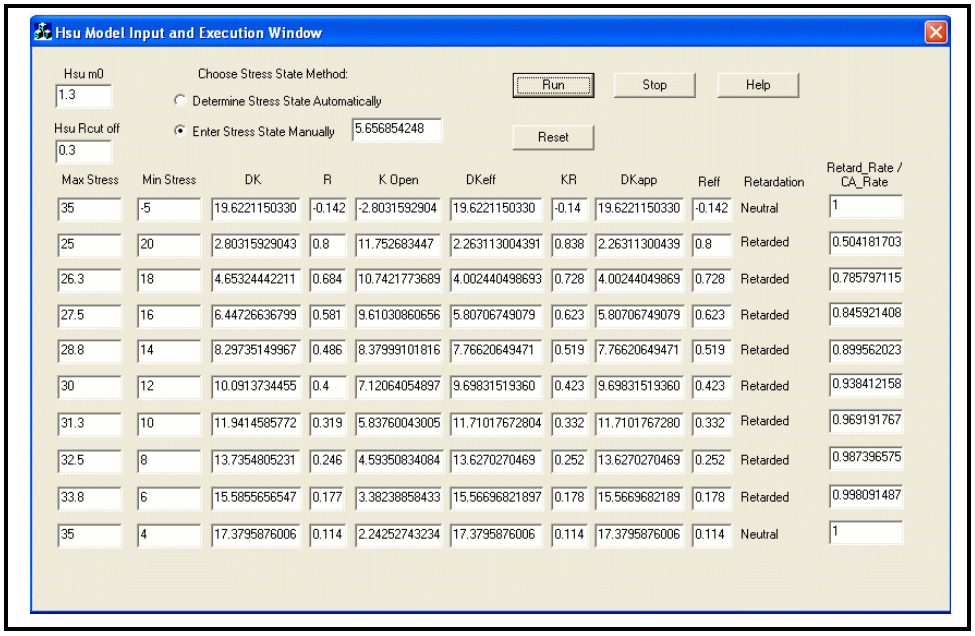
Max Stress	Min Stress	DK	R	K Open	DKeff	KR	DKapp	Reff	Retardation	Retard_Rate / CA_Rate
27	3	13.4551645940	0.111	1.6818955742	13.4551645940	0.111	13.4551645940	0.111	Neutral	1
20	-1	11.2126371617	-0.05	1.5844931086	9.628144053025	0.141	9.62814405302	0.141	Retarded	0.724824423
-4	-23	10.6520053036	5.75	-12.894532735	0	-999	0	-999	No Growth	1
-8	-24	8.97010972937	3	-13.455164594	0	-999	0	-999	No Growth	1
-10	-23	7.28821415511	2.3	-12.894532735	0	-999	0	-999	No Growth	1
-2	-22	11.2126371617	11	-12.333900877	0	-999	0	-999	No Growth	1
3	-15	1.68189557425	-5	1.68189557425	0	1	0	0.8	No Growth	0
12	-6	6.72758229703	-0.5	-1.7224335146	6.72758229703	-0.25	6.72758229703	-0.256	Retarded	0.946429438
21	6	8.40947787129	0.285	3.36379114851	8.40947787129	0.285	8.40947787129	0.285	Neutral	1
26	4	12.3339008778	0.153	2.24252743234	12.3339008778	0.153	12.3339008778	0.153	Neutral	1

Example 1 Screen Capture – m0 = 1.3



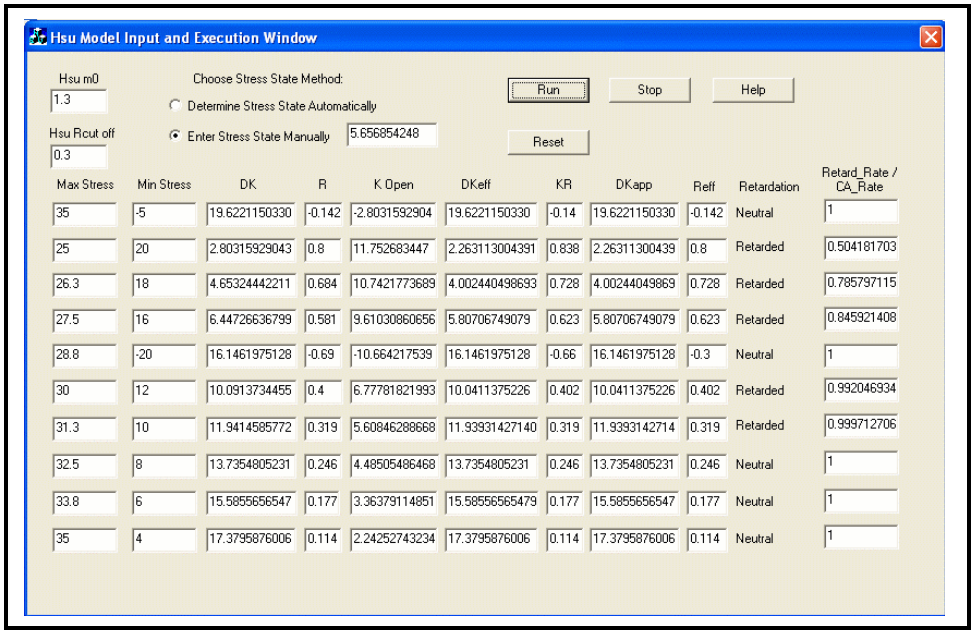
Max Stress	Min Stress	DK	R	K Open	DKeff	KR	DKapp	Reff	Retardation	Retard_Rate / CA_Rate
-10	-23	7.28821415511	2.3	-12.894532735	0	-999	0	-999	No Growth	1
-2	-22	11.2126371617	11	-12.333900877	0	-999	0	-999	No Growth	1
3	-15	1.68189557425	-5	-8.4094778712	1.681895574258	-5	1.68189557425	-0.3	Neutral	1
12	-6	6.72758229703	-0.5	-3.3637911485	6.72758229703	-0.5	6.72758229703	-0.3	Neutral	1
21	6	8.40947787129	0.285	3.36379114851	8.40947787129	0.285	8.40947787129	0.285	Neutral	1
26	4	12.3339008778	0.153	2.24252743234	12.3339008778	0.153	12.3339008778	0.153	Neutral	1
27	3	13.4551645940	0.111	1.68189557425	13.45516459406	0.111	13.4551645940	0.111	Neutral	1
20	-1	11.2126371617	-0.05	1.58449310869	9.62814405302	0.141	9.62814405302	0.141	Retarded	0.724824423
-4	-23	10.6520053036	5.75	-12.894532735	0	-999	0	-999	No Growth	1
-8	-24	8.97010972937	3	-13.455164594	0	-999	0	-999	No Growth	1

Example 2 Screen Capture – m0 = 1.3



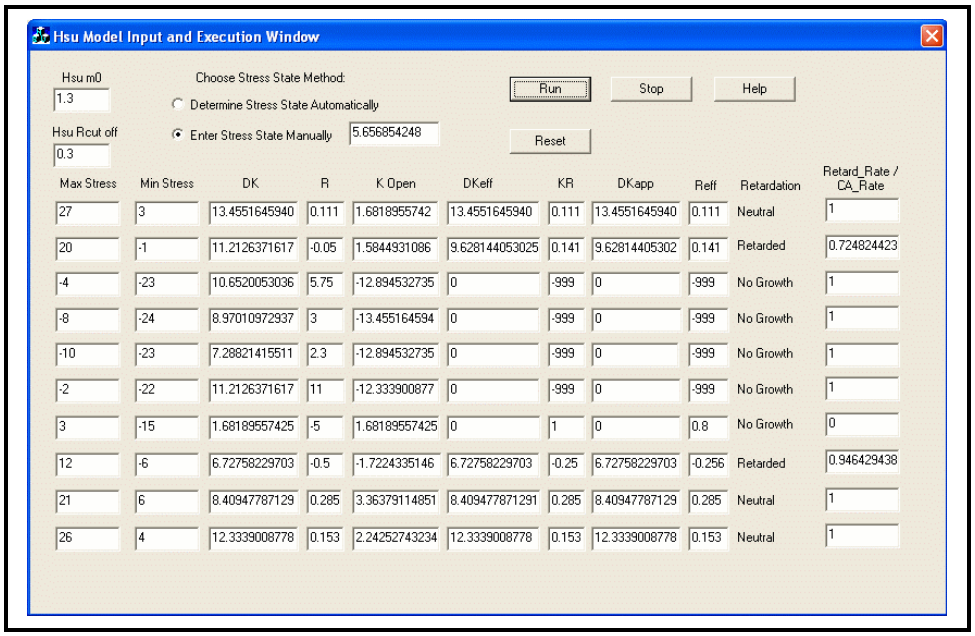
Max Stress	Min Stress	DK	R	K Open	DKeff	KR	DKapp	Reff	Retardation	Retard_Rate / CA_Rate
35	-5	19.6221150330	-0.142	-2.8031592904	19.6221150330	-0.14	19.6221150330	-0.142	Neutral	1
25	20	2.80315929043	0.8	11.752683447	2.263113004391	0.838	2.26311300439	0.8	Retarded	0.504181703
26.3	18	4.65324442211	0.684	10.7421773689	4.002440498693	0.728	4.00244049869	0.728	Retarded	0.785797115
27.5	16	6.44726636799	0.581	9.61030860656	5.80706749079	0.623	5.80706749079	0.623	Retarded	0.845921408
28.8	14	8.29735149967	0.486	8.37999101816	7.76620649471	0.519	7.76620649471	0.519	Retarded	0.899562023
30	12	10.0913734455	0.4	7.12064054897	9.69831519360	0.423	9.69831519360	0.423	Retarded	0.938412158
31.3	10	11.9414585772	0.319	5.83760043005	11.71017672804	0.332	11.7101767280	0.332	Retarded	0.969191767
32.5	8	13.7354805231	0.246	4.59350834084	13.6270270469	0.252	13.6270270469	0.252	Retarded	0.987396575
33.8	6	15.5855656547	0.177	3.38238858433	15.56696821897	0.178	15.5669682189	0.178	Retarded	0.998091487
35	4	17.3795876006	0.114	2.24252743234	17.3795876006	0.114	17.3795876006	0.114	Neutral	1

Example 3 Screen Capture – m0 = 1.3



Max Stress	Min Stress	DK	R	K Open	DKeff	KR	DKapp	Reff	Retardation	Retard_Rate / CA_Rate
35	-5	19.6221150330	-0.142	-2.8031592904	19.6221150330	-0.14	19.6221150330	-0.142	Neutral	1
25	20	2.80315929043	0.8	11.752683447	2.263113004391	0.838	2.26311300439	0.8	Retarded	0.504181703
26.3	18	4.65324442211	0.684	10.7421773689	4.002440498693	0.728	4.00244049869	0.728	Retarded	0.785797115
27.5	16	6.44726636799	0.581	9.61030860656	5.80706749079	0.623	5.80706749079	0.623	Retarded	0.845921408
28.8	-20	16.1461975128	-0.69	-10.664217539	16.1461975128	-0.66	16.1461975128	-0.3	Neutral	1
30	12	10.0913734455	0.4	6.77781821993	10.0411375226	0.402	10.0411375226	0.402	Retarded	0.992046934
31.3	10	11.9414585772	0.319	5.60846288668	11.93931427140	0.319	11.9393142714	0.319	Retarded	0.999712706
32.5	8	13.7354805231	0.246	4.48505486468	13.7354805231	0.246	13.7354805231	0.246	Neutral	1
33.8	6	15.5855656547	0.177	3.36379114851	15.58556565479	0.177	15.5855656547	0.177	Neutral	1
35	4	17.3795876006	0.114	2.24252743234	17.3795876006	0.114	17.3795876006	0.114	Neutral	1

Example 4 Screen Capture – m0 = 1.3



Max Stress	Min Stress	DK	R	K Open	DKeff	KR	DKapp	Reff	Retardation	Retard_Rate / CA_Rate
27	3	13.4551645940	0.111	1.6818955742	13.4551645940	0.111	13.4551645940	0.111	Neutral	1
20	-1	11.2126371617	-0.05	1.5844931086	9.628144053025	0.141	9.62814405302	0.141	Retarded	0.724824423
-4	-23	10.6520053036	5.75	-12.894532735	0	-999	0	-999	No Growth	1
-8	-24	8.97010972937	3	-13.455164594	0	-999	0	-999	No Growth	1
-10	-23	7.28821415511	2.3	-12.894532735	0	-999	0	-999	No Growth	1
-2	-22	11.2126371617	11	-12.333900877	0	-999	0	-999	No Growth	1
3	-15	1.68189557425	-5	1.68189557425	0	1	0	0.8	No Growth	0
12	-6	6.72758229703	-0.5	-1.7224335146	6.72758229703	-0.25	6.72758229703	-0.256	Retarded	0.946429438
21	6	8.40947787129	0.285	3.36379114851	8.409477871291	0.285	8.40947787129	0.285	Neutral	1
26	4	12.3339008778	0.153	2.24252743234	12.3339008778	0.153	12.3339008778	0.153	Neutral	1

Additional data is available in a folder app_D, that gives detail variable values for the above solutions but was thought to be to lengthy to provide here.

9 APPENDIX E – Crack Growth Methodology

This section briefly describes the crack growth methodology used on this project. The computer program AFGROW was used for all analysis. AFGROW is a PC based crack growth analysis tool distributed free by AFRL/VASM at the http://fibec.flight.wpafb.af.mil/fibec/afgrow.html web site. Mr. Jim Harter is responsible. AFGROW also conducts fatigue or crack initiation. Fatigue is based on local notch strain closed hysteresis methods far superior to simple S-N based methods. Complete material databases are available, either the Harter material or NASGRO which then utilizes a Forman based integrator.

A brief history of AFGROW. ASDGRO was developed in the early to mid 1980's. This became MODGRO by incorporating a different method for calculating stress intensity. 1987 saw MODGRO Version 1.X emerge by adding tabular crack growth rate database as well as Newman and Raju's part-through solutions, and other standard closed form solutions. 1989 MODGRO Version 2.X translated the coding language used and added plasticity based closure model, and changed beat calculations to user defined crack increments. 1994 AFGROW Version 3.X renamed MODGRO Version 2.X.

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